연세경제연구 제 Ⅵ 권, 제 1 호
Yonsei Economic Studies, Vol. Ⅵ, No. 1 (March 1999)

# The Theory of Index Numbers and Its Applications to Developing Countries

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This paper has attempted to show that the theory of index numbers can be extended to provide the Diewert-superlative index numbers for prices as well as quantities that are suitable for developing countries. One example is the cost of living index while the other is the monetary index, both of which utilize the respective user cost concepts that differ from previous studies. Although the theory is inherently complex, the resulting index numbers are rather simple and also desirable in view of the related empirical studies carried out for the developing country like Korea.

"One must not expect to be able to make the naive measurements that untutored common sense always longs for; we must accept the sad facts of life and be grateful for the more complicated procedures economic theory provides."

Samuelson, P. A. and S. Swamy [27, p. 592]

## Ⅰ. Introduction

Despite the development of an increasingly sophisticated literature on the index

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number theory since the turn of this century, it has not been adequately reflected in the actual measurements of economic aggregates, particularly in developing countries. One area of concern is the quantity index and the other is the price index. A good example for the former is the questions on what money is and how its quantity is measured among the variety of monetary components. The common practice for obtaining monetary aggregates is to simply add the monetary assets that are thought to be near-moneys. Since the "nearness" of assets to money implies a preference ordering, the monetary index, instead of a simple-sum index, may be defined on the basis of the consumer's choice in utility optimization. Thus, utilizing the choice-theoretic approach to index numbers by Diewert [6], Barnett [3] developed a method of constructing monetary indexes such as Divisia quantity index. Following this, many western countries (See Barnett *et al.* [4]) have compiled the necessary data and actually constructed such monetary indices. A similar effort, however, has not been made in developing countries although Lee ([17], [18], [19]) has demonstrated the usefulness of constructing such an index for Korea.

Perhaps more critical is the other question on the Consumer Price Index or the related price index (called CPI, hereafter) that has been constructed by almost all countries including Korea. The CPI is essentially based on the Laspeyres formula that measures the changes in prices with fixed weights for the individual goods and services consumed. Since the fixed-weight ignores the individual behavior of consuming less on items that become relatively more expensive, the CPI does not reflect the rational consumer behavior — i.e., it is not based on the choice-theoretic approach to index numbers. Moreover, the CPI does not have the theoretical base to explain how the initial cost of a durable good can be spread over the life of the durable good. For owner-occupied housing as an example, what is the "objective" rental cost of housing, what is the correct opportunity cost of capital and what should be done with the capital gain (or loss)? For these reasons, the rental cost of owner-occupied housing is often measured on an ad hoc basis in many countries including the U.S. (See e.g. U.S. [31]). Even for an ad hoc procedure, it is more difficult in developing countries due to the lack of adequate rental or mortgage markets.[1] As a result, the cost of owner-occupied housing

1) For instance, without an adequate rental housing market it is not possible to obtain the equivalent rental value of owner-occupied housing whereas the homeownership cost cannot be imputed from regular mortgage payments if the housing mortgage market is not available.

has been often omitted from the CPI or measured incorrectly in developing countries. For these and other reasons, there exist considerable doubts as to the accuracy of the CPI in measuring the true cost of living.[2] Moreover, accurately measuring pries and their rate of change, inflation, is central to almost every economic issue in any country.[3] For accurate measurements of the CPI, the U.S. Senate Finance Committee appointed an advisory committee, called Boskin Commission (Boskin *et al.* [5]), to study the CPI. In the United Kingdom, the Bank of England devised a housing-adjusted price index as an alternative to the consumer price index (See Allen [1]) published by the Office for National Statistics of the British government. Unfortunately, however, a parallel effort has not been made by the government agencies of many developing countries.[4]

To resolve these problems, it is important to define an appropriate cost of using durable goods and to incorporate such a user cost in a choice-theoretic model with a flexible utility function. For the theoretical consistency, however, it would be more desirable to derive and include the user cost at the same time within a given framework of utility optimization model. This was initially done in our earlier paper (Lee and Chung [20]) with an application to the Korean annual data and is later elaborated further in a paper (Lee and Chung [21]) with additional user-cost components, employing the quarterly data. While our user costs of durables are similar to those of monetary aggregates by Barnett [3], the user-cost evaluated cost of living index is derived, following the index number theory developed by Diewert [6]. The index number thus derived is "superlative" in the sense of Diewert [6], that it is derived exactly from a flexible utility function which provides a second-order approximation to any linear homogeneous function.

In this paper I shall outline our approach to the cost of living index, make some references to the quantity index and discuss the related applications of both indexes to

2) In Korea for an example, the reasons for doubts range from the outright disbelief (See e.g. Kim [15]) in the government figures on the CPI to the failure of incorporating quality changes in the CPI components including rental housing (See e.g. Han [13]).

3) Some examples include economic growth and productivity; government taxes and spending; budget deficits and debt; monetary policy; real interest rates, wages and incomes; real exchange rates and international trades and finances; and the comparative performance of economies.

4) For instance, the Korean government attempted only recently to include the cost of owner-occupied housing from the 1995 base year, which is more than 15 years behind the U.S. Yet, there are neither the published studies nor the academic conferences sponsored by the government to discuss the alternative methods of measuring the CPI for Korea.

developing countries. The plan of this paper is as follows. Section Ⅱ presents our overall theoretical framework of the cost of living index. Section Ⅲ identifies the user cost of durables such as housing, derives a superlative cost of living index in the form of Fisher ideal index and goes on to present a method of estimating the expected values required for the user cost concept. Section Ⅳ discusses the usefulness of the cost of living index and monetary index for developing countries. Finally, section Ⅴ completes the paper with the concluding remarks.

## Ⅱ. Theoretical Framework

The "cost of living" index may be defined as the ratio of the minimum costs of a given level of living (or utility) in two different price situations (See e.g. Samuelson and Swamy [27]). In the theory of consumer behavior, the minimum cost can be derived as dual to the problem of maximizing utility function of consumption goods subject to a given budget constraint. When the goods are consumable within a reference period, the utility function and its budget constraint are defined only for that period. But when the consumption involves consumer durables, both the utility function and the budget constraint must be extended to multi-period to allow the consumption flows from durable good over time.

Let $t$ be the current period and $T$ be the length of planning horizon for which the consumer makes an optimal decision. For each period $s$ ($t \leq s \leq t+T$), let the relevant variables be defined as follows:

$q_{is}$ = $i$-$th$ consumption good ($i = 1, \cdots, n$) during period $s$
$q_s = [q_{1s}, q_{2s}, \cdots, q_{ns}]'$
$Q_{cs}$ = chonsei housing stock at the end of period $s$
$Q_{hs}$ = owner-occupied housing stock at the end of period $s$
$Q_s = [Q_{cs}, Q_{hs}]'$
$p_{is}$ = price of $i$-$th$ consumption good ($i = 1, \cdots, n$) during period $s$
$p_s = [p_{1s}, p_{2s}, \cdots, p_{ns}]'$
$P_{cs}$ = price of chonsei housing stock at the end of period $s$

$P_{hs}$ = price of owner-occupied housing stock at the end of period $s$

$P_s = [P_{cs}, P_{hs}]'$

$g^e_{hs}$ = expected capital gain (loss) on owner-occupied housing for the coming period $(s+1)$ with the expectation formed at the end of period $s$

$m_{cs}$ = repair and maintenance cost relative to chonsei housing, $Q_{cs}$, at the end of period $s$

$\gamma_{hs}$ = miscellaneous costs of maintaining owner-occupied housing, $(\delta_{hs} + \tau_{hs} + m_{hs})$ relative to the housing stock at the end of period $s$,

where $\delta_{hs}$ = rate of depreciation,

$\tau_{hs}$ = property tax rate and

$m_{hs}$ = repair and maintenance cost relative to owner-occupied housing

$a_s$ = Per capita holding of real benchmark asset at the end of period $s$

$R^e_s$ = expected rate of return on the benchmark asset $a_s$ for the coming period $(s+1)$ with the expectation formed at the end of period $s$

$Y_s$ = nominal wage income during period $s$

$P^*_s$ = cost of living index during period $s$

For a multi-period analysis, we let $u_t$ be the consumer's current intertemporal, $T$-period, utility function. We assume that $u_t$ is weakly separable in each period's consumption of goods $(q_s', Q_s')$ for $s = t, \cdots, (t+T)$ and terminal benchmark asset $a_{t+T}$, so that $u_t$ can be written in the form,

$$\begin{aligned} u_t &= u_t[(q_t', Q_t'), \cdots, (q_{t+T}', Q_{t+T}'); a_{t+T}] \\ &= U_t[v_t(q_t', Q_t'), \cdots, v_{t+T}(q_{t+T}', Q_{t+T}'); a_{t+T}] \end{aligned} \tag{1}$$

where the overall utility function, $U_t$, is a monotonically increasing function of the sub-utility functions, $v_t, \cdots, v_{t+T}$ that are monotonically increasing, linearly homogeneous and strictly quasiconcave. Here we assume that the real benchmark asset does not serve as the store of value at an interior point of current planning horizon and as a result, it does not appear in the sub-utility functions.

The consumer's decision problem is to maximize the overall utility function (1) subject to the corresponding $T$-period budget constraint. To derive the overall $T$-period

budget, the budget for each period $s(t \leq s \leq t+T)$ may be defined as,

$$Y_s + (1 - m_{c,s-1})P_{c,s-1}\,Q_{c,s-1} + (1 + g^e_{h,s-1} - \gamma_{h,s-1})P_{h,s-1}\,Q_{h,s-1} + (1 + R^e_{s-1})P^*_{s-1}a_{s-1} = \sum_{i=1}^{n} p_{is}\,q_{is} + P_{c,s}\,Q_{c,s} + P_{h,s}\,Q_{h,s} + P^*_s\,a_s \quad (2)$$

where $Y$ is nominal wage income, $m_{c,s-1}$ is the rate of repair and maintenance cost on "chonsei" housing, $g^e_{h,s-1}$ is the expected rate of capital gain (or loss) on owner-occupied housing and $R^e_s$ is the expected rate of return on benchmark asset and $P^*_s$ is the cost of living index to be defined later.[5] Here, similar to the expected rate of capital gain $g^e_{h,s-1}$, $\gamma_{h,s-1}$ indicates miscellaneous housing costs as a fraction of the value of housing stock at the *end* of period $(s-1)$, which is defined by

$$\gamma_{h,s-1} = \delta_{h,s-1} + \tau_{h,s-1} + m_{h,s-1} \quad (3)$$

where $\delta_{h,s-1}$ and $\tau_{h,s-1}$ are, respectively, the depreciation rate and property tax rate on owner-occupied house; and $m_{h,s-1}$ is the repair and maintenance cost relative to the value of owner-occupied housing stock at the *end* of period $(s-1)$.[6] From the left hand side of eq(2), it can be seen that the real value of the cost adjusted chonsei and owner-occupied housing plus the benchmark asset carried over from the prior planning horizon is

$$(1 - m_{c,t-1})\,Q_{c,t-1} + (1 + g^e_{h,t-1} - \gamma_{h,t-1})\,Q_{h,t-1} + (1 + R^e_{t-1})\,a_{t-1}$$

5) There are two types of rental housing in Korea. One is the ordinary rental with regular rent payment and the other is so-called "chonsei" which is a rental contract with a lump sum deposit to be repaid on leaving the rental housing. The former is related to renting a room or a small apartment while the latter usually involves a medium or large-size apartment, comprising 16.4% and 20.3%, respectively, of the Korean households in 1990. The rental cost index included in the Korean CPI is calculated by a weighted average of the two with percentage increases in rent and chonsei for each period.

6) In eq(2), since the expectation on $g^e_{h,s-1}$ is formed at the *end* of period $(s-1)$, the expected capital gain is for the coming period of $s$. Similarly, since the housing costs such as $m_{c,s-1}$ and $\gamma_{h,s-1}$ are given relative to the value of previous housing stock at the *end* of period $(s-1)$, they indicate the cost for the coming period of $s$. Also in eq(2), since the monthly rental of housing can be considered as the monthly flow of housing consumption, it is included as non-durable goods.

and the real value of the consumer's provisions for the later planing horizon is

$$(1 - m_{c,t+T})Q_{c,t+T} + (1 + g^e_{h,t+T} - \gamma_{h,t+T})Q_{h,t+T} + (1 + R^e_{t+T})a_{t+T}$$

To derive the overall budget for the current planning horizon, let $\rho_s$ be the discount factor defined by

$$\rho_s = \begin{cases} 1 & s = t \\ \prod_{u=t}^{s-1}(1+R^e_u) & t+1 \le s \le t+T \end{cases} \tag{4}$$

First, we solve eq(2) for $a_s$ for each $s$ between $t$ and $t+T$. Then, insert $a_t$ in $a_{t+1}$ and $a_{t+1}$ in $a_{t+2}$ successively to obtain $a_{t+T}$. The resulting equation after rearranging gives the overall ($T$-period) budget evaluated at current period $t$ as:

$$\begin{aligned} &\sum_{s=t}^{t+T}(Y_s/\rho_s) + (1-m_{c,t-1})P_{c,t-1}Q_{c,t-1} + \\ &(1 + g^e_{h,t-1} - \gamma_{h,t-1})P_{h,t-1}Q_{h,t-1} + (1 + R^e_{t-1})P^*_{t-1}a_{t-1} \\ &= \sum_{s=t}^{t+T}\sum_{i=1}^{n}(p_{is}/\rho_s)q_{is} + \sum_{s=t}^{t+T}\left[\frac{1}{\rho_s} - \left(\frac{1-m_{cs}}{\rho_{s+1}}\right)\right]P_{cs}Q_{cs} + \\ &\sum_{s=t}^{t+T}\left[\frac{1}{\rho_s} - \left(\frac{1+g^e_{hs}-\gamma_{hs}}{\rho_{s+1}}\right)\right]P_{hs}Q_{hs} + \left[\frac{1-m_{c,t+T}}{\rho_{t+T+1}}\right]P_{c,t+T}Q_{c,t+T} + \\ &\left[\frac{1+g^e_{h,t+T}-\gamma_{h,t+T}}{\rho_{t+T+1}}\right]P_{h,t+T}Q_{h,t+T} + (P^*_{t+T}/\rho_{t+T})a_{t+T} \end{aligned} \tag{5}$$

The overall budget constraint so derived makes economic sense. On the left hand side of eq(5), the total wealth is given by the sum of the discounted total nominal wage income plus the nominal values of chonsei deposit, owner-occupied housing and benchmark asset that are passed on to the start of the current planning horizon from the end of the last planning horizon. On the right hand side of eq(5), the total consumer expenditures are given in two main categories: One category consists of the discounted non-durable consumption expenditures and the discounted two expenditure flows of user-cost evaluated chonsei housing and owner-occupied housing stock, whereas the

other category contains the discounted values of both chonsei housing deposit and owner-occupied housing stock, and the discounted benchmark asset, all of which would be passed on to the next planning horizon. Thus we have the overall ($T$-period) budget constraint (5) for current planning horizon that corresponds to the $T$-period intertemporal utility function (1).

## Ⅲ. Derivation of Index Number

### 1. The User Cost Concept

Since the consumer expenditure for any commodity is the price times quantity, the price can be identified from the amount of expenditure when the quantity is specified. Thus, from eq(5) we can immediately identify the user cost of owner-occupied housing for period $s$ as,

$$\pi_{hs} = \left[\frac{1}{\rho_s} - \left(\frac{1+g^e_{hs}-\gamma_{hs}}{\rho_{s+1}}\right)\right]P_{hs}$$

By substituting eq(3) for $\gamma_{hs}$ and eq(4) for $\rho_s$ and $\rho_{s+1}$ while setting $s=t$ in the above equation, we can now see the current user cost of owner-occupied housing, $\pi_{ht}$, as

$$\pi_{ht} = \left[\frac{R^e_t - g^e_{ht} + \delta_{ht} + \tau_{ht} + m_{ht}}{1+R^e_t}\right]P_{ht} \tag{6}$$

Similarly from eq(5), we can also identify the user cost of chonsei housing for period $s$ as

$$\pi_{cs} = \left[\frac{1}{\rho_s} - \left(\frac{1-m_{cs}}{\rho_{s+1}}\right)\right]P_{cs}$$

and hence obtain the current user cost of chonsei housing, $\pi_{ct}$, as

$$\pi_{ct} = \left[\frac{R_t^e + m_{ct}}{1+R_t^e}\right] P_{ct} \tag{7}$$

which includes repair and maintenance cost on chonsei housing, $m_{ct}$, as it is usually born by the chonsei tenant in Korea. Since the property depreciation and tax are paid by the property owner, they are presumably allowed for in the unit price ($P_{ct}$) of lump-sum chonsei deposit paid by the chonsei tenant.

The user cost thus derived is similar to the opportunity cost defined in terms of expenditure flows. For instance, $\pi_{ht}$ in (6) is the discounted value of the interest forgone minus capital gain (or loss) plus tax, depreciation, and repair and maintenance cost by holding a unit of owner-occupied housing. Since it is the cost of using the housing services generated from homeownership, it is the user cost of owner-occupied housing. Such a user cost is analogous to those of Jorgenson [14] and Donovan [10] or the user cost of monetary assets defined by Barnett ([2], [3]).

## 2. The Cost of Living Index

To derive the cost of living index, the representative consumer is viewed as making an optimal decision while maximizing the intertemporal, $T$-period, utility function (1) subject to the overall $T$-period budget constraint (5). Since the intertemporal utility function is assumed to be homogeneous and blockwise weakly separable, following Green's [12] theorem (4), the consumer can be viewed as making consistent two-stage decision.

In the first stage, the consumer optimally allocates the overall $T$-period budget among each period aggregate consumer expenditures and the terminal benchmark asset. In the second stage, he further allocates each period aggregate expenditure separately over the individual consumer goods. Accordingly, the allocation of individual goods for current consumption is simply to maximize the current sub-utility function subject to the current one-period aggregate consumption expenditure, $E_t$, i.e.,

$$\begin{aligned} &\max \{ v_t(q_t', Q_t') \} \\ &\text{subject to } E_t = p_t' q_t + \Pi_t' Q_t \end{aligned} \tag{8}$$

where $p_t$ is the vector of prices as $p_t = [p_{1t}, p_{2t}, \cdots, p_{nt}]'$ and $\Pi_t$ is the vector of user costs of housing defined by $\Pi_t = [\pi_{ct}, \pi_{ht}]'$.[7] Dual to this maximization is the minimization problem involving

$$\begin{aligned} &\min \{p_t' q_t + \Pi_t' Q_t\} \\ &\text{subject to } v_t(q_t', Q_t') = v^\circ \end{aligned} \tag{9}$$

where $v^\circ$ is a given level of sub-utility. Solving (9), we can obtain the minimum current expenditure, min $\{E_t\}$, as a function of unknown parameters:

$$\min \{E_t\} = m(v^\circ, p_t', \Pi_t') \tag{10}$$

Assuming that the consumer replans in each time-period, the minimum expenditure for $(t+1)$ period is derived in the same form as (10) by replacing subscript $(t+1)$ for $t$.

Comparing the living costs between period $t$ and $(t+1)$ is to take the ratio of minimum expenditures for two periods,

$$\frac{\min\{E_{t+1}\}}{\min\{E_t\}} = \frac{m(v^\circ, p_{t+1}', \Pi_{t+1}')}{m(v^\circ, p_t', \Pi_t')} \tag{11}$$

To eliminate the unknown parameters, following Diewert [6], we assume that the sub-utility function is homogeneous quadratic mean of order two,

$$v_t = [(q_t', Q_t')\, A\, (q_t', Q_t')']^{1/2} \tag{12}$$

where $A$ is a $(n+2)$ dimensional symmetric matrix with $(n+1)$ zero or negative eigenvalues and one positive eigenvalue. Then, the minimum expenditure ratio (11) with unknown parameters can be reduced to a parameter-free Fisher ideal price index:

$$\frac{\min\{E_{t+1}\}}{\min\{E_t\}} = \left[\frac{p_{t+1}' q_t + \Pi_{t+1}' Q_t}{p_t' q_t + \Pi_t' Q_t} \frac{p_{t+1}' q_{t+1} + \Pi_{t+1}' Q_{t+1}}{p_t' q_{t+1} + \Pi_t' Q_{t+1}}\right]^{1/2} \tag{13}$$

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7) Although we will not elaborate here, it seems obvious that similar user costs may be defined for other durable goods for their inclusion in the cost of living index derived in this paper.

Given the normalization of min $\{E_t\} = 100$ and multiplying through the subsequent chain indexes, we can generate the cost of living index in chain form, which is used as $P_s^*$ for period $s$ in eq(2).

The cost of living index thus derived has a number of desirable properties. First of all, unlike the fixed-weight CPI, our cost of living index allows the effect of substitution among consumer goods as it is derived from a choice-theoretic model of utility optimization. Secondly, it incorporates the user costs of durable goods such as housing that are derived from the same framework of utility optimization, thus avoiding an ad hoc concept of homeownerhip cost used by others. Thirdly, since our cost of living index takes account of house prices, it may be used as an inflation target of monetary policy which has to be concerned with speculation in house prices that often occurs in rapidly developing countries. Fourthly, our cost of living index is superlative in the sense of Diewert [6] that it is exact for a flexible homothetic utility function that can approximates any homothetic function. Fifthly, as shown for the Fisher ideal index by Diewert [7], our cost of living index using Fisher formula satisfies more reasonable "tests" than any other index numbers, including the Laspeyres or even the Divisia index. Finally, as pointed out by Diewert [6], Fisher ideal index like ours is consistent with the "axioms" of the revealed preference theory and hence our cost of living index is more general than is implied by the specific form of utility function (12). For these reasons, our cost of living index is superior to any other consumer price indexes that are currently being used by developed as well as developing countries around the world.

## 3. Formulation of Expectation

Since an expectation on capital gain or on the rate of return on asset is usually formed from the past observations, it should be modeled for estimation so that the expected values may be used in obtaining the user costs for measuring the overall cost of living index. For this purpose, let $x_t^e$ be the expected capital gain or rate of return with the expectation formed at the end of period $t$ for the expected value for the coming period $(t+1)$. Following Milbourne [25] and Lee and Chung [21], we combine adaptive and extrapolative expectations mechanisms by the maintained hypothesis:

$$x_t^e - x_{t-1}^e = (1-\lambda)(x_t - x_{t-1}^e) + \beta(x_t - x_{t-1}) \tag{14}$$

where the change in expectation is, in part, adapted to the discrepancy between the observed and the expected rate, $(x_t - x_{t-1}^e)$, and, in part, extrapolated by the difference in the observed rates $(x_t - x_{t-1})$. Rewriting the equation gives

$$x_t^e = (1-\lambda)\, x_t + \lambda\, x_{t-1}^e + \beta\,(x_t - x_{t-1}) \tag{14}'$$

where $\lambda$ is the adaptation coefficient with a value of $0 < \lambda < 1$ and $\beta$ is the extrapolation coefficient with $\beta$ being any value.

If consumers are supposed to behave as in eq(14)′, they will attempt to estimate the parameters to form the best unbiased estimates of $x_t^e$ and $x_{t-1}^e$. Consequently, if $\epsilon_{s+1}$ is a random error term, it follows that $\lambda$ and $\beta$ will be estimated so that $E(\epsilon_{s+1}) = 0$ for $s=t$ and $t-1$ where $\epsilon_{s+1} = x_{s+1} - x_s^e$. Inserting this in eq(14)′ and rearranging gives the estimating equation:

$$x_{t+1} = (1+\beta)\, x_t - \beta\, x_{t-1} - \lambda\epsilon_t + \epsilon_{t+1} \tag{15}$$

Therefore, $\beta$ and $\lambda$ may be estimated by the maximum likelihood method with the obvious linear restrictions on the coefficients on $x_t$ and $x_{t-1}$ and with the moving average parameter being $\lambda$.

By lagging eq(14)′ with successive substitutions and replacing the estimates for the unknown parameters, the equation for forecasting $x_t^e$ is obtained by

$$\hat{x}_t^e = (1-\hat{\lambda})\, x_t + \hat{\lambda}\,(1-\hat{\lambda})\, x_{t-1} + \hat{\lambda}^2(1-\hat{\lambda})\, x_{t-2} + \cdots +$$
$$\hat{\beta}\,(x_t - x_{t-1}) + \hat{\lambda}\,\hat{\beta}\,(x_{t-1} - x_{t-2}) + \hat{\lambda}^2\hat{\beta}\,(x_{t-2} - x_{t-3}) + \cdots \tag{16}$$

where $\hat{x}_t^e$ for expected capital gain or rate of return on benchmark asset is predicted by $x_{t-i} = g_{t-i}$ or $R_{t-i}$ for $i = 0, 1, \cdots, \infty$. Since the estimate of $\lambda$ is likely to be a positive fraction, the last terms of lagged $x_t$ or $(x_t - x_{t-1})$ become trivial as they approach zero.

# Ⅳ. Applications to Developing Countries

## 1. Consumer Price Index

The CPI attempts to measure the changes over time in the price of a fixed market basket of goods and services consumed in a selected base period. For non-durables, the CPI uses data on the market prices paid because purchase and consumption are almost synonymous. For durables, the full cost of a durable good must be spread out over the useful life of the durable good since it cannot be attributed to the period of purchase. For an example, the homeownership cost is measured in the past by two alternative approaches — rental equivalence and user cost. The first is to use the rent of rental house as a proxy for homeownership cost when the rental house has housing characteristics similar to owner-occupied housing. The second involves building up the user cost of shelter services from its components — interest cost, property tax, repair and maintenance cost, etc.

The rental equivalence approach has been used for the CPI by the U.S. Bureau of Labor Statistics (See e.g. Gillingham [11]). But it should involve an "objective" judgment (Diewert [8]) on what the equivalent housing means. In the U.S., it is difficult to compare the uniqueness of owner-occupied housing to the high quality of housing services in the rental market. In particular, it would be more difficult to use this approach in developing countries where the rental markets are not fully developed as in the case of Korea (See e.g. Kim [16]).

The user cost approach is in part adopted by the Office for National Statistics of England which calculates housing mortgage interest payments for inclusion in the so-called Retail Price Index for the British consumers. However, it does not include the opportunity cost of the equity tied up in housing, not to mention other components such as the cost of property depreciation. To partially amend this, the Bank of England (See e.g. Allen [1]) has devised Housing-Adjusted Retail Prices in which the user cost includes the cost of servicing a mortgage, plus the opportunity cost of the housing equity, plus depreciation and maintenance costs. While the new index seems to be better, it is not constructed from the theoretical basis as compared to our cost of living

index. Moreover, no such data as mortgage payments are available in developing countries where the mortgage markets are not yet adequately developed. On the other hand, McFadyen and Hobart [24] applied the Jorgenson's user cost concept [14] for physical capital to the case of owner-occupied housing for the Canadian CPI. But since the rate of return on housing was not observable, they used the mortgage interest rate, as in other western countries, instead of alternative interest rate for the opportunity cost of capital embodied in owner-occupied house.

In Korea, the cost of owner-occupied housing had been omitted by the National Statistical Office from the domain of the CPI up until 1995. Since then, homeownership cost has been estimated for inclusion in the Korean CPI but neither the conceptual approach nor the statistical procedure appears to be appropriate.[8] For instance, the CPI's calculated by the National Statistical Office with and without the owner-occupied housing are almost identical despite the fact that over the years house prices have increased substantially higher than have other prices. Indeed when our method is applied to the 1975 ~ 1993 annual observations (Lee and Chung [20]), the actual inflation in Korea had been substantially higher than previously indicated by the government CPI. Moreover, when we used the quarterly data for a recent period (Lee and Chung [21]), an increase in the price level has been quite large since the implementation of the IMF financial-reform package as compared to the official CPI that includes its own measure of homeownership cost. Thus, our method in this paper appears to be quite useful in measuring the true cost of living for countries where the rental or the mortgage markets for housing are not fully developed.

Finally, it should be noted that in constructing our cost of living index, there is a practical difficulty of obtaining information on quantities currently being consumed that is typically available with a lag of one year or more through the sample survey. Such a difficulty makes it unsuitable to produce monthly estimates of the cost of living index

8) Since the National Statistical Office of the Republic of Korea does not make an explicit account of the procedure, it is difficult to assess the detailed procedure used. The 1996 Annual Report on the CPI by the National Statistical Office [30] simply footnotes that "Owner-occupied housing is imputed from the services generated by the opportunity cost (mortgage rate)," (translated from p. 479). However, since they do not use the market price of owner-occupied house, it is not the opportunity cost of the capital embodied in owner-occupied house like ours. Neither is the traditional built-up cost of using shelter services as it does not account for the home equity, depreciation, repair and maintenance cost etc. Moreover, in absence of adequate housing mortgage market in Korea, their mortgage rate refers, not to the cost of owning a home, but to the cost of a loanable fund in using the owner-occupied house as the collateral.

like the CPI. Fortunately, however, Shapiro and Wilcox [28] found a method of overcoming this problem by approximating the cost of living index like Fisher ideal, based on the index formula that Lloyed [23] and Moulton [26] derived from the CES utility function. Their empirical analysis for the U.S. indicates that the CES index *using the lagged expenditure weights of two years ago* exactly captures the trend rate of growth of a U.S. superlative index (like the Fisher ideal) provided that the elasticity of substitution for the CES utility function should be 0.7. This method can be applied to any developing country by estimating the elasticity of substitution for the CES index for that country to approximate the true cost of living index. For this purpose, however, the cost of living index developed in this paper must first be constructed as has been carried out for the case of Korea (See Lee and Chung [20], [21]).

## 2. Monetary Quantity Aggregate

The simple summation conventionally used to aggregate over monetary components ignores different degrees of moneyness among the components and hence, it does not capture the flows of monetary services to the economy. For this reason, Barnett [3] derived the user cost concept of monetary components and developed the method of aggregating user-cost evaluated monetary quantities, using the theory of index numbers by Diewert [6]. Since our cost of living index is derived as dual to the quantity index, the approach by Barnett [3] for monetary index is similar to our cost of living index while there are some notable differences.

In monetary index, all monetary components are the stocks like durable goods that are evaluated by the user costs of monetary components to indicate the flows of monetary services. Secondly, the user cost of monetary asset is an opportunity cost concept like the user cost of housing but it involves interest yields on monetary components rather than *expected* capital gain, depreciation, and repair and maintenance cost. Lastly, the monetary index is derived as a superlative index in the form of Fisher ideal index similar to ours but it is the quantity index rather than the price index.

Although the monetary indexes proposed by Barnett [3] have been constructed in many countries (See the Federal Reserve Bank of St. Louis for monthly publications of *Monetary Trends* [29], and also see Barnett *et al.* [4] for other examples), their

applications to developing countries have been quite limited due perhaps to the lack of understanding the particular issues in economic development. For instance, in developing countries like Korea, the government often controls interest rates and directs the bank loans to specific firms and industries at low interest rates while the true market rate is determined elsewhere in the underground (curb) money market. Thus, when the monetary control shifts the structure of interest rates, the flows of services from monetary components, given the money total, would also change due to a shift in the user costs. Such a change in monetary services can be captured by the user-cost evaluated monetary index but not by the simple-sum index used by the government. Moreover, since the curb market rate is quite high and variable relative to other interest rates, it is important to incorporate the curb market rate as the rate of opportunity cost so that the presence of curb market may be taken into account in the monetary index.[9]

By taking these into account, several studies have been carried out to construct the monetary index that is appropriate for the developing country like Korea. Initially, Lee [17] discussed the usefulness of monetary index for the cases of developing countries and suggested the use of such an index for Korea. Subsequently, Lee [18] actually constructed the monetary index on the basis of the Korean quarterly data, showing that it is more reliable than the conventional simple-sum monetary aggregate. A further study by Lee [19] analyzed the comparative static result of a change in the curb market rate to verify that the presence of curb market should be reflected in monetary aggregation for the developing country. On the other hand, Lee and Han [22] developed a method, based on the theories of index number, monetary aggregation and money supply, by which changes could me measured in the relative size of the underground to the regulated money market. When this method was applied to Korea as an example, the result indicated that the method was quite reliable in measuring the relative size of the relevant money markets. Therefore, all of these results seem to indicate that there are promises in the application and extension of monetary index for developing countries.

9) In theory, any measurement of the benchmark rate of return may be viewed as a proxy for the unknown rate of return on human capital. However, in the empirical studies (Barnett [3], Lee [17], [18], [19], Lee and Han [22] and Barnett *et al.* [4]), the benchmark rate has been measured by the maximum rate on any asset available in the economy. The highest return available in Korea is the curb market rate which we have used in all of our studies (Lee [17], [18], [19], Lee and Han [22], Lee and Chung [20]) cited in this paper except one recent study [21] where the 3-year corporate bond rate is utilized since the curb market rate is no longer available.

While individuals could further explore such a study, the government agencies must be involved in generating the data required for these research since the data are quite limited in developing countries.

## V. Concluding Remarks

In this paper, we have attempted to show that despite the complexities involved in index number theory, it can be utilized to derive a new index for the cost of living or to extend the existing method of monetary index for purposes of practical applications to developing countries.

Using a choice-theoretic model, we first derived the user cost concept for durables such as housing and then incorporated it in the overall cost of living in the form of a Diewert-superlative, Fisher-ideal, index number. Compared to the fixed-weight CPI, the cost of living index thus derived has the main advantage in that it includes the user cost of owner-occupied housing without using the unobservable rental value or mortgage rate, while allowing the commodity substitutions for changes in relative prices. The usefulness of this index is illustrated from the actual measurements for Korea which as expected show a substantially higher inflation than does the CPI.

This paper also shows that the method of monetary index by Barnett [3] is useful for the cases of developing countries. The applications to Korea indicate that it is important in monetary aggregation to take into account not only the presence of underground money market but also a growing importance of non-bank financial intermediary liabilities. Also by using the underlying theory of a monetary index, a method may be derived to measure the relative size of the underground to regulated money market which cannot be actually observed.

All of these results, therefore, lead us to believe that there are promises for further research in the theory and application of index number methodology, particularly in relation to developing countries. Finally, it is well to recall, as suggested by our predecessors, that untutored naive measurements must not preside over the more complicated procedures provided by economic theory, especially when such a procedure can be simplified and extended as has been attempted in this paper.

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