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Contest between David and Goliath

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Abstract

This paper considers a model of contest between two asymmetric agents in which reputation effects work in different directions. If David wins, he gains substantial fame, and he has nothing to lose but his effort even if he loses. By contrast, it is just a natural outcome for Goliath to win, but if he loses, then he faces substantial shame. We characterize the conditions for a unique pure-strategy Nash equilibrium and conduct comparative statics to examine how efforts and expected payoffs respond to variations in contest parameters. From this analysis, we understand the structure of interactions between fame, shame, ability, and effort in the contest.

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Ⅰ. Introduction

The media have compared the Smartwatch battle between Pebble and Apple to that between David and Goliath.[1)] Another comparison has been made for the competition in music-streaming services between Spotify and Apple.[2)] The household furniture sales

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1) "While the rest of the smartwatch market waits on the sidelines for Apple to show its hand, Pebble has leap into the limelight by creating a classic David and Goliath story." (Financial Times, Feb. 27, 2015)

competition between Hanssem (a local manufacturer in Korea) and IKEA (a new entrant in Korea) is another good example. There are many other versions of the story about David versus Goliath. A medical malpractice lawsuit between an unsuccessful, alcoholic lawyer and the high-priced legal team with strong support from the hospital in question is the story in the 1982 movie entitled "*The Verdict*", which fits the battle between David and Goliath. The match between Manchester United and FC Seoul[3] and the game of "go" between a six-year-old prodigy and the world's top-ranking grandmaster are representative examples of the contest between David and Goliath. As these examples demonstrate, there are many David and Goliath stories and this observation raises the question of what are the distinctive features of contests between David and Goliath. And the purpose of this paper is to answer to this question using a simple model.

This paper is closely related to previous studies of asymmetric contests in the sense that, broadly speaking, they consider the situation where players draw their different valuations from the prize, have different abilities to convert their effort into the probability of winning, or both. Examples of such include Baik (1994), Fonseca (2009), Hurley (1998), Nti (1999, 2004), Siegel (2009a,b), Stein (2002), Xiao (2015), among others. Baik (1994) considers contests with two asymmetric players both in the simultaneous-move form and in the endogenous-timing framework, focusing on effort expended by the players. Nti (1999) analyzes contests with asymmetric valuations for a variable range of the returns to scale parameter in the contest success function, and a condition for a unique pure strategy Nash equilibrium is established.

2) "The stage is set for a battle in which Spotify, the David to Apple's Goliath, is the incumbent and market leader." (Financial Times, June 9, 2015)

3) There were two times of matches in 2007 and in 2009 during Manchester United's pre-season trip to Korea.

Equilibrium effort level and equilibrium payoffs are derived, and the issue on rent dissipation is commented. Nti (2004) considers the problem of designing a contest to elicit maximum aggregate effort from players with asymmetric valuations. Optimal designs for different classes of contest technologies are computed and characterized. Stein (2002) studies a contest with N asymmetric agents where each of contestant may have a different valuation or a different ability. A pure strategy Nash equilibrium is obtained and its consequences are investigated. Siegel (2009b) considers contests where N players compete for one of M identical prizes by choosing a score. With this situation, he allows for differing production technologies, costs of capital, prior investments, attitudes toward risk, and others. A closed-form formula for players' equilibrium payoffs are provided. Besides, the asymmetric contest literature is still extending in the several other directions. For instance, Siegel (2009a) and Xiao (2015) consider asymmetric contests and provides an algorithm that constructs the equilibrium. Fonseca (2009) investigates asymmetric contests through an experiment in the laboratory, and reports on an experimental test of the effects of asymmetry in the contest success function both in the simultaneous and sequential move frameworks.

Here in this paper, we analyze a contest between two asymmetric agents in which reputation effects work in different directions. As in the papers mentioned above, we characterize Nash equilibria and examine how individual and total effort levels at the Nash equilibrium and equilibrium payoffs respond when contest parameters change. However, two important features of the current paper different from these existing papers are in order. First, this paper explicitly considers players' reputation effect. In our setting, reputation effects play an important role and more importantly, reputation effects work in different directions. If David wins, then he gains substantial fame, but he has little to lose even if he loses. By contrast, it is just a natural outcome for Goliath to win, but if he loses, then he faces great shame. For this reason, the payoff structure is modeled different from the literature, employing fame and shame parameters. We choose to do this way, because it is possible that interesting factors and their interactions may cancel each other out if we treat the competition between David and Goliath like other asymmetric contests without an explicit consideration. Second, unlike a standard asymmetric contest model which has an asymmetric logit-form probability-of-winning function with constant returns to scale technology and/or asymmetric valuations, the participation constraints play a crucial role.[4] In the standard model, both agents are willing to participate in the contest. In the model considered in this paper, however, Goliath may not want to participate for fear of shame he faces when he loses.[5]

The rest of the paper is organized as follows. Section 2 sets up the proposed model, characterizes the pure-strategy Nash equilibrium, and provides the conditions for the pure-strategy Nash equilibrium to be a unique interior solution of the game. Section 3 conducts comparative statics to examine how efforts and expected payoffs respond to variations in contest parameters. Section 4 concludes.

Ⅱ. The Model

Consider a contest in which two risk-neutral agents D (David) and

4) By a "standard" asymmetric contest we mean a contest which has an asymmetric logit-form probability-of-winning function with constant returns to scale technology and/or asymmetric valuations.

5) If the game of contest has a variable range of the returns to scale parameter in the probability-of-winning function, the participation has a role as in our model. (See, for instance, Nti (1999))

G (Goliath) compete with each other to win a prize. The prize is worth V in nominal value and is to be awarded to one of the two agents. Let x_D and x_G represent the two agents' irreversible effort levels measured in the same unit as the prize. Let p_D represent the probability that agent D wins the prize if the agents' effort levels are x_D and x_G. The probability that agent G wins the prize is then $p_G = 1 - p_D$. The contest success function for agent D is given by the following logit form:

$$p_D(x_D, x_G) = x_D / (x_D + \gamma x_G), \tag{1}$$

where the parameter γ represents agent G's ability in the contest relative to that of agent D, and, for this reason, it is set $\gamma > 1$ by assumption. Here p_i is equal to 1/2 if $x_D = x_G = 0$.

Suppose that agent G firmly establishes a brand name and currently enjoys a high level of reputation and popularity for some reasons. Although the nominal value of the prize is the same for the two agents, reputation effects work differently according the outcome of the contest. Agent D gains substantial fame if he wins the contest, while there is no shame even if he loses. On the other hand, it is just a natural outcome for agent G to win, but he faces great shame if he loses. Here assume that reputation effects can be measured in a unit commensurate with the prize.

Let π_i represent the expected payoff for agent i. Then we have

$$\begin{aligned}\pi_D &= p_D(1+f)V - x_D \\ &= \frac{x_D}{x_D + \gamma x_G}(1+f)V - x_D \end{aligned} \tag{2}$$

and

$$\pi_G = p_G V + (1 - p_G)(-sV) - x_G$$

$$= \frac{\gamma x_G}{x_D + \gamma x_G} V - \frac{x_D}{x_D + \gamma x_G} s V - x_G \tag{3}$$

where $0 < f < 1$, $0 < s < 1$. The parameter f represents the reputation effect for agent D (that is, fame) if he wins the contest, and the parameter s represents that for agent G (that is, shame) if he loses. Assume that all of this is common knowledge. It should be noted that p_D appears in both agents' payoffs. Assume that the reservation payoff agents get from an outside option is zero, which plays a role of participation constraint.

First-order conditions for maximizing π_D and π_G can be reduced to

$$(1+f)(x_D + \gamma x_G) V - (1+f) x_D V = (x_D + \gamma x_G)^2 \tag{4}$$

and

$$\gamma(x_{D+} \gamma x_G) V - \gamma^2 x_G V + s \gamma x_D V = (x_D + \gamma x_G)^2 . \tag{5}$$

From (4) and (5), the following reaction functions can be obtained:

$$x_D = - \gamma x_G + \sqrt{(1+f)\gamma x_G V} \tag{6}$$

and

$$x_G = [- x_D + \sqrt{(1+s)\gamma x_D V}]/\gamma . \tag{7}$$

By solving the pair of simultaneous equations (6) and (7), the closed-form solution of the contest can be obtained. Here second-order sufficiency conditions are also satisfied.[6)]

Proposition 1 summarizes the results obtained above and their

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6) $\partial^2 \pi_D / \partial x_D^2 = \{-2(1+f)\gamma x_G V\} / \{x_D + \gamma x_G\}^3 < 0$ and $\partial^2 \pi_G / \partial x_G^2 = \{-2(1+s)\gamma^2 x_D V\} / \{x_D + \gamma x_G\}^3 < 0$. However, this does not automatically guarantee the existence of an interior solution.

relationships.

Proposition 1. (a) *Let* (x_D^*, x_G^*) *denote the interior Nash equilibrium if it exists. Then we have*

$$x_D^* = \frac{(1+f)^2(1+s)\gamma}{\{(1+f)+(1+s)\gamma\}^2}V,$$
$$x_G^* = \frac{(1+f)(1+s)^2\gamma}{\{(1+f)+(1+s)\gamma\}^2}V.$$

(b) *The relationship between the equilibrium effort level and the agent's reputation-corrected valuation of the prize can be represented as follows:*

$$\frac{x_D^*}{(1+f)V} = \frac{x_D^*}{(1+s)V} = \frac{(1+f)(1+s)\gamma}{\{(1+f)+(1+s)\gamma\}^2}.$$

Proposition 1 implies the following. First, from (a), the equilibrium effort level depends on the ability asymmetry parameter γ, reputation parameters f and s, and each agent's reputation-corrected valuation of the prize (that is, $(1+f)V$ and $(1+s)V$). Second, from (b), the agent who places higher value on reputation makes more effort in equilibrium because $x_D^* > x_G^*$ iff $f > s$ and $x_D^* < x_G^*$ iff $f < s$. More importantly, however, both agents allocate the same fraction of their reputation-corrected valuation to the contest. Therefore, if both parties think highly of their reputation, then the competition is bound to be aggravated.

The agent's winning probability and expected payoff are calculated. Equilibrium winning probabilities for the agents are

$$p_D^* = \frac{(1+f)}{(1+f)+(1+s)\gamma} \tag{8}$$

and

$$p_G^* = \frac{(1+s)\gamma}{(1+f)+(1+s)\gamma}. \tag{9}$$

Expected payoffs are

$$\pi_D^* = \frac{(1+f)^3}{\{(1+f)+(1+s)\gamma\}^2} V \tag{10}$$

and

$$\pi_G^* = \frac{(1+f)^2\gamma^2 - 2s(1+f)(1+s)\gamma - s(1+f)^2}{\{(1+f)+(1+s)\gamma\}^2} V. \tag{11}$$

For the participation of contest, equilibrium payoffs in equations (10) and (11) must be positive. Otherwise, the agent receiving a negative payoff chooses to exit the game, thereby destroying the equilibrium.

Here $\pi_D^* > 0$ is easily checked. The condition of equation (11) being positive can be reduced to the condition of the numerator being positive. Therefore, this require that $\xi = (1+s)^2\gamma^2 - 2s(1+f)(1+s)\gamma - s(1+f)^2$ is positive under the condition $\gamma > 1$.[7] According to the discriminant $s(1+f)^2(1+s)^3 > 0$, this quadratic equation has two real roots. And, from the constant term $-s(1+f)^2 < 0$, one is positive, and the other is negative. Therefore, this quadratic equation is always positive in the area greater than the positive root γ^*. Then we need to consider two subcases depending on the value of γ^* compared with the assumption $\gamma > 1$.

<u>Case 1</u>: $\gamma^* \leq 1 \quad \Leftrightarrow \quad 1+f \leq (1+s)/(s+\sqrt{s(1+s)})$.

7) Recall that the condition $\gamma > 1$ must be satisfied by assumption because this parameter represents Goliath's ability.

If $1+f \leq (1+s)/(s+\sqrt{s(1+s)})$, then $\xi > 0,\ \forall \gamma > 1$. This condition is denoted by Condition (a).

Condition (a): $(1+f) \leq (1+s)/(s+\sqrt{s(1+s)})$[8)]

<u>Case 2</u>: $\gamma^* \geq 1 \quad \Leftrightarrow \quad 1+f \geq (1+s)/(s+\sqrt{s(1+s)})$.

If $1+f \geq (1+s)/(s+\sqrt{s(1+s)})$, then $\xi > 0$ requires $\gamma > \gamma^*$, where $\gamma^* = \{(1+f)(s+\sqrt{s(1+s)})\}/(1+s)$. The condition is called Condition (b).

Condition (b): $(1+f) \geq (1+s)/(s+\sqrt{s(1+s)})$ and
$\gamma > \gamma^* = \{(1+f)(s+\sqrt{s(1+s)})\}/(1+s)$

Then, equilibrium payoff for Goliath is greater than zero iff Condition (a) or Condition (b) is satisfied. Intuitions behind these conditions are as follows. Condition (a) is satisfied for low values of the shame parameter s, and Goliath's expected payoff for this value is always positive. So, Goliath has a strong incentive to participate in the contest. On the other hand, Condition (b) is satisfied for high values of the shame parameter s. But in this case, it requires a high value of ability parameter γ in order for Goliath to participate in the contest, because the high ability compensates the high risk of shame.

Proposition 2 highlights the results obtained above.

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8) Characterizing the relevant values of f and s satisfying the condition $(1+f) \leq (1+s)/(s+\sqrt{s(1+s)})$ is not mathematically difficult but cumbersome. For a very small value of s, the condition is satisfied for almost all possible value of $f \in (0,1)$. As the value of s grows, the range of f satisfying the condition shrinks. Finally, for a larger value than a certain threshold, the condition is not satisfied for any value of $f \in (0,1)$. If the value of s goes beyond this point, then it is the realm of Condition (b).

Proposition 2. *Suppose Condition (a) or Condition (b) is satisfied. Then the contest between two players, D and G, has a unique pure strategy Nash equilibrium.*

(Sketch of) Proof

Since the expected payoff of both players is positive by the assumption (or by the participation constraint), any pure strategy equilibrium involves positive effort levels for both players. Proposition 1 has shown that there is a unique solution (x_D^*, x_G^*) to the first order conditions. In addition, the second order sufficiency condition for both players is, as shown in footnote 6, $\partial^2 \pi_D / \partial x_D^2 < 0$ and $\partial^2 \pi_G / \partial x_G^2 < 0$. Therefore, David is maximizing against x_G, and similarly for Goliath. Thus the sufficiency condition implies a unique pure strategy equilibrium. □

Proposition 2 has interesting implications. Reputation or popularity which is not equipped with solid ability makes the agent fear failure and thus avoid the contest. Here, the productive role of the shame parameter can be found. Shame after a failure is painful, but this encourages agent G to improve his ability, and this make G remain in the contest.

Consider the case of Apple in the smartwatch fight with Pebble and the competition in music-streaming services with Spotify. Apple is a latecomer with strong reputation. If it loses, it loses a lot. What drives Apple to the competition may be its superior technological ability and its effort to reduce the probability of failure.

Ⅲ. Comparative Statics

Basic comparative statics are conducted to analyze how efforts and payoffs respond to variations in contest parameters. First, examine the response of effort level to three parameters:

$$\frac{\partial x_D}{\partial f} > 0, \quad \frac{\partial x_D}{\partial s} \begin{matrix} > 0 & \text{if}(1+f)V > (1+s)\gamma V \\ < 0 & \text{if}(1+f)V < (1+s)\gamma V \end{matrix},$$

$$\frac{\partial x_D}{\partial \gamma} \begin{matrix} > 0 & \text{if}(1+f)V > (1+s)\gamma V \\ < 0 & \text{if}(1+f)V < (1+s)\gamma V \end{matrix},$$

and

$$\frac{\partial x_G}{\partial f} \begin{matrix} > 0 & \text{if}(1+f)V < (1+s)\gamma V \\ < 0 & \text{if}(1+f)V > (1+s)\gamma V \end{matrix}, \quad \frac{\partial x_G}{\partial s} > 0,$$

$$\frac{\partial x_G}{\partial \gamma} \begin{matrix} > 0 & \text{if}(1+f)V > (1+s)\gamma V \\ < 0 & \text{if}(1+f)V < (1+s)\gamma V \end{matrix}.$$

This proves the next proposition.

Proposition 3. (a) *The effort level of agent D increases with his fame parameter. The effort level of agent D increases with the shame parameter and ability asymmetry parameter of agent G, as long as the reputation-corrected value for agent D is greater than the reputation-ability-corrected value for agent G.*
(b) *The effort level of agent G increases with his shame parameter. The effort level of agent G increases with the fame parameter for agent D, as long as reputation-ability-corrected value for agent G is greater than the reputation-corrected value agent D. However, the effort level of agent G decreases with the ability asymmetry parameter if his reputation-ability-corrected value is greater than the reputation-corrected value for agent D.*

A brief explanation is provided as follows: Agent D has little to lose even if he fails, but he gains substantial fame if he succeeds.

Therefore, the greater amount of fame is enjoyed by agent D, the higher level of effort he makes. In addition, agent D responds to variations in other parameters by increasing the level of his effort, as long as his gain exceeds that of G. The problem of agent G is also similar. The greater the shame agent G faces when he fails, the higher the level of effort he expends to keep his reputation. Agent G increases his effort level even if the level of fame increases for D, as long as his gain is larger than that of D. However, agent G reduces his effort in response to his increased ability if his gain is greater than that of D because D also reduces his effort level.

Finally, the response of expected payoffs to parametric variations is examined.

$$\frac{\partial \pi_D}{\partial f} > 0\,, \quad \frac{\partial \pi_D}{\partial s} < 0\,, \quad \frac{\partial \pi_D}{\partial \gamma} < 0$$

and

$$\frac{\partial \pi_G}{\partial f} < 0\,, \quad \frac{\partial \pi_G}{\partial s} < 0\,, \quad \frac{\partial \pi_G}{\partial \gamma} > 0\,.$$

This proves the next Proposition.

Proposition 4. (a) *The expected payoff of agent D increases with his fame parameter but decreases with the shame parameter and the ability asymmetry parameter for agent G.*
(b) *The expected payoff of agent G decreases with his shame parameter and the fame parameter for agent D, but increases with the ability asymmetry parameter for agent G.*

Note that an increase in the fame parameter for D and the ability asymmetry parameter for G has the opposite effect on the agents' expected payoffs. For instance, an increase in the level of fame for agent D is beneficial to him but has an adverse effect on agent G. Noteworthy is that an increase in the shame parameter for G has a negative effect on both agents. An increase in the shame parameter has a direct negative effect on the expected payoff of G, and an increase in the shame parameter makes agent G expend more effort, indirectly leading to a negative effect on the expected payoff of D.

Ⅳ. Conclusions

This paper has considered a model of a contest between two asymmetric agents in which reputation effects work in different directions, characterizes the conditions for a unique pure-strategy Nash equilibrium. From this analysis, the structure of interactions between fame, ability and shame in the contest is further understood. For instance, reputation without solid ability makes the agent fear failure and thus avoid the contest. And, the shame parameter plays a productive role. Shame after a failure is painful, but this encourages agent G to improve his ability and remain in the contest. Finally, comparative statics are considered to examine how efforts and expected payoffs respond to variations in contest parameters.

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다윗과 골리앗의 경쟁

이 용 주*

논문초록

본 논문은 경기자간의 평판효과가 다른 방향으로 작용하는 비대칭적 경쟁을 모형화한다. 능력이 뒤쳐지는 다윗의 경우, 경쟁에서 승리하면 엄청난 명성을 얻게 되지만 지더라도 잃을 것이 거의 없다. 반면, 골리앗의 경우, 이기는 것은 당연한 결과로 여겨지지만, 만약 지게 되면 엄청난 치욕에 직면하게 된다. 본 논문은 이러한 비대칭적 상황에서 유일한 순수전략 내쉬균형이 존재하게 되는 조건을 구하고, 비교정학을 통해 파라미터의 변화에 따른 노력수준과 기대효용의 변화를 살펴본다. 이러한 분석을 통해 명성, 능력, 치욕과 노력수준간의 상관관계 구조를 직관적으로 이해하게 된다.

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