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Monopoly Quality Differentiation with Top-quality Dependent Fixed Costs

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Abstract

This paper analyzes monopoly quality differentiation in an environment where production of quality-differentiated goods involves fixed investment costs that depend solely on the level of the highest quality. The presence of such top-quality dependent fixed costs always leads to a pooling of some high-type customers. In contrast with the standard model, all consumer types (including the highest type) experience quality distortion, and the firm may offer a product line which is narrower than the efficient one.

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Ⅰ. Introduction

The seminal paper by Mussa and Rosen (1978) provided us with a good understanding of the optimal strategy of a monopolist selling a range of vertically differentiated goods. They study a market where a good can be produced in a number of quality varieties, so-called a

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product-line. Consumers buy at most one unit among a variety of qualities. The unit production cost is constant for a given quality, which is increasing in quality. There is no fixed costs. They analyze how a monopolist price-discriminates consumers by offering an array of qualities. The main findings are i) the optimal strategy involves downward quality distortions except for the highest quality (*efficiency-on-the-top*), ii) the monopolist enlarges the quality range toward the low end of quality, relative to the efficient outcome (*downward enlargement of quality spectrum*).

This paper extends the standard model to an environment where the provision of a product-line requires an initial fixed investment in common assets, which are shared over the entire product-line. This initial investment may be necessary for production facilities, distribution channels, or more generally R&D of core technologies. The cost of common assets depends only on the highest of the quality spectrum. Notable examples are network industries such as telecommunications. To provide various communication services such as voice, data, and video, a firm needs to construct a transmission network and switching facilities, which are usually shared overall different types of services. The quality of the common facilities determines the range of quality the firm can provide. A network constructed for voice telephony is not capable of delivering moving images properly. The opposite, however, is feasible almost costless. The same interpretation is possible when communication services are differentiated by transmission speed or reliability. Extending the concept of common assets to intangible assets such as technologies, the framework can be easily applied to all kinds of high-tech industries. Once a firm has invested in a technology capable of producing a certain quality, it can produce products of lower qualities by using or degrading existing technologies, but not conversely.

We analyze the effect of top-quality dependent fixed costs on monopoly quality differentiation. The presence of top-quality dependent fixed costs creates a public good-like feature over a range of high qualities, restraining the firm from complete screening of consumers. As a result, bunching always appears in a range of high types. This bunching differs from those observed in the standard model due to some irregularity of demand conditions (e.g. a violation of the monotonic hazard rate condition). At the optimum, all consumers, including the highest type, are provided with inefficiently low qualities, in contrast with the 'efficiency-on-the top' in the standard model. We also compare the profit-maximizing and first-best quality provisions in respect of product diversity.

Closely related are papers by Gabszewicz *et al*. (1986) and Shitovitz *et al*. (1989). Gabszewicz *et al*. study how a natural monopolist chooses the optimal quality range. They assume a unit variable cost of quality which rises only slowly with improvements in quality (a feature leading to the 'natural' monopoly). In this case, bunching generally occurs in a range of high types when the dispersion of consumer willingness-to-pay is small compared with the feasible quality range. At the extreme, the monopolist may choose to offer only a single quality, the highest in the feasible range. Shitovitz *et al*. examine a similar problem with a finite number of consumers, and show that offering the top quality only is optimal when both the utility function and the marginal cost of quality are linear. In both cases, the authors rely on an initially-given range of feasible qualities to obtain the bunching result. However, it is not obvious why quality range is given exogenously.[1)]

1) Also, there has been a line of research on relaxing Mussa-Rosen's assumptions on consumers preferences. Srinagesh and Bradburd (1989) examined a case where there is a negative correlation between total and marginal utility of quality across consumer types (the opposite of the single-crossing condition). They show that quality may be enhanced, not

Ⅱ. The Model

Consider a monopolist producing a quality-differentiated spectrum of goods. Quality is measured by a one-dimensional attribute q. To provide a range of qualities, the firm needs to invest in assets, which are shared over all qualities. The investment cost, which depends only on the top quality q_t, is given by $I(q_t)$ where $I(0)=0, I'(\cdot)>0$ and $I''(\cdot)>0$. Once the firm has invested in the common assets capable of producing quality up to q_t, it can provide goods of lower quality $\hat{q}\leq q_t$ only with its unit provision cost. The constant marginal cost for quality q is $c(q)$, where $c(0)=0$, $c'(\cdot)>0$ and $c''(\cdot)>0$. There is a continuum of consumers, whose type is represented by a one-dimensional parameter $\theta\in[\underline{\theta},\overline{\theta}]$ with $\underline{\theta}>0$. Here θ is private information and distributed according to distribution function $F(\theta)$ with density $f(\theta)$. Consumers have unit demands. A type-θ consumer, when purchasing a good of quality q, gets a gross surplus of θq. Hence θ is her marginal willingness-to-pay for quality.

We first characterize the first-best outcome as a benchmark. Suppose that the firm has complete information about θ. It is

degraded, at the higher ends of quality while there is no distortion at the lowest quality. Donnenfeld and White (1988) have obtained a similar result. Srinagesh *et. al.* (1992) considered a situation where consumers' reservation utility is increasing with the marginal utility of quality, and show that the profit-maximizing strategy can involve bidirectional distortions: quality degradation at the low end of the spectrum and quality enhancement at the high end. Also, de Meza (1997) considered the possibility that the highest type's participation constraint is binding, and shows that a monopolist may have the incentive to boost top-of-the-range quality above the efficient level. Rochet and Stole (2002) proposed a more general framework where consumers have random outside options. They find that the profit-maximizing quality provision produces either no distortion on the boundaries in a fully separating equilibrium or efficiency-on-the-top with bunching over a range of low types. See also the excellent survey on nonlinear pricing by Stole (2008).

convenient to solve the problem in two stages. First, the firm invests in common assets incurring costs of $I(q_t)$, and then chooses individual quality allocations subject to the given top quality q_t. Given q_t the second-stage individual quality allocation problem is:

$$S(q_t) = \max \int_{\underline{\theta}}^{\overline{\theta}} [\theta q(\theta) - c(q(\theta))] f(\theta) d\theta \tag{1}$$

subject to $q(\theta) \leq q_t$ for all θ. Define θ^* such that $\theta^* = c'(q_t)$. The pointwise maximization of the integrand subject to $q(\theta) \leq q_t$ yields:

$$q^f(\theta) = \begin{cases} 0 \ for \ \theta < \alpha \\ q \ such \ that \ \theta = c'(q) \ for \ \alpha \leq \theta \leq \theta^* \\ q_t for \ \theta > \theta^* \end{cases} \tag{2}$$

where α is the exclusion point such that $\alpha = c'(0)$. The consumers of type less than α are excluded from the market. The quality allocation is fully separating if $\overline{\theta} \leq \theta^*$, completely pooling if $\theta^* \leq \underline{\theta}$, and partially pooling if $\underline{\theta} < \theta^* < \overline{\theta}$. The maximum profit (or equivalently social welfare) is then

$$\begin{aligned} B(q_t) &\equiv S(q_t) - I(q_t) \\ &= \int_{\underline{\theta}}^{\theta^*(q_t)} [\theta q^f(\theta) - c(q^f(\theta))] f(\theta) d\theta \\ &\quad + \int_{\theta^*(q_t)}^{\overline{\theta}} [\theta q_t - c(q_t)] f(\theta) d\theta - I(q_t). \end{aligned}$$

Define $\overline{q}$ such that $\overline{\theta} = c'(\overline{q})$, i.e. the efficient quality for type $\overline{\theta}$. A useful fact is that the first-best top quality is strictly less than $\overline{q}$. From (2) it is obvious that increasing quality beyond $\overline{q}$ does not affect S. Then, $B(\cdot)$ is strictly decreasing in $[\bar{q}, \infty)$ given $I'(\cdot) > 0$. Furthermore, the first-order effect of reducing top quality from $\bar{q}$ on S is negligible. So it is always profitable to reduce the top quality from $\bar{q}$.

Proposition 1 At the first-best optimum, bunching always occurs in a range of high types ($\theta^* \le \bar{\theta}$).

Now we choose the first-best top-quality. Since the function $B(\cdot)$ is concave, the first-best top-quality q_t^f is determined by the first-order condition:

$$\int_{\theta^*(q_t^f)}^{\bar{\theta}} \theta f(\theta) d\theta = [1 - F(\theta^*(q_t^f))] c'(q_t^f) + I'(q_t^f) \tag{3}$$

A marginal increase of top-quality has no effect on quality allocation for consumers of type $\theta \le \theta^*(q_t)$, but the increased top quality is offered to those of type $\theta > \theta^*(q_t)$. The LHS of (3) denotes the sum of the surplus increases over the high types in the bunching region, and the RHS the corresponding cost of quality increase (the sum of the incremental marginal and fixed costs). The way the optimal top-quality is determined is similar to capacity choice in peak-load pricing. Also, condition (3) resembles the condition for the efficient provision of a price-excludable public good. The nonrivalry of common assets across all feasible qualities creates a public-good like feature, although each product here is actually rivalrous in consumption.

Ⅲ. Incomplete information

Now suppose the firm cannot distinguish consumer type. Given tariff $t(q)$, the net surplus a type-θ consumer obtains is

$$v(\theta) = \max : \theta q - t(q). \qquad (4)$$

Given q_t, the firm solves the individual quality allocation problem. Define the revenue function (including marginal costs) as

$$R(q_t) = \max \int_{\underline{\theta}}^{\overline{\theta}} [t(q(\theta)) - c(q(\theta))] f(\theta) d\theta$$

subject to

i) $v'(\theta) = q(\theta)$ and $q'(\theta) \geq 0$ (incentive compatibility constraints)

ii) $v(\theta) \geq 0$ (participation constraints)

iii) $q(\theta) \leq q_t$ (top-quality constraints).

It must be that $v(\underline{\theta}) = 0$ at the optimum, since the firm offers zero utility to the lowest type. Substituting t using (4) and eliminating v by integration by parts, we obtain

$$R(q_t) = \max \int_{\underline{\theta}}^{\overline{\theta}} [(\theta - \frac{1 - F(\theta)}{f(\theta)}) q(\theta) - c(q(\theta))] f(\theta) d\theta \qquad (5)$$

subject to $q'(\theta) \geq 0$ and $q(\theta) \leq q_t$. We will initially ignore the monotonicity constraint, and later show that it is satisfied at the optimum under a certain condition. Maximizing the integrand pointwise subject to the top-quality constraint, the optimal quality schedule is given by

$$q^{\pi}(\theta)=\begin{cases}0\,for\ \ \theta<\beta\\ q\,such\ \ that\ \ \theta-\dfrac{1-F(\theta)}{f(\theta)}=c'(q)\ \ for\ \ \beta\leq\theta\leq\theta^{**}\\ q_t\,for\ \ \theta>\theta^{**}\end{cases}\quad(6)$$

where β is the exclusion point such that $\beta-\dfrac{1-F(\beta)}{f(\beta)}=c'(0)$, and θ^{**} is such that $\theta^{**}-\dfrac{1-F(\theta^{**})}{f(\theta^{**})}=c'(q_t)$. The consumers of types less than β are excluded from the market, and there can be a bunching in a range of high types if $\theta^{**}<\overline{\theta}$. The quality schedule is nondecreasing as required under the condition that

$$\theta-\frac{1-F(\theta)}{f(\theta)}\ \text{ is increasing in }\theta. \quad (7)$$

Using the optimal quality schedule in (6), the maximum profit is given by

$$\begin{aligned}\Pi(q_t)&\equiv R(q_t)-I(q_t)\\ &=\int_{\underline{\theta}}^{\theta^{**}(q_t)}[(\theta-\frac{1-F(\theta)}{f(\theta)}q^{\pi}(\theta)-c(q^{\pi}(\theta))]f(\theta)d\theta\\ &\quad+\int_{\theta^{**}(q_t)}^{\overline{\theta}}[(\theta-\frac{1-F(\theta)}{f(\theta)})q_t-c(q_t)]f(\theta)d\theta-I(q_t).\end{aligned}$$

The profit-maximizing top quality is strictly less than $\overline{q}$ such that $\overline{\theta}=c'(\overline{q})$. An increase of top quality beyond $\overline{q}$ only incurs additional costs without increasing the firm's revenue. The top-quality dependent fixed cost, however small it is, restricts the firm from fully screening consumers, and therefore bunching generally appears.

Proposition 2 At the profit-maximizing optimum, bunching always occurs in a range of high types ($\theta^{**} < \bar{\theta}$).

The profit-maximizing top quality q_t^{π} must satisfy the first-order condition:

$$\int_{\theta^{**}(q_t^{\pi})}^{\bar{\theta}} (\theta - \frac{1-F(\theta)}{f(\theta)}) f(\theta) d\theta = [1 - F(\theta^{**}(q_t^{\pi}))] c'(q_t^{\pi}) + I'(q_t^{\pi}) \quad (8)$$

$$\Rightarrow \theta^{**}(q_t^{\pi})[1 - F(\theta^{**}(q_t^{\pi}))] = [1 - F(\theta^{**}(q_t^{\pi}))] c'(q_t^{\pi}) + I'(q_t^{\pi}), \quad (9)$$

which is sufficient since the profit function is concave. The quality allocations for $\theta \le \theta^{**}(q_t)$ are not affected by increasing top quality over q_t. The LHS of (9) denotes the increase in marginal revenue collected from consumers of type $\theta > \theta^{**}(q_t)$, and the RHS the corresponding incremental cost. A change of q_t affects the value of θ^{**}, but its effect on the firm's revenue is negligible at the margin. With incomplete information on consumer type, the firm determines the top-quality level based on the marginal consumer (type-θ^{**})'s marginal valuation for quality rather than the average marginal valuation for quality across consumers of types higher than θ^{**} who purchase the top quality. In such a partially pooling equilibrium, the quality allocation exhibits a kinked pattern. A complete pooling equilibrium may occur if the unit production cost increases slowly in quality improvements so that $\theta^{**}(q_t^{f}) \le \underline{\theta}$, as found by Gabszewicz *et al*. (1986).

Now we compare the profit-maximizing solution with the first-best outcome.

Proposition 3 i) The profit-maximizing top-quality is less than the first-best top quality ($q_t^{\pi} < q_t^{f}$). ii) All consumer types, including the highest type, experience quality distortion ($q^{\pi}(\theta) < q^{f}(\theta)\ \forall\ \theta > c'(0)$).[2]

With top-quality dependent fixed costs, even the highest type faces downward quality distortion, in contrast with the efficiency-on-the-top result in the standard model. Here two different sources of inefficiency are involved. The top-quality under-provision for a range of high types is associated with the generic inefficiency in the monopoly quality provision with incomplete information. Quality distortions for the other low types are a consequence of consumer screening.

Another interesting question is how the monopolist's choice of product diversity compares with the social optimum. Two different results emerge depending on whether the lowest type is served or excluded in the first-best.

Corollary 4 If $\underline{\theta} \leq c'(0)$, the firm provides a less diverse product line relative to the first-best outcome. Otherwise, the comparison of quality diversity is ambiguous.

First, suppose that $q^{f}(\underline{\theta}) = 0$. It is clear from the definitions of α and β that $q^{\pi}(\underline{\theta}) = 0$ as well. Then, given that $q_t^{\pi} < q_t^{f}$, the profit-maximizing quality spectrum is completely nested in the first-best quality spectrum, with the shrink occurring from the high end of the quality spectrum. So, a less diverse product line is offered relative to the first best outcome. Second, suppose that $q^{f}(\underline{\theta}) > 0$. Then we have $q^{\pi}(\underline{\theta}) < q^{f}(\underline{\theta})$ and $q_t^{\pi} < q_t^{f}$. Given that $q^{\pi}(\theta)$ and $q^{f}(\theta)$ are nondecreasing in $[\underline{\theta}, \overline{\theta}]$, we cannot say which regime provides a greater diversity.

2) The proof is omitted and can be provided on request.

Special cases with complete pooling: In many network industries (e.g. Internet service provision and cable TV), the unit provision cost is linear in quality or is a nonnegative constant independent of quality. Not surprisingly, in such cases we have a complete pooling in equilibrium both in the first-best and the profit-maximization regimes. A simple analysis shows that the social planer and the firm will provide a fixed quality to the types higher than or equal to some exclusion point and exclude all the other low types. The monopolist serves a smaller market relative to the first-best equilibrium. Also, the profit-maximizing quality is smaller than the first-best quality. (The proof can be offered on request.)

Ⅳ. Conclusion

This paper extended the standard quality discrimination model to an environment with top-quality dependent fixed costs. It has been shown that, in contrast with the standard model, i) pooling always appears in a range of high types, due to the public-good like feature in the provision of top quality, ii) all types of consumers, even the highest type, are served inefficiently low qualities in the profit-maximizing equilibrium, and iii) the firm may provides a less diverse product line relative to the first-best outcome. This result sheds some light on quality differentiation in industries subject to top-quality specific sunk investments prior to production of an array of qualities.

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◈ *References* ◈

De Meza, D. (1997), "Product Diversity under Monopoly: Two High Quality Results," *Bulletin of Economic Research*, 49, 169-171.

Donnenfeld, S. and L. J. White (1988), "Product Variety and the Efficiency of Monopoly," *Economica*, 55, 393-401.

Gabszewicz, J., Shaked, A., Sutton, J. and J. Thisse (1986), "Segmenting the Market: The Monopolist's Optimal Product Mix," *Journal of Economic Theory*, 39, 273-289.

Mussa, M. and S. Rosen (1978), "Monopoly and Product Quality," *Journal of Economic Theory*, 18, 301-317.

Rochet, J. C. and L. A. Stole (2002), "Nonlinear Pricing with Random Participation Constraints," *Review of Economic Studies*, 69, 277-311.

Shitovitz, B., Spiegel, M. and S. Weber (1989), "Top of the Line' Quality as an Optimal Solution," *European Economic Review*, 33, 1641-1652.

Srinagesh, P. and R. Bradburd (1989), "Quality Distortion by Discriminating Monopolist," *American Economic Review*, 79, 96-105.

Srinagesh, P., R. Bradburd and H.-W. Koo (1992), "Bidirectional Distortion in Self-selection Problems," *Journal of Industrial Economics*, 40(2), 223-228.

Stole, L. (2008), Price Discrimination in Competitive Environments, in the Handbook of Industrial Organization, Vol III.

독점기업의 품질차별화 전략: 최상위 품질에만 의존하는 고정비용이 존재하는 경우

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논문초록

본 논문은 생산에 따른 고정비용이 최상위 품질수준에만 의존하는 경우에 독점기업의 품질차별화 전략을 분석하였다. 이와 같은 비용구조의 변화는 품질에 대한 선호도가 높은 소비자그룹의 번칭(bunching)을 유도하고, 모든 소비자들에게 비효율적인 품질수준을 제공하며, 경우에 따라 사회적 최적에 비해 품질수준의 다양성을 낮추는 결과를 초래한다. 이는 고정비용을 전혀 고려하지 않는 기존 모형 에서 도출되는 완전한 소비자들의 완전한 스크리닝(full screening), 선호도가 가장 높은 소비자에 대한 최적 품질 제공(efficiency-on-the-top), 사회적 최적에 비해 다양한 품질수준 등의 결과와 대비된다.

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