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〈강의자료 Teacher's Corner〉

A Pedagogic Note on Testing Hypotheses Concerning Point Trinomial Parameters

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Hypotheses concerning point binomial parameters are tested based on general binomial probability mass functions for small samples. The case of point multinomial parameters is, from statistical view point, not very interesting.

However, from the view point of economic statistics the story becomes different. The statistical model of a duopoly can be formulated into a point binomial mass function, where its parameter represents the market share of a duopolist.[1] An oligopoly can be modelled statistically into a point multinomial probability mass function. Here, we pick up a tripoloy.[2]

Let $f(x_1, x_2)=\theta_1^{x_1}\theta_2^{x_2}(1-\theta_1-\theta_2)^{1-x_1-x_2}$, where $0<\theta_i<1$, the market share of the ith tripolist, $i=1, 2$;

Suppose the null hypothesis $H_0 : \theta_1=\frac{1}{3}$, $\theta_2=\frac{1}{3}$, the alternative hypothesis $H_1 : \theta_1=\frac{1}{2}$, $\theta_2=\frac{1}{3}$, and the sample size $n=20$. Then,

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1) Suppose a beer market is divided between Brewers A and B, and A's "market share" is $\theta(=0.8)$, then from the retailer's view point, the market share is a "decision urn" ; it contains 8 black balls and 2 white balls, and for each purchase the customer draws a ball from the urn : if he draws a black ball, he buys A's brand; if he draws a white ball, he buys B's brand. Hence, the customer's decision rule is described by a point binomial probability mass function as follows:

$$f(x)=0.8^x 0.2^{1-x}, \quad x=1, \text{ if he draws a black ball}$$
$$x=0, \text{ if he draws a white ball}$$

2) See Appendix I.

$$f(y_1, y_2|H_0) = \frac{20!}{y_1!y_2!(20-y_1-y_2)!}\left(\frac{1}{3}\right)^{y_1}\left(\frac{1}{3}\right)^{y_2}\left(\frac{1}{3}\right)^{20-y_1-y_2}$$

$$f(y_1, y_2|H_1) = \frac{20!}{y_1!y_2!(20-y_1-y_2)!}\left(\frac{1}{2}\right)^{y_1}\left(\frac{1}{3}\right)^{y_2}\left(\frac{1}{6}\right)^{20-y_1-y_2}$$

The question is : How to construct the best critical region for a level of significance, say $\alpha=0.05$?

The Neyman-Pearson ratio for y_1, y_2 :

$$\gamma_{y_1,y_2} = \frac{f(y_1, y_2|H_1)}{f(y_1, y_2|H_0)}$$

$$= \frac{\frac{20!}{y_1!y_2!(20-y_1-y_2)!}\left(\frac{1}{2}\right)^{y_1}\left(\frac{1}{3}\right)^{y_2}\left(\frac{1}{6}\right)^{20-y_1-y_2}}{\frac{20!}{y_1!y_2!(20-y_1-y_2)!}\left(\frac{1}{3}\right)^{y_1}\left(\frac{1}{3}\right)^{y_2}\left(\frac{1}{3}\right)^{20-y_1-y_2}}$$

$$= \left(\frac{3}{2}\right)^{y_1}\cdot 1^{y_2}\cdot\left(\frac{1}{2}\right)^{20-y_1-y_2}$$

There is no straightforward way to rank the Neyman-Pearson ratios. However, a simple PC program does the job. The following probabilities are generated by a PC program developed by Doris Yang Soo Kim.[3)]

The results are summarized as follows :

The level of significance $\alpha=0.05$

	The number of elements in the best critical region / The total number of the elements in the sample space	Type I error probability	The power of the test
$n=20$	64 / 231	0.04936	0.59338
$n=30$	156 / 496	0.04949	0.75122

〈Appendix Ⅰ〉

The proposed statistical technique can be applied to the following cases, too.

i. *A* trigarchy prevails in a country. It has been believed that the three trigarchs, *A*, *B*,

3) See Appendix Ⅱ.

and C share the "power" equally. However, this belief is lately challenged by a new allegation that "power" sharing is such that $A : B : C = 3 : 2 : 1$. In a recent mid-term appointment of 30 delegates, the A-faction obtained 14, the B-faction 15, and the C-faction 1. Which hypothesis, old or new, does this appointment outcome support?

This question presuppose that the appointment of a delegate is negotiated among the trigarchs in a smoke-filled back-room and decided by the "average bargaining strength" of each faction. Hence, to an outsider, "power" is synonymous with "average bargaining strength," the appointment of a delegate is a stochastic phenomenon, and successive negotiations are not related.

ii. It is accepted that the racial composition in an area is such that Black : Yellow : White = 1: 1: 1. Now a demographer asserts that the composition has changed such that Black : Yellow : White = 3 : 2 : 1 due to recent massive migrations. To counter the assertion, a census bureau employee picked up 30 people randomly and checked their racial backgrounds. The result was : Black 14 ; Yellow 15 ; White 1. Does the result support the assertion?

iii. In a crime-ridden metropolitan area, three factions A, B, and C, engage in bloody rivalry for drug-trafficking. The "prestige" of a faction is measured by the number of its pushers. The police has assumed that the "prestige" ratio is $A : B : C = 1 : 1 : 1$. However, lately it is often reported that the "prestige" ratio has changed to $A : B : C = 3 : 2 : 1$. The police chief checked recent arrest records, and found out that A 14, B 15, and C 1. Which one, the old assumption or the new record, does the finding support?

iv. Visitors to a certain sight-seeing spot can be classified according to their origins : A ; B ; C. The ratio of visitors from each origin is taken for $A : B : C = 1 : 1 : 1$. However, a recent marketing survey suggests that the ratio has changed to $A : B : C = 3 : 2 : 1$ due to abrupt economic changes in the three origins.

To judge the credibility of the suggestion, an employee from the sight-seeing bureau randomly picked up 30 arrivals at the airport and checked their origins. The result was : A 14 ; B 15 ; C 1. Does the result support the suggestion?

〈Appendix Ⅱ〉

Readers of this note can get an easy access to the fortran source program using the file transfer protocol (ftp). One can login a public domain file server of Yonsei University by entering a following command at the dialog box which pops up as a result of clicking "start" and "run" sequentially in Windows 95.

ftp ftp.yonsei.ac.kr

The login name is anonymous and password is one's e-mail address. At the ftp prompt one needs to change the directory as follows ;

ftp>cd pub/stat

One can use "get" command to down load in his A:\diskette the fortran source program, npratio. for as follows ;

ftp>get npratio.for a:\npratio.for
ftp>quit