연세경제연구 제 Ⅶ 권, 제 1 호
Yonsei Economic Studies, Vol. Ⅶ, No. 1 (March 2000)

Fairness and Satisficing in Ultimatum Bargaining Game Experiments

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We re-examine subjects' behavior in the ultimatum bargaining experiments studied by Roth *et al.* [24]. In our model, un-modeled factors including rationality determine players' initial aspirations, and a modified version of case-based learning process governs subsequent adaptations.

The calibration results of this paper show that "satisficing" can explain the actual subjects' behavior surprisingly well. More precisely, it is shown that 77.7 to 96.7% of the observed behavior is consistent with our model prediction. We also argue that the closer are the initial aspiration levels to the perfect equilibrium payoff, the lower is the mean or modal offer in the actual experiments.

Ⅰ. Introduction

Experimental studies make it clear that models of game behavior demanding perfect rationality of the players seem to perform poorly with even very simple games. Consider the situation in which rational players play a finite-horizon extensive form game of perfect information. Traditional theory can generically propose a sharp prediction, called the subgame perfect equilibrium. The trouble is that experimental

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The author thanks Joon-Mo Yang and Hong-Taek Lim for their research assistance on computer programming. The paper benefited from discussions with Tilman Borgers, Kalyan Chatterjee, David Easley, Peter Klibanoff, Hamid Sabourian, and seminar participants at Churchill College Cambridge and the Seventh World Congress of the Econometric Society, Tokyo. Financial support from the Korea Research Foundation(1998-003-C00176) is gratefully acknowledged. The usual caveats apply.

studies have reported drastically different behavior in some well-known games, such as ultimatum bargaining games, and the centipede game as studied by McKelvey and Palfrey [19]. The class of games we study is the one-stage ultimatum bargaining game.[1)]

In this game, player 1 proposes to divide 10 US dollars. Player 2 then either accepts or rejects this offer. If accepted, the money is divided as proposed; if rejected, each player receives zero. The perfect equilibria of this game prescribe that the proposer offers almost nothing to the responder and that the outstanding proposal is accepted. Contrary to this extreme prediction of perfect equilibrium, experimental studies reported that the outcomes tend toward an approximately fair division (around 4 out of 10).

We cast doubt on perfect rationality of players and, in particular, the backward induction argument. This paper focuses on the ultimatum bargaining game and investigates how well "satisficing" players do in place of rational players. The model can be described as follows. Players form their own initial aspiration levels right after being informed about the game but before playing it. This is meant to capture that, without explicitly modeling how, each subject's initial aspiration level is determined by factors such as the informational condition, the first mover advantage or disadvantage, the subject's model of opponents' play, his experience of having faced similar situations before and his knowledge about game theory. Once initial aspiration levels are formed, players adapt in subsequent periods to their experience. The dynamic model of this paper is heavily based on Gilboa and Schmeidler's [11], [12] case-based decision theory (CBDT henceforth).

In our approach, a player chooses an action to maximize the objective function that is the sum of the differences between experienced payoffs and the current aspiration level. The objective function, which Gilboa and Schmeidler derived from a set of axioms, has an element of bounded rationality and habit persistence. The aspiration level evolves over time. Using Binmore's [1] terms, in our model the 'eductive' process determines the initial aspiration level, and the 'evolutive' process governs the subsequent learning process. Traditional theory, to verify whether a particular outcome is indeed an equilibrium, often presupposes an introspection process that seems beyond the cognitive limit of human subjects. On the other hand, recent developments in learning and evolution, showing how an equilibrium gets to be played, often assume agents who can

1) Roth [22] provided a comprehensive survey of ultimatum bargaining games. Also refer to Guth [13] for some resolutions.

do nothing but make occasional adaptations to previous history.[2] Our model may be an approach to incorporate these two features into one framework.[3]

This paper shows that satisficing can explain the subjects' observed behavior surprisingly well. We define the minimally inconsistent path(s) to be the one that, among the paths predicted by the model, is least different from the actual play. Here, 'least' is measured in terms of the number of periods. The calibration results show that the proportion of periods at which the observed behavior is consistent with our model ranges from 77.7 to 96.7% depending on the payoff size and the player's role. The result does not change significantly regardless of whether the stake is small or large. We also define the minimally imperfect initial aspiration (MIPA, in short) to be one that, among the initial aspiration levels consistent with any minimally inconsistent paths, is nearest to the perfect equilibrium payoff. If there is some minimally inconsistent aspiration equal to the perfect equilibrium payoff, we can not reject the hypothesis that the subject made his initial choice on the basis of rational expectations. We show that more than half of MIPAs are concentrated on the perfect equilibrium payoff and the remaining MIPAs lie around fair divisions.

The intuition behind the dynamics follows. Responders accept larger offers more often than smaller offers. Thus, it is likely that generous offers are accepted while greedy offers are rejected. These unintentional rewards and punishments make proposers' aspiration levels converge in the middle. We also argue that the closer are the MIPAs to the perfect equilibrium payoff, the lower is the mean or modal offer in actual experiments'. This implies that MIPAs exert a persistent influence on the distribution of offers in the intermediate run.

It has been recognized that satisficing can capture many salient features of experimental results. Examples include repeated prisoner's dilemma (Selten and Stoecker [25]), bargaining problems (Mitzkewitz and Nagel [20]), a repeated zero-sum game (Mookherjee and Sopher [21]), and so forth. The most widely applied framework of satisficing behavior is the stochastic learning mechanism due to Bush and Mosteller [4] and Harley [14]. In this approach, each player is assumed to adapt his strategy from

2) Representative examples include evolutionary models, such as Foster and Young [7] and Kandori, Mailath and Rob [15].

3) Kim [16], [17] applied the similar framework to other classes of games, namely the best-shot public goods provision games and the centipede games. Kim [18] analyzed convergence of the CBDT learning process in a particular class of normal form game, namely game of common-interest.

one round to the next by increasing the probability assigned to the chosen strategy if it resulted in a payoff exceeding the aspiration level, and decreasing it otherwise. Fixing the aspiration level to be zero, this approach presupposes positive payoff or stimulus. Roth and Erev [23] showed by simulations that the intermediate term predictions of modified Harley's models track well the observed behavior in three games: a sequential best-shot game, a market game, and an ultimatum bargaining game. Roth and Erev suggest that subjects used essentially the same learning rules regardless of game structure and location and that 'the observed differences reflect different patterns of adaptations'.

We apply a modified version of CBDT on the following ground. Above all, we believe that the environment CBDT presumes is similar to the environment subjects often face in actual laboratory experiments. Gilboa and Schmeidler remark that CBDT is particularly appropriate in analyzing situations involving *ignorance*, which refers to the situation in which neither the states of the world nor probabilities on them are naturally defined.[4] In many experimental studies, subjects are asked to make certain decisions, while being placed in unfamiliar situations especially in initial stages. Hence, this paper attempts to test how well CBDT works in situations that we believe are the most pertinent. Second, the deterministic nature of our model enables us to check whether and to what extent actually observed paths of subjects' choices are consistent with the theory. This is in sharp contrast to Roth and Erev which relied heavily on simulations, although our approach can convey the same insight and intuition. Last, the informational requirement of CBDT is less demanding relative to other learning models. A case-based player needs to know nothing about others' choices or characteristics, nor the structure of the game at hand. All he must remember are the number of times that each strategy was chosen, if ever tried, and its average performance.

The structure of the paper is organized as follows. The next two sections build up the model and explain central notions for subsequent analysis. Section 4 reports the calibration results and their discussion. The final section concludes.

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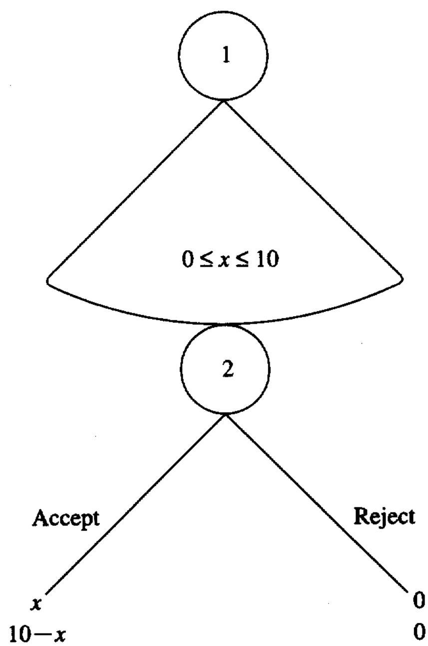
4) Gilboa and Schmeidler added the notion of ignorance to the Knightian distinction between risk and uncertainty. To demonstrate how backward induction breaks down in some repeated strategic form games, Dow and Werlang [6] applied concept of non-additive probabilities, which is a model of uncertainty.

Ⅱ. The Model

1. The Stage Game

In the one-round ultimatum bargaining game, player 1 proposes to divide 10 US dollars of the form $(x, 10-x)$. Proposals take discrete values using the unit of a quarter dollar. Player 2 then either accepts or rejects this offer. If accepted, the money is divided as proposed; if rejected, each player receives zero. The set of player 1's pure strategies is $S^1 = \{x = 0.25n \mid n = 0,1,...,40\}$ and the set of player 2's pure strategies is the set of "Accept" and "Reject" contingent on all possible offers. Let S denote the set of pure strategy profiles. The stage game is an extensive form game of perfect information, which is depicted in 〈Figure 1〉. There are two subgame perfect equilibria, depending on whether responders accept the null offer. These equilibria prescribe that player 1 demands almost all money, and that player 2 accepts. Hence, the set of subgame perfect

〈Figure 1〉 The One-round Ultimatum Bargaining Game



equilibrium payoffs is $\Pi^1_{spe} = \{9.75, 10\}$, and $\Pi^2_{spe} = \{0, 0.25\}$.

Experimental studies did not support this prediction. What happened is that player 1 predominantly offered player 2 much larger shares (from 3.58 to 4.53). More interestingly, player 2 turned down considerably low offers, albeit better than nothing. This result inspired a debate about the predictive role of game theory and, more specifically, about how fairness influences decision behavior.

According to game theory tradition, a strategy is defined as a complete contingent plan. In belief-based learning approaches, it is important to specify off-the equilibrium beliefs and plays. However, in our model of bounded rationality, it is more sensible to assume that players do not distinguish between two pure strategies that lead to the same terminal node. For this reason, a "choice" in this paper refers to a pure strategy which remains after realization equivalent strategies are merged in an equivalent class.

2. The Dynamic Process

Imagine that a pair of randomly matched players repeatedly play the stage game at dates $t = 1,2,...,T$, where T may be finite or infinite. Player i updates his strategy at period t, according to player i's payoff realized up to $(t-1)$, relative to his present aspiration level. Let H^i_t denote player i's aspiration level at the beginning of the period t. The set of T-histories experienced by player i is a subset of $\Omega' \equiv (I \times S \times I)^3$, where I is the interval [0,10] and $\mathfrak{I}$ is the set of natural numbers up to T. A T-history $\omega^i = ((H^i_t, s^i_t, \pi^i_t))_{1 \le t \le T} \in \Omega'$ will be interpreted as follows : for all $t \ge 1$, the aspiration level is H^i_t at the beginning of the period, a strategy s^i_t is chosen, and the strategy profile $s_t = (s^1_t, s^2_t)$ yields a payoff $\pi_t = (\pi^1(s_t), \pi^2(s_t))$. The projection functions $s^i_t : \Omega' \rightarrow S^i$ and $H^i_t : \Omega' \rightarrow I$ have the obvious meaning.

Player 1 at t demands a maximizer x of the functional form.[5]

$$U^1(x,t) = \begin{cases} A^1(x,t)x - K^1(x,t)H^1_t, & \text{if } K^1(x,t) > 0 \\ 0, & \text{if } K^1(x,t) = 0 \end{cases} \tag{1}$$

5) To better explain experimental data on the two-stage ultimatum bargaining game, Bolton [3] postulated that subjects behave as if they are negotiating over both absolute and relative money. Our work shares the spirit of Bolton, in the sense that the underlying objective function is modified.

where

$K^1(x,t)$ = the number of times that player 1 has proposed the amount x before t,
$A^1(x,t)$ = the number of times that the demand x was accepted before t, and
$R^1(x,t)$ = the number of times that the demand x was rejected before t.

Obviously, $A^1(x,t) + R^1(x,t) = K^1(x,t)$ holds for all x and t.

Player 1's aspiration is a weighted average of its previous value and the best average performance over all offers. That is,

$$H_t^1 = (1 - \alpha^1)H_{t-1}^1 + \alpha^1 \max_{x \in S^1} \left\{ x \frac{A^1(x,t)}{K^1(x,t)} \right\} \tag{2}$$

where $\alpha^1 \in [0,1]$ and $H_1^1 \in [0,10]$ are exogenously given.

Now consider the responder's behavior rule. Let $K^2(x,t)$ denote the number of times that the responder has been offered $(10-x)$ dollars before period t. Also let $A^2(x,t)$ and $R^2(x,t)$, respectively, denote the number of times that the responder has accepted and rejected the offer $(10-x)$ before t. Player 2's objective at period t is to accept (resp. reject) the offer $(10-x)$ if

$$A^2(x,t)[(10 - x) - H_t^2] \geq (\text{resp.} \leq) R^2(x,t)[-H_t^2] \tag{3}$$

subject to the aspiration revision rule

$$H_t^2 = (1 - \alpha^2) H_{t-1}^2 + \alpha^2 (10 - x_{t-1}) \tag{4}$$

The dynamic is deterministic, given parameter values. Markovian property does not apply, if the 'state' is defined as the current pure strategy profile and aspiration levels. In other words, paths matter.

3. Heuristic Examples

Several heuristic examples follow to instruct the reader how the dynamic process works. First, consider the situation in which a proposer has the initial aspiration level of 7 and chooses a demand of 8 (equivalently, an offer of 2) at the first period. If this offer was accepted, any offer other than 2 at period two will be inconsistent with the theory since $U^1(x=8,t=2)=1\times 8-1\times 7=1>0$. If the first period offer was rejected, choosing $x=8$ again at $t=2$ will be clearly inconsistent since $U^1(x=8,t=2)=-7<0$. Since we allow the players to apply any deterministic tie-breaking rule, any offer other than $x=8$ would not be inconsistent. Notice that, since we do not impose the condition that a proposer's demand be no smaller than his current aspiration level, aspiration affects one's choice only through its effect on the objective function.

The second example may help the reader understand the difference between maximizing the modified case-based function and maximizing the average payoff. Consider the situation in which a proposer, with the current aspiration of 4, faces a strategy choice problem at $t=9$. Assume that he demanded $x=5$ six times, of which five were accepted, and $x=8.5$ twice, of which only one responder accepted. A case-based player will choose $x=5$ at $t=9$, since $U^1(x=5,t=9)=5\times 5-6\times 4=1$ is larger than $U^1(x=8.5,t=9)=1\times 8.5-2\times 4=0.5$ and $U^1(x,t=9)=0$ for all $x\neq 5$, 8.5. However, an average-payoff maximizing player never chooses $x=5$ at $t=9$, since the average payoff from $x=5(5\times 5/6\approx 4.17)$ is smaller than the average payoff from $x=8.5(8.5\times 1/2=4.25)$.

One might be tempted to think that the case-based agent chooses what he did before if the payoff is above the aspiration level and otherwise choose something else at random. The last example demonstrates that this 'win-stay-lose-shift' type agent may choose different action from the case-based player. Suppose that a proposer endowed with $H_t^1=5$ and $\alpha^1=0.25$ chose $x=6$(respectively, $x=8$) one hundred times(resp. thirty two) out of which the offer was accepted 90 (resp. 24) times. At period t, the case-based player chooses $x=6$ since $U^1(x=6,t)=100\times(5.4-5)=40$ is larger than $U^1(x=8,t)=32\times(6-5)=32$. If this offer is accepted, the aspiration level at $t+1$ will be 5.25. Now it is easy to check that the case-based player will switch to $x=8$ while the win-stay-lose-shift type player stays at $x=6$.

Ⅲ. The Analysis

In this section we develop the theoretical framework. Experimental data at individual level provide the sequence of realized outcomes. Since there is a set of strategies which is consistent with a given outcome, corresponding to a sequence of realized outcomes there is the set of the sequences of strategies that is consistent with the actual data. We call it as the set of actual paths. Let $\hat{S}^i$ and $\hat{s}^i = (\hat{s}^i_t)_{1 \le t \le T}$ be the set of actual paths for player i and its typical element, respectively.

Now we want to characterize the paths of strategies which are consistent with our model, namely the sequences of strategies that a player who behaved as if he was a modified CBDT decision-maker would have chosen. To this end, fix the initial actual choice $\hat{s}^i_1$, are compatible with U^i-maximization and aspiration revision rule. Formally,

$$\Omega = \{\omega \in \Omega' \mid s^i_1(\omega) = \hat{s}^i_1,\ s^i_t(\omega) \in \arg\max_{s^i \in S^i} U^i(s^i, t),\ \forall t \ge 2, \text{ and}$$
$$\exists\, a^i \in [(0,1) \text{ such that } (H^i_t)_{1 \le t \le T} \text{ evolves by Eq.(4)}\} \qquad (5)$$

Focus on a particular subject playing player i's role, so we suppress the superscript i whenever there is no confusion.

We say that a play of player i is *consistent with the model* if $\hat{s}^i \in \{s^i(\omega) \mid \omega \in \Omega\}$, where $s^i(\omega)$ is the projection function. Suppose that the observed play of a particular player is not perfectly consistent with the model. We postulate that his intended play is the path minimizing the number of periods at which he behaved inconsistently. More formally,

Definition Define the set $\Omega^*(\hat{s}^i)$ to be the set of *minimally inconsistent* (MIC, in short) histories that is associated with the actual path $\hat{s}^i$, where

$$\Omega^*(\hat{s}^i) \equiv \{\omega \in \Omega \mid \#\{t \mid \hat{s}^i_t \ne s^i_t(\omega)\} \le$$
$$\#\{t \mid \hat{s}^i_t \ne s^i_t(\omega')\},\ \forall \omega' \in \Omega\} \qquad (6)$$

Let $\Omega^* \equiv \bigcup_{\hat{s}\in\hat{S}} \Omega^*(\hat{s})$ be the set of all MIC histories. Define $\sigma^i[\Omega^*(\hat{s}^i)]$ to be the set of MIC paths that is associated with the actual path $\hat{s}^i$, where s_t^i, is the projection function from histories to strategy choices. Clearly, $\Sigma^i := \bigcup_{\hat{s}\in\hat{S}} \sigma^i[\Omega^*(\hat{s})]$ is the set of all MIC paths.

Corresponding to each MIC path, there exist pairs of the initial aspiration level and aspiration revision coefficient, (H_1, α), that are consistent with the given path. Let a minimally inconsistent aspiration level(MICA, in short) be the initial aspiration level that is compatible with some MIC path for some revision coefficient, α.

The following story is what we keep in mind about how people behave. Subjects form their initial aspiration levels after being informed of the game but before ever playing it. We do not attempt to model how players form their initial aspirations. We believe that un-modeled factors, such as rationality of players, the first mover (dis)advantage, and whether the player has faced a similar situation before affect the level of initial aspiration. For this reason, if there is some MICA equal to the perfect equilibrium payoff, we may not be able to reject the hypothesis that the subject made his initial choice on the basis of rational expectations. Hence, we are interested in the MICA that is nearest to the perfect equilibrium payoff. We call it to be the minimally imperfect initial aspiration level(MIPA, in short) and the MIC paths associated with MIPA to be the minimally imperfect (MIP, in short) paths.

We want to formally define these notions. Let $\Lambda^i = \{H^i(\omega)\ [0,10] \mid \omega\ \Omega\}$ be the set of player i's initial aspiration levels that are compatible with some MIC path. Now the notion of minimal imperfection is defined as follows.

Definition Define the *minimally imperfect initial aspiration level* (MIPA, in short) to be :

$$\lambda^i = \arg\inf_{\substack{\pi^i\in\Pi^i_{spe} \\ H^i\in\Lambda^i}} |\pi^i - H^i| \tag{7}$$

where Π^i_{spe} is the set of perfect equilibrium payoffs to player i and the argument is taken over H^i's. Also define a *minimally imperfect path* to be an element of the set $\{s^i \in \Sigma^i \mid H^i(\omega) = \lambda^i\}$, where $H^i(\cdot)$ is the projection function from histories to initial aspiration levels.

Ⅳ. The Main Results

1. Calibration Results

We calibrate the above learning model to the ultimatum bargaining experimental data of Roth *et al.* [24].[6] The variation in experimental treatment is the country in which the experiment was conducted : the US, Japan, Israel and Yugoslavia. In addition, in the US, an experiment was conducted with stakes three times those indicated above. This treatment of "the US high payoff" is also included in the US data.

〈Table 1〉 summarizes the calibration results. Each cell indicates the number of

〈Table 1〉 Calibration Results

(a) Proposers

	Israel	Japan	Yugoslavia	US
none	9	7	4	4
1	7	10	6	5
2	4	5	11	8
3	4	5	1	8
4 or more	6	2	8	4
%	80.3	84.8	77.7	77.9

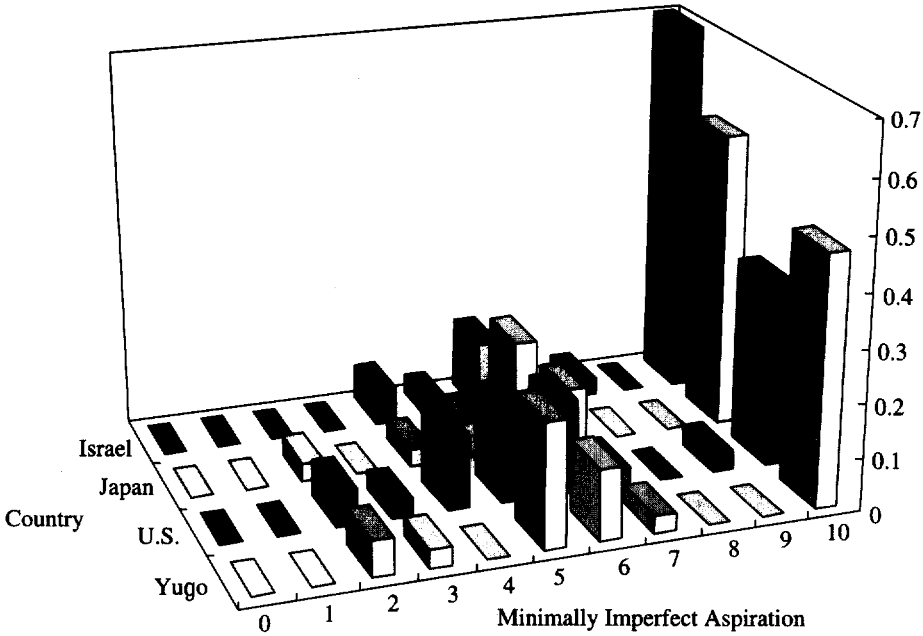
(b) Responders

	Israel	Japan	Yugoslavia	US
none	21	19	22	17
1	8	6	6	9
2	1	3	2	2
3 or more		1		1
%	96.7	94.8	96.7	94.5

Note : Each cell indicates the number of subjects whose choices were consistent with some MIC paths except for the number of periods shown in the first column.

6) Roth *et al.* [24] reported only the aggregate statistics. Our analysis are based on the individual level panel data, which was personally provided by Al Roth.

〈Figure 2〉 Minimally Imperfect Aspiration Levels



Note : Those MIPAs that(is in the inteval between ($z-0.5$) and ($z+0.5$) are grouped and Cabeled as 'z', where z = 0, 1, 2, ⋯, 10.

subjects whose choices were consistent with a MIC path except for the number of periods shown in the corresponding row. For instance, actual plays of 9 Israeli subjects out of 30 were perfectly consistent with the model. The last row indicates the aggregate proportions of periods at which plays are consistent with the model. 〈Table 1〉 (a) shows that 77.7%(Yugoslavia) to 84.8%(Japan) of the proposers' choices were consistent with the model. 〈Table 1〉 (b) shows that our hypothesis fares better with the responders' behavior. In particular, 79 responders out of the total 118 chose choices exactly as our hypothesis would predict.

〈Figure 2〉 reports the MIPAs of the proposers. For a clearer demonstration, the label "z" bunches all those MIPAs in the interval (z − 0.5, z + 0.5), where z = 0,1,...,10. The distribution is clearly bi-modal. Most frequencies are concentrated either around the fair division, *i.e.* x=4 to 6 dollars, or on the subgame perfect equilibrium payoff, *i.e.* x=10 dollars. Remind that the distribution of MIPAs is the one as tightened as possible to the degenerate distribution on the equilibrium payoff x=10. Hence, the unobservable distribution of actual initial aspirations would be at least as widely dispersed as the distribution of MIPAs.

Interestingly, there is a systematic cross-country relationship between the distribution of MIPAs and the average proposal of actual data. Precisely, the closer the distribution of proposers' MIPAs is to the degenerate distribution on the perfect equilibrium payoff, the lower is the average or modal proposal to responders especially in the last few rounds. To investigate this observation, we summarize the salient features of actual experimental data in Roth *et al.*'s Figure 3 and 4 (pp. 1083 ~ 1084). The distribution of proposals aggregated over all rounds shows that, in Israel and Japan the modal proposal is 4, while in Yugoslavia and the US the modal proposal is 5. Moreover, the last round distribution shows that the mean proposals rank upwards Israel, Japan, the US and Yugoslavia. ⟨Figure 2⟩ makes it clear that this ranking roughly coincides with how close the distribution of MIPAs is to the degenerate distribution on $x=10$.

This rule-of-thumb comparison can be confirmed by calculating the degrees of minimal imperfection as follows :

$$\sum_{\lambda} \inf_{\pi \in \Pi_{spe}} p(\lambda) \mid \lambda - \pi \mid$$

where $p(\lambda)$ denotes the fraction of the subjects whose MIPA is equal to λ. The degree of minimal imperfection is 1.27 in Israel, 1.97 in Japan, 2.50 in Yugoslavia, and 3.31 in the US out of the stake 10. This ranking roughly coincides with the mean or modal proposal of actual proposers. Intuitively, the more ambitious are proposers in the beginning of the experiment, the larger does the average demand turn out to be at least in the intermediate term. These findings suggest that the number of proposers who initially aspire the equilibrium payoff ($x=10$ dollars) exerts a persistent influence on the mean or modal offer in the intermediate run.

The emergence of fairness can be intuitively explained as follows. A responder accepts any offer larger than his own aspiration level possibly except once. This "unintentional reward" makes generous proposers revise their aspirations upwards. If the proposed offer falls short of a responder's current aspiration level, the responder may accept (respectively, reject) the offer whenever the right-hand side of Eq.(3) exceeds (respectively, falls short of) the left-hand side. We claim that the responder tends to reject a lower offer more often than a higher offer. Letting Eq.(3) hold with equality and rearranging the resulting equation give rise to :

$$\frac{R^2(x,t)}{K^2(x,t)} \equiv r(x,t) \approx \frac{H_t^2-(10-x)}{2H_t^2-(10-x)}$$

where $r(x,t)$ is the rejection rate of the offer $(10-x)$ at period t. It is easy to check that rejection rate is higher if the amount of offer, $(10-x)$, is lower. In particular, the rejection rate is highest if the system gets close to the perfect equilibrium. This "unintentional punishment" makes greedy proposers revise their aspirations downwards. As time passes, offers converge somewhere in the middle. To recapitulate, a wide dispersion of initial aspirations discourages extreme greed and encourages extreme generosity and that, moreover, if adaptation occurs quickly enough, convergence to moderate behavior becomes possible.

Gale, Binmore and Samuelson [10] apply the stochastic replicator dynamic framework to reach a similar conclusion, and offer an aspiration-based learning story to justify the replicator dynamics. Consider a strictly positive offer of, say, one dollar, which is weakly dominated for proposers. There will be some evolutionary pressure against this strategy because, in a noisy population, the set of proposers who make such low offers is continually renewed. However, if this fraction of the proposing population becomes sufficiently small, the pressure will be negligible compared with the drift engendered by the noise in the responding population. This argument holds under the crucial assumption that responders tend to be noisier when proposers are making low offers. We derive this behavior, rather than assume it.[7)]

2. Comparison with the Average-payoff Maximization Hypothesis

There are a large number of other simple-minded adaptive rules, including the average-payoff maximization and the stimulus response. In this subsection, we investigate how the average-payoff maximizing player is doing and show that our model fares better than the average performance maximization hypothesis.

Focus on the proposers' side of the ultimatum bargaining game. According to this

7) Moreover, Gale *et al.*'s approach is powerless in the class of games in which a unique Nash equilibrium exists, such as the centipede game as studied by McKelvey and Palfrey [19].

〈Table 2〉 Comparison between the Gitting Performances of two Hypotheses (Aggregated over All 118 Subjects of the Proposer's Role)

	The Modified CBDT Hypothesis	The average-payoff maximization hypothesis
None	24	33
1	28	16
2	28	17
3	18	14
4 or more	20	38
%	80.2	73.8

alternative hypothesis, a choice is consistent if the player chooses a strategy with the best average performance. Also a choice would not be inconsistent if the player proposes an offer that has never been chosen and as long as the demanded share is no less than the highest average payoff at the moment.

〈Table 2〉 compares the fitting performances of two hypotheses. Each cell indicates the number of proposers whose offers were consistent with the corresponding hypothesis except the number of periods shown on the corresponding row. There are 33 subjects whose plays were perfectly consistent with the average-payoff maximization, which exceeds 24 of the modified CBDT. Except that, our model generally fares better than the alternative hypothesis. At the aggregate level, the proportion of periods in which plays were consistent with our model is 80.2%, which is significantly larger than 73.8% of the counterpart.

V. Final Remarks

Our model is deterministic. Many seminar participants and referees suggest that the right direction is to allow tie-breaking randomization and to estimate the maximum likelihood parameters. Although I agree this MLE method makes sense, I intentionally did not follow their advice. The reason is that, from the traditional theorist's viewpoint, the decision-maker in our model may be hard to accept. Decision-makers maximize an objective function that incorporates aspiration and cumulative payoffs, but not the average realized payoff. Aspiration evolves in an adaptive manner but not in a forward-looking way. If we allow players to randomize or tremble, the parsimony of the model might be extremely problematic.

Besides the aspiration update, we could have introduced another type of dynamics, namely expansion of the set of choices. Roughly speaking, if strategies that were available resulted in unsatisfactory outcomes, the decision-maker begins searching for new available strategies, which leads to an enlarged set of choices. If we modify the model in this way, it turns out that the set of actions available each period, S^i, can be treated as a choice variable of the outside analyzer who is free to choose S^i to make the data fit the theory. To put it polemically, if some player would always choose the same strictly dominated strategy this could be justified as being completely rational under the approach pursued here by simply assuming that the player fails to realize that other strategies are available. However, in order to make the model as parsimonious as possible, we do not formalize this idea.

One obvious shortcoming of this paper directly stems from lack of data. The existing experiments were conducted for nine to ten periods, on which our analysis of consistency is based. This is clearly not sufficient for a study of dynamic processes. The high performance of our model might be even due to the fact that it fits only ten-period time series data.

A number of questions are raised about works that are currently being undertaken, and problems for future research. There are other competing theories available. A model of bounded rationality is the Bush and Mosteller [4] type stochastic learning approach. There are also attempts to reconcile experimental data with more traditional game-theoretic predictions. Camerer and Weigelt [5] and McKelvey and Palfrey [19] use the home-made priors approach, but such a method seems too difficult to generalize to other games. Fudenberg and Levine [9] argue that, using the self-confirming equilibrium notion introduced by Fudenberg and Levine [8], the proportion of players that would need to have irrational payoffs to generate the observed path is small. They report that the average loss of a player is \$0.03 to \$0.64 in a game involving stakes between \$2 and \$30. From a methodological point of view, while Fudenberg and Levine's analysis are based on the aggregate probability distribution over outcomes, we consider the complete history of each individual player's choices. Telser [26] argues that experimental results of the ultimatum game are consistent with and can be explained by the law of demand.

It will be interesting to compare which approach fares best in applicable experimental data. Kim [16], [17] exhibits high performance of our model in the centipede game and the sequential best-shot game. Again, a critical limitation is that the existing experimental studies provide very short periods data. An agenda for future research is to conduct laboratory experiments for much longer periods and to test various theories by using new experimental data.

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