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A Review of Measuring Undernutrition with Variable Calorie Requirements

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The first objective of this paper is to suggest sensible ways of estimating undernutrition in the absence of the ideal solution. To that end, methologies are developed to deal with different levels of information available on the distribution of requirement.

The second objective of this paper is to develop a new class of undernutrition measures. A proper measure of undernutrition should take into account not only the proportion of people undernourished, but also the gap between the calorie requirement and intake of each individual.

Ⅰ. Introduction

A major objective of this paper is to suggest sensible ways of estimating undernutrition in the absence of the ideal solution. To that end, methodologies are developed to deal with different levels of information available on the distribution of requurement. Obviously, the less we know about requirements, the less we can tell about the state of nutrition. But it is our contention that we can still say something of value; the objective of our methodologies is to enable us to do so by making the most of the available information, and without committing either a type Ⅰ or a type Ⅱ error.

A second objective of the paper is to develop a new class of undernutrition measures.

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Most of the recent literature is centered around estimating the proportion of population that is undernourished, without taking note of the degree of severity of their undernourished undernutrition. A proper measure of undernutrition should take into account not only the proportion of people undernourished, but also the gap between the calorie requirement and intake of each individual. We will develop a class of such measures.

The remainder of the paper is organized in five sections. Section Ⅱ, the usual average norm approach is discussed. Then in Section Ⅲ the new class of severity-sensitive measures of undernutrition is developed. Next, we develop methodologies for comparing undernutrition across populations under different levels of information on requirements where both mean and variance of requirements are known in Section Ⅳ, and where only mean of requirements is known in Section Ⅴ. Finally, Section Ⅵ presents the major conclusions of the study.

Ⅱ. The Approach of the Average Calorie Norm

Suppose that the calorie intake x of an individual is a random variable with mean μ and the probability density function $f(x)$. If the calorie requirement of an indivisual is a given number R and his calorie intake is x, then the person is said to be suffering from undernutrition if $x < R$.

Since the energy requirement of an individual is not fixed, it can be assumed that requirement follows a probability distribution with density function $g(R)$ with mean $\overline{R}$.

Let $P(x)$ be the probability that a person with calorie intake x suffers from undernutrition. This probability must depend on $g(R)$ and is given by

$$P(x) = \int_x^b g(R)\, dR = 1 - G(x)$$

where $a \leq R \leq b$ and $G(x)$ is the probability distribution function of calorie requirement. An index of undernutrition is then given by

$$M = \int_0^\infty [1 - G(x)] f(x)\, dx \qquad (1)$$

which is interpreted as the probability that a randomly selected person in the population suffers from undernutrition. This index is referred to as the head-count of the measure of undernutrition.

An approximate procedure to estimate the extent of undernutrition is to calculate the proportion with energy intake less than $\overline{R}$, *i.e.*

$$F(\overline{R}) = \int_0^{\overline{R}} f(x)\,dx \tag{2}$$

where $F(x)$ is the probability distribution function of calorie intake. This approach will be referred to as the average calorie norm approach.

Sukhatme [31] and also Srinivasan [28] have questioned the average calorie norm approach on the grounds that it leads to considerable overestimation of the degree of undernutrition in the population. Their charge of overestimation is based on the grounds of both inter-individual variation and intra-individual variation. In this section we argue that the extent of overestimation is small.[1] In order to assess the extent of overestimation, let us integrate (1) by parts

$$M = \int_0^{\infty} F(x)\,g(x)\,dx \tag{3}$$

Then using Taylor's expansion gives

$$F(x) = F(\overline{R}) + (x - \overline{R})f(\overline{R}) + \frac{1}{2}(x - \overline{R})^2 f'(\overline{R}) \tag{4}$$

where $f'(x)$ is the first derivative of $f(x)$ and the terms of higher order of smallness have been omitted. Hence (4) is an approximate relationship. Combining (3) and (4) gives approximate relationship

$$M = F(\overline{R}) + \frac{1}{2}\sigma_R^2 f'(\overline{R}) \tag{5}$$

1) Note that this result is based on the assumption of independence between intake and requirement distributions. Once this assumption is relaxed, as in section Ⅳ, there can be either overestimation or underestimation, and its extent may be large or small, depending on the nature of correlation between intake and requirement.

where σ_R^2 is the variance of the requirement distribution.

Like any income distribution, we may assume that the distribution of calorie intake is a skewed distribution with a single mode. One characteristic of such distributions is that the mean is greater than the mode. If $\overline{R}$ (the average calorie requirement) is greater than the mode of the calorie intake distribution, which will generally be the case, $f'(\overline{R})$ (the slope of the calorie intake density function) will be negative. Thus, equation (5) implies that $F(\overline{R}) > M$; *i.e.*, the average calorie norm approach tends to overestimate the extent of undernutrition. The extent of such overestimation depends on the second term in the right-hand side of (5). It is expected that $f'(\overline{R})$ will be of smaller order of magnitude than σ_R^2; thus, we conjecture that the degree of overestimation is not large.

Ⅲ. A New Class of Undernutrition Measures

The aggregate measure of undernutrition M given in (1) is interpreted as the probability that a randomly selected individual in the population suffers from undernutrition. This measure provides an estimate of the proportion of population that is undernourished. Thus, it may be called a head-count measure of undernutrition(the term used in the measurement of poverty literature).[2] The measurement of the degree of undernutrition must take into account the gap between the calorie requirement and intake for each individual. If it does not, it can lead to perverse results.

Suppose the distribution of requirements is given by a vector $R = (1, 1.5, 10)$, which is fixed for a population. Suppose that there are three individuals in the population whose calorie intakes are given by the vector $x = (2, 2.5, 3)$. It can be easily verified that $M = \frac{1}{3}$; *i.e.*, the proportion of population suffering from undernutrition is 33.3 percent.

Let us now suppose that there is a threefold increase in the individuals' calorie intake. It can be verified that M is still equal to $\frac{1}{3}$ despite the fact that intensity of hunger (or undernutrition) is considerably reduced. In order to rectify such a defect, we have developed below a class of undernourished individuals as well as the extent of their

2) Sen calls the head-count ratio a very crude index of poverty because an unchanged number of people below the poverty line may go with a sharp rise in the extent of the short-fall of income from the poverty line.

deprivation.

Let $h(x, R)$ be the degree of undernutrition suffered by an individual with calorie intake x and requirement R. Since R follows a probability distribution, the expected undernutrition suffered by an individual with intake x is given by

$$E(U / x) = \int_0^\infty h(x, R)\, g(R)\, dR \tag{6}$$

Since the individual suffers from undernutrition only if $R > x$, we must have

$$H(x, R) = 0 \qquad \text{if} \qquad x \geq R$$

$$H(x, R) > 0 \qquad \text{if} \qquad x < R$$

Then (6) should be written as

$$E(U / x) = \int_x^\infty h(x, R)\, g(R)\, dR$$

In order to make this idea empirically operational, it is necessary to specify the function $h(x, R)$. One simple specification in terms of one parameter is given by

$$h(x, R) = \left[\frac{R - x}{\bar{R}}\right]^\alpha$$

$\bar{R}$ being the average calorie norm, which gives

$$E(U / x) = \int_x^\infty \left[\frac{R - x}{\bar{R}}\right]^\alpha g(R)\, dR$$

The average undernutrition suffered by the population is given by

$$K(\alpha) = \int_0^\infty \left[\int_x^\infty \left[\frac{R - x}{\bar{R}}\right]^\alpha g(R)\, dR \right] f(x)\, dx \tag{7}$$

where α is the parameter to be specified. If $\alpha = 0$, $K(\alpha)$ is equal to the measure M and when $\alpha = 1$, $K(\alpha)$ becomes

$$K = \frac{\overline{R}-\mu}{\overline{R}} + \frac{1}{\overline{R}}\int_0^\infty x\,G(x)f(x)\,dx - \int_0^\infty G_1(x)f(x)\,dx \tag{8}$$

where $G_1(x) = \frac{1}{\overline{R}}\int_0^x R\,g(R)\,dR$

It can be proved that the sum of the two integrals in the right-hand side of (8) is non-negative, which implies that $K > \frac{\overline{R}-\mu}{\overline{R}}$.

A common procedure to determine undernutrition at aggregate level is to compare the average per capita availability of energy with per capita energy needs. When requirement exceeds availability, the country or region is classified as inadequately nourished. Such a measure may be given by $\frac{\overline{R}-\mu}{\overline{R}}$ which has many well-known limitations.

Since the measure K derived above takes into account the distribution of calorie intake among different individuals as well as the distribution of calorie requirements, it may be considered a suitable measure of undernutrition at aggregate level. It can be seen that the commonly used measure, *i.e.* $\frac{\overline{R}-\mu}{\overline{R}}$, underestimates the degree of undernutrition, which implies that undernutrition will still exist.

This observation was made at the UNWFC held in Rome in 1974, where it was considered that energy supplies in the developing regions should be at least 10 percent above aggregate requirements to allow for maldistribution. The figure of 10 percent was arrived at on an ad hoc basis, but now equation (8) can be used to estimate the magnitude of underestimation.

Further, note that when $\alpha = 1.0$ the degree of undernutrition suffered by an individual is given by the exact amount of his calorie shortfall. It would be more appriate to give a higher weight to the larger calorie shortfall, which implies that α should be greater than unity. How much greater it should be is a matter of value judgement.

If the distribution of $g(R)$ collapses at the mean $\overline{R}$, it can be proved that $K(\alpha)$ becomes

$$K(\alpha) = \int_0^\infty \left[\frac{R-x}{\overline{R}}\right]^\alpha f(x)\,dx$$

which is an expression for the class of decomposable poverty measures proposed by Foster *et al.* [13] with poverty line $\overline{R}$. Thus, $K(\alpha)$ provides a generalization of their poverty measure when the poverty line is not a fixed number but follows a probability distribution.

Ⅳ. Estimation When Only the Mean and Variation of Requirements are Known

When only the mean and variance of requirements are known, we may try to make a range of estimates by using alternative forms of two parameter distributions. We performed the computations on the basis of two distributions, namely uniform and normal.

First, we assume that $g(R)$ is uniformly distributed with mean $\overline{R}$ and standard deviation σ_R. It can be shown that

$$
\begin{aligned}
1 - G(x) &= 1 && \text{if} \quad x < \overline{R} - \sqrt{3}\sigma_R \\
&= \frac{\overline{R} + 3\sigma_R - x}{2\sqrt{3}\sigma_R} && \text{if} \quad \overline{R} - \sqrt{3}\sigma_R \le x \le \overline{R} + 3\sigma_R \\
&= 0 && \text{if} \quad x > \overline{R} + \sqrt{3}\sigma_R
\end{aligned}
$$

which, on substituting in (1), gives

$$
M = F(\overline{R} - \sqrt{3}\sigma_R) + \frac{\overline{R} + \sqrt{3}\sigma_R}{2\sqrt{3}\sigma_R}[F(\overline{R} + \sqrt{3}\sigma_R) - F(\overline{R} - \sqrt{3}\sigma_R)] -
$$

$$
\frac{\mu}{2\sqrt{3}\sigma_R}[F_1(\overline{R} + \sqrt{3}\sigma_R) - F_1(\overline{R} - \sqrt{3}\sigma_R)] \qquad (9)
$$

where $F(x)$ is the probability distribution function of the distribution of calorie intake and

$$
F_1(x) = \frac{1}{\mu}\int_0^x x f(x)\, dx
$$

is the first-moment distribution function, which is interpreted as the proportion of calories consumed by the people who have calorie consumption less than or equal to x.

Further, it can be verified that the class of undernutrition measures $K(\alpha)$ derived in (7) is given by

$$K(\alpha) = \frac{1}{2\sqrt{3\sigma_R}\,(\alpha+1)}\int_0^{\overline{R}+\sqrt{3\sigma_R}}(\overline{R} + \sqrt{3\sigma_R} - x)^{\alpha+1} f(x)\,dx - \frac{1}{2\sqrt{3\sigma_R}\,(\alpha+1)}\int_0^{\overline{R}-\sqrt{3\sigma_R}}(\overline{R} - \sqrt{3\sigma_R} - x)^{\alpha+1} f(x)\,dx \qquad (10)$$

which leads to M when we substitute $\alpha = 0$.

Assuming that $g(R)$ is normally distributed with mean $\overline{R}$ and standard deviation σ_R than

$$G(x) = Q\left(\frac{x - \overline{R}}{\sigma_R}\right) \qquad (11)$$

where $Q(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$

Thus, M will be given by

$$M = \int_0^{\infty}\left[1 - Q\left(\frac{x - \overline{R}}{\sigma_R}\right)\right] f(x)\,dx$$

which can be readily computed given the distribution of calorie intakes.

K given in (8) can be computed if we know $G(x)$ and $G_1(x)$. $G_1(x)$ for a normal distribution is given by

$$G_1(x) = \frac{1}{\overline{R}\sqrt{2\pi}\,\sigma_R}\int_0^{x} Re^{-\frac{1}{2}\left(\frac{R - \overline{R}}{\sigma_R}\right)^2}\,dR$$

and $G(x)$ is derived in (11). So, substituting (11) and this equation in (8), we can obtain an estimate of K given the distribution of calorie intake.

All the measures of undernutrition presented are based on the assumption that the calorie intake x and the calorie requirement R, both measured per consumer unit, are independently distributed. This was the suggestion given by Sukhatme [30], who argued that, since the available evidence indicates small correlation between the two variables, x and R can be assumed to be independently distributed for practical evaluation. Let us consider a bivariate density between intake x and requirement R, $f(x, R)$. Then the head-count measures of undernutrition, which is the probability that a randomly selected person in the population suffers from undernutrition, is given by

$$M^* = \int\int f(x, R)\, dx\, dR, \qquad R < x \tag{12}$$

In practice, the bivarite density functionn $f(x, R)$ is not known, One common procedure is to assume that it is a bivariate normal density. This approach has two major limitation. First, the distribution of calorie intake is expected to be skewed whereas the bivariate normal distribution implies that it is symmetric. Second, the entire distribution of calorie intake is characterized by only two parameters, *i.e.* μ and σ^2; therefore, it cannot provide good fit to the entire distribution of calorie intake.

In order to solve this difficulty, let us write

$$f(x, R) = g(R/x) f(x)$$

where $g(R/x)$ is the conditional density of R given x, and $f(x)$ is the marginal density of x. Then M^* can be written as

$$M^* = \int_0^\infty \int_x^b g(R/x) f(x)\, dR\, dx$$

where the density function $f(x)$ can be obtained from the given data on calorie intakes. To compute M^*, it will be necessary to specify the density function $g(R/x)$. So let us assume that $g(R/x)$ follows a univarite normal distribution with mean and variance (Var) as

$$E(R/x) = \overline{R} + \rho\frac{\sigma_R}{\sigma}(x - \mu)$$

and

$$Var\,(R\,/\,x) = \sigma_R^2\,(1 - \rho^2)$$

respectively, where ρ is the correlation coefficient between x and R. Then M^* will given by

$$M^* = 1 - \int_0^{\infty} Q\left[\frac{x - \overline{R} - \rho\frac{\sigma_R}{\sigma}(x - \mu)}{\sigma_R\sqrt{(1-\rho^2)}}\right] f(x)\,dx \tag{13}$$

where $Q(x)$ is defined in (11) and $f(x)$ can be derived from the actual data on calorie intakes.

V. Estimation When Only Mean Requirement is Known

When only mean requirement is known, we may still obtain upper and lower bounds on the measure of undernutrition, provided we can assume symmetry of the requirement distribution.

Let us write M as

$$M = \int_0^{\overline{R}} [1-G(x)]\,f(x)\,dx + \int_{\overline{R}}^{\infty} [1-G(x)]\,f(x)\,dx$$

It is obvious that

$$\int_0^{\overline{R}} [1-G(x)]\,f(x)\,dx \geq [1-G(\overline{R})]\int_0^{\overline{R}} f(x)\,dx = [1-G(\overline{R})]\,F(\overline{R}) \tag{14}$$

and

$$\int_{\overline{R}}^{\infty} [1-G(x)]\,f(x)\,dx \geq 0 \tag{15}$$

which are derived from the fact that the distribution function $G(x)$ is a non-decreasing function in its domain.

Combining (14) and (15), we obtain

$$M \geq [1-G(\overline{R})]\, F(\overline{R})$$

and if we assume $g(R)$ to be symmetrically distributed around its mean $\overline{R}$, $G(\overline{R}) = \frac{1}{2}$, which gives

$$M \geq \frac{1}{2} F(\overline{R})$$

which provides a lower bound on M and can be obtained by knowing $\overline{R}$ and the distribution of calorie intake.

Similarly, it can be seen that

$$\int_{\overline{R}}^{\infty} [1-G(x)]\, f(x)\, dx \;\leq\; [1-G(\overline{R})]\,[1-F(\overline{R})] \tag{16}$$

and

$$\int_{\overline{R}}^{\infty} [1-G(x)]\, f(x)\, dx \;\leq\; F(\overline{R}) \tag{17}$$

which gives $M \leq F(\overline{R}) + [1-G(\overline{R})]\,[1-F(\overline{R})]$; and if $g(R)$ is symmetric around its mean, we obtain an upper bound on M as

$$M \leq \frac{1}{2} + \frac{1}{2} F(\overline{R})$$

This lead to the following proposition.

Proposition If the distribution of calorie requirement is symmetric around its mean, then

$$\frac{1}{2} F(\overline{R}) \leq M \leq \frac{1}{2} + \frac{1}{2} F(\overline{R})$$

Ⅵ. Conclusions

The objective of this paper was two fold: (1) to develop sensible methodologies for comparing the nutritional status of different populations, given the fact that within each population nutritional requirements vary from person to person, and (2) to develop measures of undernutrition which would take account not only of the number of people undernourished but also of the severity of their undernutrition. Some of the major findings of the study are summarized as follows.

(1) The average calorie norm approach that has been used by the World Bank and the FAO to estimate undernutrition at global level overestimates the degree of undernutrition in the event of independence between intakes and requirements, but the extent of the overestimation may be small.

(2) A common procedure to determine undernutrition at aggregate level is to compare the average per-capita availability of energy with the averge per-capita energy needs. It has been demonstrated that this procedure underestimates the degree of undernutrition, which implies that, even if the average per-capita calorie intake of a country is exactly equal to its average per-capita calorie requirements, undernutrition will still exist.

(3) The study also explores the measurement of undernutrition when calorie intake and requirement are correlated. It is shown that the estimates of undernutrition are not too biased if the correlation is assumed to be zero.

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