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<강의자료 Teachers' Corner>

Applications of the Cauchy Distribution

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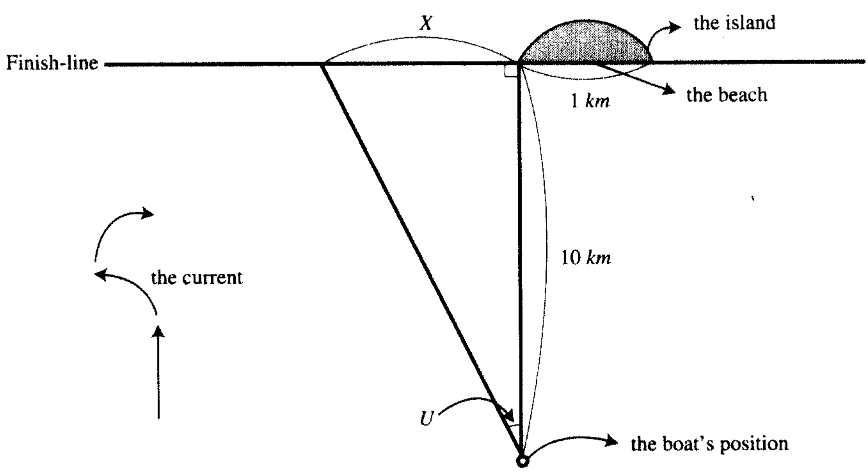
1. A ship sails northwards to an island. The ship hits a stray mine and sinks immediately. All of the crew except one drown. The surviving sailor barely succeeds in boarding a lifeboat. As he is severely wounded, he leaves his fate to the mercy of the currents. The physical configuration of the situation is as follows :



The current rapidly moves northwards but incessantly changes its direction. So, we have no idea at what point the boat will cross the Finish-line. Will the boat reach the beach?

U in the above figure is a continuous uniform random variable, and $f(u) = \frac{1}{\pi}$, $-\frac{\pi}{2} < u < \frac{\pi}{2}$. And then, X in the above figure is a Cauchy random variable, and $f(x) =$

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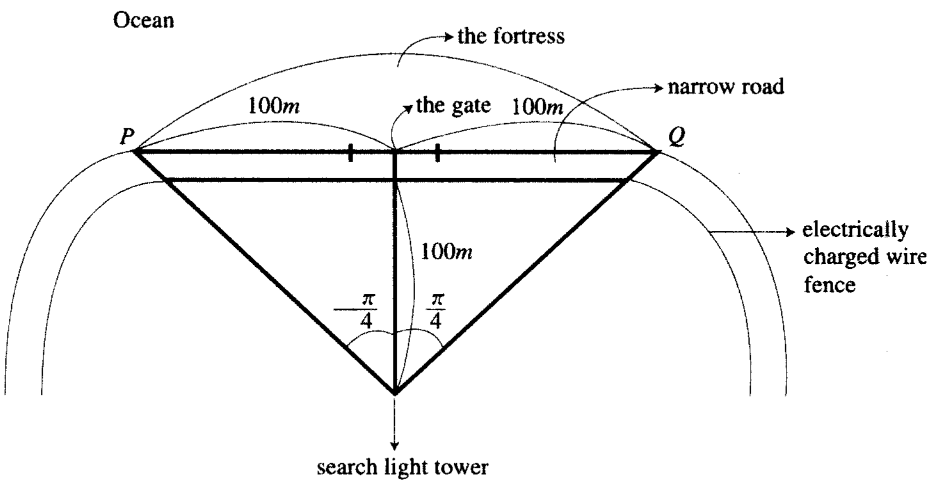
$\frac{1}{\pi}\frac{1}{1+x^2}, \ -\infty < x < \infty.$[1]

The probability that the boat will reach the beach is $\int_0^{0.1}\frac{1}{\pi}\frac{1}{1+x^2}dx = \frac{1}{\pi}Tan^{-1}0.1.$

2. We present here a stochastic model for the escape of an agent after his infiltration into an enemy' s fortress.[2] Agent K of Country A penetrates into a fortress of enemy Country B. On accomplishing his mission, he takes an escape route. The physical configuration of the situation is as follows.



The search light rotates from Q to P, and then from P to Q with uniform speed, 120 seconds per round. A crew of A in a boat waits for a signal from K near Q on the ocean. The crew can reach Q in 60seconds on receiving the signal but can wait up to 15 seconds at Q. K is at P. He must signal immediately and pass through $\overline{PQ}$. His speed is $200m/30$seconds $= 100m/15$seconds $= 6.67m/$second, normal running speed for ab physical configuration. What is the probability of K' s successful escape?

Let the location of a spot on the wall hit by the search light be X. Then, X is a Cauchy random variable, and

1) Freeman, Harold, *Introduction to Statistical Inference*, Addison-Wesley, 1963, pp. 134 ~ 136.

2) The model is rather unrealistic. We enjoy James Bond novels, though they are unrealistic.

$$f(x) = 2\frac{1}{\pi}\frac{1}{1+x^2}, \qquad -1 < x < 1 \text{ (The unit of } x \text{ is } 100m)$$

When K is at P, the search light may approach or may depart. Let us suppose that it approaches. Then, if $-1 < x < -0.75$, K can wait 15seconds until the search light hits P, and then he can jog closely behind the search light as far as Q and join the crew. The probability is $\frac{1}{2} \times \frac{0.25}{2} = \frac{1}{16}$. When K is at P, the search light may depart. If $-1 < x < 0$, he can jog or run closely behind the search light as far as Q and join the crew. The probability is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

$$\frac{1}{16} + \frac{1}{4} = \frac{5}{16}$$