한국경제학보 제22권 제1호
The Korean Journal of Economics Vol. 22, No. 1 (Special Issue 2015)

A Test on the Normality in the Tobit Model*

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Abstract

We propose an alternative Hausman test for normality in the Tobit model. Unlike the previous tests by Ruud (1984) or Newey (1987), our test compares the Tobit estimator to an estimator which is consistent *both* under the null hypothesis and under the alternative hypothesis. To do so, it utilizes a purely nonparametric estimator for the Tobit model proposed by Newey (1999) and Jeong (2004).

KRF Classification : B030104
Keywords : Tobit Model, Normality Test, Bootstrap

Ⅰ. Introduction

The maximum likelihood estimation of censored regression model has been named 'Tobit' after Tobin (1958). It is well known that the validity of the Tobit estimator depends on the assumption of

* We are grateful for the helpful comments on the earlier draft from Graduate Seminars on Contemporary Economics at Yonsei University. The authors are members of 'Brain Korea 21' Research Group of Yonsei University. This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2013S1A3A2053586).
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normality. Arabmazar and Schmidt (1982) and Goldberger (1983) among others show that the Tobit estimator becomes inconsistent when the normal distribution assumption of the disturbance is not satisfied. A number of alternative tests for the normality assumption in the Tobit model have been suggested. One direction of the studies is the Lagrange Multiplier, or score test. Lin and Schmidt (1984), Bera, Jarque and Lee (1984), and Chesher and Irish (1987) propose various types of score tests. Another line of research is Hausman test approach. Nelson (1981) first proposes a Hausman test for the normality in the Tobit model. He compares a method of moment estimator and the maximum likelihood estimator to construct a Hausman test.[1] Ruud (1984) uses a probit estimator, truncated regression estimator, and the Tobit estimator in constructing a similar pseudo-Hausman test.[2] Newey (1987) proposes the same Hausman-type test using the difference between Powell's (1986) symmetrically censored least squares (SCLS) estimator and the Tobit estimator.

However, the finite sample performance of the Hausman tests for normality is far from perfect. Ericson and Hansen (1999) examine the finite sample performance of Newey's (1987) Hausman test through a Monte Carlo simulation. Even with 640 observations, Newey's test severely over-rejects the true null hypothesis, and significantly under-rejects the false null hypothesis. For example, it rejects 2.7% of the true null (at 1% nominal size), and rejects only 1.7% of the false null (t_3 distribution). Holden (2004) evaluates the performance of Ruud's (1984) Hausman test which compares the probit estimator

1) Nelson's test is not exactly Hausman test because his second estimator is not consistent under the alternative hypothesis. Although a Hausman-type test could be defined with an inconsistent second estimator, the original Hausman test statistic uses a consistent estimator under both the null hypothesis and the alternative hypothesis for the second estimator.

2) Nelson's (1981) test could be interpreted as a special case of Ruud's (1984).

and the Tobit estimator. The size turns out inaccurate and the power is very low. For example, when the sample size is medium (150 observations), the rejection rate of the true null is 6.05% (5% nominal size) and the rejection rate of the false null (t distribution) is 14.35%. Even when the sample size is large (900 observations), the size and power are not satisfactory.

The size distortion and power reduction of the Hausman tests are probably due to the bad design of the test statistics. Ruud's test compares the probit estimator and the Tobit estimator. Both the probit and Tobit estimators are consistent under the normality (H_0). However, under nonnormailty (H_1), *both* the estimators are inconsistent. The original Hausman test compares two estimators, one of which is consistent only under H_0, and the other of which is consistent both under H_0 and H_1. By doing so, the Hausman test statistic is bounded in probability under H_0, and is guaranteed to diverge under H_1. Of course, as Nelson (1981) and Ruud (1984) argue, it is possible that Hausman test could be well defined even with estimators both are inconsistent under H_1. If the probability limits of the estimators are different under H_1, Hausman test is feasible. However, the small sample performance depends on the actual difference of the two estimates even in such a case.

Newey's test also has a limitation. Newey's test employs the SCLS estimator for the second estimator to be compared with the Tobit estimator. It is well known that the SCLS estimator is not consistent when the error distribution is asymmetric. Thus, if the underlying distribution happens to be an asymmetric nonnormal distribution, just like Ruud's test, *both* the SCLS estimator and the Tobit estimator are inconsistent under H1. Again, of course, asymptotically Hausman test has no problem if the probability limits of the estimators are different under H_1. However, the finite sample properties of the test could be distorted as shown by Ericson and Hansen (1999).

This paper proposes an alternative Hausman test for normality in the Tobit model. As the proposed test does not have the problems that Ruud's test or Newey's test, its finite sample properties are expected to be better than those earlier tests. It utilizes a purely nonparametric estimator for the Tobit model to avoid the inconsistency of the second estimator under H_1.

Ⅱ. A Nonparametric Estimation of Tobit Model

Let us consider the following (generalized) Tobit model. However, the test can also be applied to the simpler version of Tobit model where $X_i = Z_i$.[3)]

$$y_i^* = X_i\beta + u_i \qquad i = 1, 2, \cdots, n \tag{1}$$

$$d_i^* = Z_i\gamma + v_i \tag{2}$$

$$y_i = y_i^* \text{ and } d_i = 1 \quad \text{if} \quad d_i^* \geq 0$$

$$y_i = d_i = 0 \qquad \text{if} \quad d_i^* < 0$$

where X_i has k variables, Z_i has m variables, and (u_i, v_i) is assumed to have a serially independent bivariate normal distribution with a mean of zero and a variance matrix of $\begin{bmatrix} \sigma^2 & \sigma_{uv} \\ \sigma_{uv} & 1 \end{bmatrix}$ for all i. With the normality assumption, the model can be estimated by the standard MLE, to produce the so-called 'Tobit' estimators.

To construct a Hausman test, we need a nonparametric estimator to be compared with the Tobit estimator. This paper uses the nonparametric estimator introduced by Jeong (2004). The estimator is

3) This simple Tobit model is sometimes called 'standard' Tobit model or 'Type 1' Tobit model after Amemiya (1985).

based on the generalized residual. The generalized residual, developed by Gourieroux, et al. (1987) and Chesher and Irish (1987), is defined to replace the unobservable residual in latent variable models. It is simply the (estimated) conditional expectation of the error given the observed data. Formally, the generalized residual, $\tilde{e}_i$, is defined as:

$$\tilde{e}_i \equiv E_{\hat{\theta}}(e_i \mid y_i) \tag{3}$$

where $\hat{\theta}$ is an asymptotically efficient estimator of the parameter vector of the model, θ. The generalized residual turns out to be useful in estimation of limited dependent variable models. When the errors of the 'observed' model have non-zero mean due to the limited observability in the model, the non-zero mean is often a source of bias in estimation. In such cases, the generalized residual can capture the bias caused by the non-zero mean of errors and make the model estimable. In the above Tobit model, if equation (1) is estimated by OLS using only the uncensored data ($d_i^* \geq 0$), the estimate of β will be biased because the conditional mean of u_i is not zero. However, the conditional expectation, $E(u_i \mid d_i^* \geq 0)$, is a linear function of the generalized residual ($\tilde{v}_i$) where $\tilde{v}_i \equiv E(u_i \mid d_i^* \geq 0)$. Thus, the error term ($e_i$) of the following regression will have a zero mean, and the equation can be estimated by OLS.

$$y = X_i\beta + \sigma_{uv}\tilde{v}_i + e_i \tag{4}$$

The functional form of $\tilde{v}_i$ depends on the underlying distribution. Assuming normality, the generalized residual becomes the well-known 'inverse Mill's ratio.'

$$\tilde{v}_i = \frac{\phi(Z_i\hat{\gamma})}{\Phi(Z_i\hat{\gamma})} \tag{5}$$

where ϕ is the pdf of the standard normal distribution, and Φ is the cdf of the standard normal distribution. In this case, the estimation of equation (4) becomes the *Heckit* or Heckman's two-step estimation.[4)]

If the normality assumption is not satisfied, the generalized residual will take an unknown form. Jeong (2004) proposes the following procedure. First, the selection equation (2) is estimated by some nonparametric estimation method, such as maximum score estimation by Manski (1975), a distribution-free maximum likelihood estimation by Cosslett (1983), smoothed maximum score estimation by Horowitz (1992), or single index estimation by Klein and Spady (1993) and Ichimura (1993), etc. Using the estimates of γ, an approximation of the conditional expectation of u_i in equation (1) is specified as follows.

$$\hat{g}_i \equiv E(u_i \mid d_i^* \geq 0) \approx \alpha_0 + \alpha_1(Z_i\hat{\gamma}) + \alpha_2(Z_i\hat{\gamma})^2 \tag{6}$$

This second-order approximation is a simpler version of the one in Newey (1999). In the second stage, $\hat{g}_i$ replaces the generalized residual term ($\sigma_{uv}\tilde{v}_i$) in equation (4).

$$y_i = x_i\beta + \hat{g}_i + e_i \tag{7}$$

Equation (7) is estimated by OLS, and then the standard error of the

4) If X and Z have exactly the same variables, β is only identified by the nonlinearity of the generalized residual. However, as the generalized residual is often close to 'linear,' such practice may create identification problem. See Little (1985).

estimated β, $\hat{\beta}_{NP}$, is computed through bootstrapping.[5] The bootstrap is employed for refined finite sample properties, although Newey (1999) derives the asymptotic variance of $\hat{\beta}_{NP}$ under somewhat strong regularity conditions.[6] The consistency and asymptotic normality of the estimator are also shown by Newey (1999). Jeong (2004) finds that the estimator performs well in finite samples, and that it has more accurate sizes and higher powers than the traditional ones when the underlying distribution is nonnormal.

Ⅲ. Test Statistics for Normality

For a test on the normality of the censored regression model, we construct a Hausman test statistic with the above nonparametric estimator, $\hat{\beta}_{NP}$, and the traditional Tobit estimator, $\hat{\beta}_{Tobit}$. It should be emphasized that is consistent *both* under H_0 and H_1, unlike the estimators Ruud (1984) and Newey (1987) use. As explained in the introduction, the Tobit estimator is consistent and efficient under H_0, and inconsistent under H_1. The Hausman statistic is:

$$H = n(\hat{\beta}_{NP} - \hat{\beta}_{Tobit})' \hat{V}^{-1} (\hat{\beta}_{NP} - \hat{\beta}_{Tobit}) \tag{8}$$

where $\hat{V}$ is an estimator of the asymptotic variance (V) of the difference, $\sqrt{n}(\hat{\beta}_{NP} - \hat{\beta}_{Tobit})$. Applying Theorem 1 of Newey (1999), it would be straightforward to derive a chi-square distribution for H. Instead of the asymptotic distribution, however, we employ the

5) Note that the intercept term in β is not identified from α_0. Only $(\beta_0 + \alpha_0)$ can be identified. Vella (1998) discusses some possible measures for separating these two intercept terms.

6) Actually, Newey's derivation is for a bigger set of estimator than $\hat{\beta}_{NP}$.

bootstrap distribution of H for the test of normality, to improve its finite sample properties. We estimate V also through a nested bootstrap. Thus, the test procedure involves a double bootstrap. First, the bootstrap estimate of V, $\hat{V}_B$, is constructed by a nested bootstrap, and then H_B is bootstrapped again in the second stage, to produce the critical values of the normality test, where $H_B = n(\hat{\beta}_{NP} - \hat{\beta}_{Tobit})' \hat{V}_B^{-1} (\hat{\beta}_{NP} - \hat{\beta}_{Tobit})$.

As the double bootstrap requires a considerable computing time, we propose an alternative bootstrap procedure. When we apply bootstrap distribution, it is not actually necessary to normalize the statistic with $\hat{V}$. Instead, the squared sum of the differences, $(\hat{\beta}_{NP} - \hat{\beta}_{Tobit})'(\hat{\beta}_{NP} - \hat{\beta}_{Tobit})$, can be bootstrapped for the test.

Ⅳ. Conclusion

We propose an alternative Hausman test for normality in the Tobit model. Unlike the previous tests by Ruud (1984) or Newey (1987), our test compares the Tobit estimator to an estimator which is consistent *both* under the null hypothesis and under the alternative hypothesis. As the inconsistency of the estimator under H1 is a potential source of distorted finite sample properties of the previous tests, the proposed test is expected to have better small sample performance. To evaluate the performance of the proposed test, a thorough Monte Carlo study is necessary. We leave the extensive simulations for a further research.

Received: March 26, 2015. Revised: April 18, 2015. Accepted: April 25, 2015.

◈ *References* ◈

Amemiya, T. (1984), "Tobit Models: A Survey," *Journal of Econometrics*, 24, pp. 3-61.

Arabmazar, A. and P. Schmidt (1982), "An Investigation of the Robustness of the Tobit Estimator to Non-normality," *Econometrica*, 50, pp. 1055-1063.

Bera, A. K., C. M. Jarque and L. Lee (1984), "Testing the Normality Assumption in Limited Dependent Variable Models," *International Economic Review*, 25, pp. 563-578.

Chesher, A. D. and A. Irish (1987), "Numerical and Graphical Residual Analysis in the Grouped and Censored Normal Linear Model," *Journal of Econometrics*, 34, pp. 33-54.

Cosslett, S. R. (1983), "Distribution-Free Maximum Likelihood Estimator of the Binary Choice Model," *Econometrica*, 51, pp. 765-782.

Ericson, P. and J. Hansen (1999), "A Note on the Performance of Simple Specification Tests in the Tobit Model," *Oxford Bulletin of Economics and Statistics*, 61, pp. 121-127.

Goldberger (1983), "Abnormal Selection Bias," in S. Karlin, T. Amemiya, and L. Goodman, eds., *Studies in Econometrics, Time Series and Multivariate Statistics*, Academic Press, New York.

Gourieroux, C., A. Monfort, E. Renault, and A. Trognon (1987), "Generalised Residuals," *Journal of Econometrics*, 34, pp. 5-32.

Holden, D. (2004), "Testing the Normality Assumption in the Tobit Model," *Journal of Applied Statistics*, 31, pp. 521-532.

Horowitz, J. L. (1992), "A Smoothed Maximum Score Estimator for the Binary Response Model," *Econometrica*, 60, pp. 505-531.

Ichimura, H. (1993), "Semiparametric Least Squares (SLS) and Weighted SLS Estimation of Single-Index Models," *Journal of Econometrics*, 58, pp. 71-120

Jeong. J. (2004), "Bootstrapping Sample Selection Models Through Generalized Residuals," YERI working paper

Jeong, J. and G. S. Maddala (1996), "Testing the Rationality of Survey

Data Using the Weighted Double-Bootstrapped Method of Moments," *Review of Economics and Statistics*, 78, pp. 116-122.

Klein, W. and R. H. Spady (1993) "An Efficient Semiparametric Estimator for Binary Response Models," *Econometrica*, Vol. 61, No. 2 (March, 1993), pp. 387-421.

Lin, T. and P. Schmidt (1984), "A Test of the Tobit Specification against an Alternative Suggested by Cragg," *Review of Economics and Statistics*, 66, pp. 174-176.

Manski, C. F. (1975), "The Maximum Score Estimation of the Stochastic Utility Model of Choice," *Journal of Econometrics*, 3, pp. 205-228.

Nelson, F. D. (1981), "A Test for Misspecification in the Censored Normal Model," *Econometrica*, 49, pp. 1317-1329.

Newey, W. K. (1987). "Specification Tests for Distributional Assumptions in the Tobit Model," *Journal of Econometrics*, Vol. 34, pp. 124-45.

______________ (1999), "Two-Step Series Estimation of Sample Selection Models," Working Paper, Department of Economics, MIT.

Powell, J. L. (1986) "Symmetrically Trimmed Least Squares Estimation for Tobit Models," *Econometrica*, 54, pp. 1435-1460.

Ruud, P. A. (1984) "Tests of Specification in Econometrics," *Econometric Reviews*, 3, pp. 211-242.

Tobin, J. (1958) "Estimation of Relationships for Limited Dependent Variables," *Econometrica*, 26, pp. 24-36.

Vella, F. (1989), "Estimating Models with Sample Selection Bias: A Survey," *Journal of Human Resources*, 33, pp. 127-169.

토빗 모형의 정규분포 검정법

정 대 영* · 정 진 욱**

논문초록

이 논문은 토빗 모형에서 오차항의 정규성을 검정하는 새로운 검정법을 제시한다. Rudd (1984)나 Newey (1987) 등 기존의 검정법들과는 달리, 이 새로운 검정법은 토빗 추정치를 귀무가설과 대립가설 모두에서 일치성을 갖는 추정치와 비교한다. 이런 비교를 위하여, Newey (1999)와 Jeong (2004)이 제시한 비모수적 추정법을 활용한다.

주제분류 : B030104
핵심 주제어 : 토빗모형, 정규분포검정, 붓스트랩

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