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Backward-Looking Preferences and Present Bias*

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Abstract

This paper presents a model of backward-looking preferences in which an agent forms a memory based on his past consumptions and his current utility depends not only on his current consumption but also on his memory. Several memory formation processes are discussed, and it is shown that under some processes backward-looking preferences generate present bias.

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Ⅰ. Introduction

An individual's current well-being is influenced not only by his current experiences but also by his past and future experiences. An economic agent derives utility from his anticipations about future events (e.g., getting a promotion, winning a lottery), while past events shape who he is including his health conditions and preferences. In this paper, we focus on the effect of the past on current utility and present a model of backward-looking preferences.

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In our model, an agent forms a memory based on his past consumptions, and his current utility depends not only on his current consumption but also on his memory. We assume that good memories contribute positively to current well-being. The basic idea is that, as an agent has more pleasant memories about his past experiences, he will be in a better mood to enjoy his current consumption. At the same time, he can obtain positive utility from reminiscing about the past. Our main result shows that, under certain memory formation processes, backward-looking preferences lead to present bias. The intuition behind this result is as follows. Since a memory about a good event can be recalled over and over again in the future, a good event in an early period counts more than one in a late period when assessing overall life-time utility. Thus, the agent puts disproportionately high importance on earlier periods than later ones, resulting in present bias. In this way, our model provides an explanation for present bias.

In the literature, various studies have emphasized forward-looking and backward-looking components of well-being and derived their implications. Elster and Loewenstein (1992) discuss the roles of memory and anticipation in determining current utility. They argue that deriving utility from consuming memories can justify immediate indulgence, which is related to our point that backward-looking behavior may generate present bias. Loewenstein (1987), Caplin and Leahy (2001), Kőszegi (2010), and Galperti and Strulovici (2014) study agents who derive utility from anticipation. Brunnermeier and Parker (2005) examine optimal expectations when an agent faces a trade-off between increased anticipatory utility and worse decision making from optimism. Habits and addiction can be considered as the influences of the past on the present (see, for example, Pollak, 1970; and Becker and Murphy, 1988). Comparison between the past and the present also affects current utility, which is referred to as the contrast effect in Elster and Loewenstein (1992) (see, for example, Ryder and Heal, 1973). If the current situation is relatively worse than the previous ones, having pleasant experiences in the past is harmful to current well-being through the contrast effect. In this paper, we ignore the contrast effect and focus on the beneficial effect of pleasant past experiences on current well-being through memory recollection, which is referred to as the consumption effect in Elster and Loewenstein (1992).

The remainder of this paper is as follows. In Section II, we introduce a model of backward-looking preferences and memory formation processes. In Section III, we show that backward-looking preferences may lead to present bias. In Section IV, we conclude.

Ⅱ. Backward-Looking Preferences

We consider an economic agent (or a decision maker) who is infinitely lived.1) Time is discrete and denoted by $t = 0, 1, \ldots$. The agent's consumption in period t is denoted by c_t, and his consumption space is denoted by C. In each period, the agent has a memory that depends on his past consumption. The agent's memory in period t is denoted by m_t. For $t \geq 1$, m_t is determined by some function $f_t : C^t \rightarrow \mathbb{R}$ so that $m_t = f_t(c_0, c_1, \ldots, c_{t-1})$, while we set $m_0 = 0$ as normalization. The agent's period t utility is given by $u(c_t, m_t)$ where $u : C \times \mathbb{R} \rightarrow \mathbb{R}$ is a period utility function. We assume that $u(c, m)$ is nondecreasing in m for any c. This is to capture the idea that the agent derives utility from consuming memories about pleasant past experiences. For simplicity, we assume that $u(c, m) =$

1) Our model and result extend straightforwardly to the case where the agent lives for a finite horizon.

$\alpha v(c)+(1-\alpha)m$ where $\alpha\in(0,1)$ and $v(c)$ denotes direct utility of consumption c. That is, current utility is a convex combination of utility from consumption and utility from memory. We further assume that overall utility is time-separable and geometrically discounted so that it takes the form $\sum_{t=0}^{\infty}\delta^t u(c_t,m_t)$ where $\delta\in(0,1)$ denotes the agent's discount factor. Note that, since m_t is determined by $(c_0,c_1,\ldots,c_{t-1})$ for each $t\geq 1$, overall utility can be written as a function of a consumption stream, i.e., $U(c_0,c_1,\ldots,c_{t-1})=\sum_{t=0}^{\infty}\delta^t u(c_t,m_t)$, given the initial memory m_0.

We assume that $m_t=f_t(c_0,c_1,\ldots,c_{t-1})$ is a weighted average of past consumption utilities $v(c_0),v(c_1),\ldots,v(c_{t-1})$. Below we present five memory formation processes.

(1) *Short-lived memory:* Let $m_t=v(c_{t-1})$. In this case, the current memory is equal to the consumption utility in the previous period. Memory is short-lived in the sense that it lasts for only one period.

(2) *Equal-weight memory:* Let $m_t=\frac{1}{t}\sum_{\tau=0}^{t-1}v(c_\tau)$. In this case, the current memory is equal to the arithmetic mean of the consumption utilities in all the past periods. That is, memory puts an equal weight on all the past periods.

(3) *Geometrically decaying memory:* Let $m_t=\frac{1-\rho}{1-\rho^t}\sum_{\tau=0}^{t-1}\rho^{t-1-\tau}v(c_\tau)$ where $\rho\in(0,1)$. In this case, memory puts a positive weight on all the past periods, but the weight on a past period decays geometrically with factor ρ as the period goes further into the past.

(4) *Best-moment memory:* Let $m_t=\max\{v(c_0),v(c_1),\ldots,v(c_{t-1})\}$. In this case, memory puts the entire weight on the period in which

the agent had the highest utility. In other words, the agent's memory is determined by the best moment in his life so far.

(5) *Worst-moment memory:* Let $m_t = \min\{v(c_0), v(c_1), \ldots, v(c_{t-1})\}$. In this case, memory puts the entire weight on the period in which the agent had the lowest utility. In other words, the agent's memory is determined by the worst moment in his life so far.

Geometrically decaying memory can be expressed as

$$m_t = \frac{v(c_{t-1}) + \rho v(c_{t-2}) + \cdots + \rho^{t-1} v(c_0)}{1 + \rho + \cdots + \rho^{t-1}}.$$

Then it is easy to see that short-lived memory and equal-weight memory are limiting cases of geometrically decaying memory as δ goes to 0 and 1, respectively.

Ⅲ. Present Bias

In this section, we explain how backward-looking preferences can lead to present bias. Obara and Park (2014) introduce $\{\beta_t\}_t$-weighted discounting in which $\{\beta_t\}_{t=0}^{\infty}$ is a bounded sequence of positive numbers and the overall utility of the payoff sequence $(\pi_0, \pi_1, \ldots) \in \mathbb{R}^{\infty}$ is given by

$$\beta_0 \pi_0 + \beta_1 \delta \pi_1 + \beta_2 \delta^2 \pi_2 + \cdots = \sum_{t=0}^{\infty} \beta_t \delta^t \pi_t.$$

This class of discounting operators is useful because it includes standard geometric discounting ($\beta_t = 1$ for all $t \geq 0$) and quasi-hyperbolic discounting ($\beta_0 = 1$ and $\beta_t = \beta$ for all $t \geq 1$) as special cases while it allows for more general forms of discounting. Obara and Park (2014) also define present bias for general discounting operators and show that $\{\beta_t\}_t$-weighted discounting exhibits present bias if and only if β_{t+1}/β_t is increasing with respect to t. Note that in this case β_{t+1}/β_t should be less than 1 for all t since $\{\beta_t\}_t$ is bounded, and thus β_t is decreasing with respect to t. Present bias implies $\beta_1/\beta_2 > \beta_2/\beta_3$, meaning that the weight on the current period relative to the next period is higher than that on the future period.

We examine the first three memory formation processes presented in the previous section. First, consider short-lived memory. With short-lived memory, overall utility can be expressed as

$$U(c_0, c_1, \dots) = \beta \sum_{t=0}^{\infty} \delta^t v(c_t)$$

where $\beta = \alpha + (1-\alpha)\delta$. We obtain standard geometric discounting that exhibits no bias. Since memory lasts for only one period, the effect of utility from period t consumption on overall utility does not depend on t and is represented by the constant β. Hence, we have the same weight β on utility from consumption in every period t. In the expression $\beta = \alpha + (1-\alpha)\delta$, α can be interpreted as the direct effect of consumption, while $(1-\alpha)\delta$ can be regarded as the indirect effect of consumption through memory.

Now consider equal-weight memory and geometrically decaying memory. Then overall utility can be expressed as

$$U(c_0, c_1, \dots) = \sum_{t=0}^{\infty} \beta_t \delta^t v(c_t)$$

where $\beta_t = \alpha + (1-\alpha)\delta\gamma_t$ and γ_t is defined by

$$\gamma_t = \frac{1}{t+1} + \delta\frac{1}{t+2} + \delta^2\frac{1}{t+3} + \cdots$$

for equal-weight memory and

$$\gamma_t = \frac{1}{\sum_{\tau=0}^{t}\rho^\tau} + \delta\rho\frac{1}{\sum_{\tau=0}^{t+1}\rho^\tau} + \delta^2\rho^2\frac{1}{\sum_{\tau=0}^{t+2}\rho^\tau} + \cdots$$

for geometrically decaying memory. Note that overall utility takes the form of $\{\beta_t\}_t$-weighted discounting. The following proposition shows that with equal-weight or geometrically decaying memory the backward-looking preferences exhibit present bias.

Proposition 1. *Assume equal-weight memory or geometrically decaying memory. Then $\{\beta_t\}_t$-weighted discounting in the overall utility function exhibits present bias.*

Proof. See the Appendix. □

The intuition behind this result is as follows. When the agent is young, there is only a small number of past periods to recall, with each past period receiving relatively high weights when forming a memory. In contrast, when the agent is old, he has been through many periods and thus puts relatively low weights on each past period. (For example, with equal-weight memory, the agent puts weight 1/3 on each of the past three periods in period 3 while he puts weight 1/100 on each of the past hundred periods in period 100.) Thus, utility from consumption in an earlier period has a stronger impact on overall utility than that in a later period. In other words, the agent puts disproportionately high weights on earlier periods, which generates present bias. We close this section with two remarks.

Remark 1. If the memory formation function f_t is stationary in the sense that the weights are independent of t, then we obtain no bias. As mentioned above, the main force that induces present bias is that the weights on past periods to form a memory are higher in early periods than in late periods. However, this force is no longer present when the weights on past periods are invariant to timing. One example is short-lived memory, as studied above, where f_t picks up utility from the most recent consumption. Another example is the process governed by $m_{t+1} = \rho m_t + (1-\rho)v(c_t)$ for $t = 0, 1, \ldots$ with m_0 given, which leads to the memory formation function

$$m_t = (1-\rho)\sum_{\tau=0}^{t-1}\rho^{t-1-\tau}v(c_\tau) + \rho^t m_0.$$

With this memory formation process, the weight on the most recent consumption is $(1-\rho)$, that on the previous one is $\rho(1-\rho)$, and so on, independently of t, and the overall utility function is given by

$$U(c_0, c_1, \ldots | m_0) = \beta\sum_{t=0}^{\infty}\delta^t v(c_t) + \frac{1-\alpha}{1-\delta\rho}m_0$$

where $\beta = \alpha + (1-\alpha)\delta(1-\rho)/(1-\rho\delta)$. Yet another example is finite-period memory in which memory lasts for M periods and the agent puts some weights on past M periods to form a memory. This can be considered as a generalization of short-lived memory, which corresponds to the case of $M = 1$.

Remark 2. In the decision theory literature, it is usually assumed that preferences are time-invariant in the sense that the agent's preference for one consumption stream over another does not change with the timing of decision making. With time-invariant preferences, present bias induces time inconsistency in that the current self has different preferences on consumption streams starting in a future period than the future self has. In other words, if we consider an agent with present-biased preferences such as (quasi-)hyperbolic discounting, it is common to assume that each period-t self has the same utility function, and the period-0 self and the period-1 self may disagree on which consumption stream from period 1 to choose. In contrast, present bias in our model leads to time-variant preferences while the current self and the future self agree on which consumption stream starting in a future period they prefer. That is, in our model, different period-t selves have different utility structures as the memory formation process depends on t, while the period-0 self and the period-1 self agree on which consumption stream from period 1 to prefer.

To be more specific, consider the overall utility function $U(c_0, c_1, \ldots) = \sum_{t=0}^{\infty} \beta_t \delta^t v(c_t)$ induced by geometrically decaying memory. Let $U_t(c_t, c_{t+1}, \ldots | c_0, c_1, \ldots, c_{t-1}) = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(c_\tau, m_\tau)$ be overall utility from consumption stream $(c_t, c_{t+1}, \ldots)$ evaluated in period t given past consumption $(c_0, c_1, \ldots, c_{t-1})$. Then we can show that

$$U_t(c_t, c_{t+1}, \ldots | c_0, c_1, \ldots, c_{t-1})$$
$$= \sum_{\tau=t}^{\infty} \beta_\tau \delta^{\tau-t} v(c_\tau) + (1-\alpha)\gamma_{t-1} \sum_{\tau=0}^{t-1} \rho^{t-1-\tau} v(c_\tau). \qquad (1)$$

Since past consumption is fixed, we can treat the second term on the right-hand side of (1) as constant and express overall utility from period t as $U_t(c_0, c_1, \ldots) = \sum_{\tau=0}^{\infty} \beta_{\tau+t} \delta^{\tau} v(c_\tau)$. This implies that, when there is a bias in discounting, we have time-variant preferences. Thus, it is possible to have $U(c_0, c_1, \ldots) > U(\tilde{c}_0, \tilde{c}_1, \ldots)$ and $U_t(c_0, c_1, \ldots) < U_t(\tilde{c}_0, \tilde{c}_1, \ldots)$ for some t, $(c_0, c_1, \ldots)$, and $(\tilde{c}_0, \tilde{c}_1, \ldots)$. However, the current self and the future self agree in the sense that $U(c_0, \ldots, c_{t-1}, c_t, c_{t+1}, \ldots) > U(c_0, \ldots, c_{t-1}, \tilde{c}_t, \tilde{c}_{t+1}, \ldots)$ for any $(c_0, \ldots, c_{t-1})$ if and only if $U_t(c_t, c_{t+1}, \ldots) > U_t(\tilde{c}_t, \tilde{c}_{t+1}, \ldots)$.

Ⅳ. Conclusion

In this paper, we have presented a model of backward-looking preferences and related it with present bias. In future research, we can apply our model to study the behavior of backward-looking agents. A scenario to which our theory is applicable is decision-making on traveling. An individual may want to travel a lot earlier in his life, because his traveling experiences will influence his utility for the remainder of his life.

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Appendix: Proof of Proposition 1

Proof. We prove the proposition with geometrically decaying memory. The case of equal-weight memory can be treated analogously.

Fix any $t \geq 1$. We want to show that $\beta_t/\beta_{t-1} < \beta_{t+1}/\beta_t$. This is equivalent to $\alpha[(\gamma_{t-1}-\gamma_t)-(\gamma_t-\gamma_{t+1})]+(1-\alpha)\delta(\gamma_{t-1}\gamma_{t+1}-\gamma_t^2)>0$. Hence, it suffices to show that $\gamma_{t-1}-\gamma_t > \gamma_t-\gamma_{t+1}$ and $\gamma_t/\gamma_{t-1} < \gamma_{t+1}/\gamma_t$. Since

$$\gamma_{t-1}-\gamma_t = \frac{1-\rho}{1-\rho^t}-(1-\delta\rho)\frac{1-\rho}{1-\rho^{t+1}}-\delta\rho(1-\delta\rho)\frac{1-\rho}{1-\rho^{t+2}}-\cdots,$$

$\gamma_{t-1}-\gamma_t > \gamma_t-\gamma_{t+1}$ is equivalent to

$$\frac{1-\rho}{1-\rho^t}-\frac{1-\rho}{1-\rho^{t+1}} > (1-\delta\rho)\sum_{k=1}^{\infty}(\delta\rho)^{k-1}\left(\frac{1-\rho}{1-\rho^{t+k}}-\frac{1-\rho}{1-\rho^{t+k+1}}\right). \quad (2)$$

Since

$$\frac{1-\rho}{1-\rho^t}-\frac{1-\rho}{1-\rho^{t+1}} > \frac{1-\rho}{1-\rho^{t+k}}-\frac{1-\rho}{1-\rho^{t+k+1}}$$

for all $k=1,2,\ldots$, the inequality (2) holds. Next, observe that

$$\gamma_t = \frac{1-\rho}{1-\rho^{t+1}}+\delta\rho\gamma_{t+1}.$$

Using this, we can show that $\gamma_t/\gamma_{t-1} < \gamma_{t+1}/\gamma_t$ is equivalent to $\gamma_t(1-\rho^t) < \gamma_{t+1}(1-\rho^{t+1})$, or

$$(1-\rho^{t})\sum_{k=0}^{\infty}(\delta\rho)^{k}\frac{1}{1-\rho^{t+k+1}} < (1-\rho^{t+1})\sum_{k=0}^{\infty}(\delta\rho)^{k}\frac{1}{1-\rho^{t+k+2}}. \quad (3)$$

Comparing term by term in each side, we can verify that the inequality (3) holds as well. □

과거회고적인 선호와 현재 편향

박 재 옥*

논문초록

본 논문에서는 과거회고적인 선호를 모형으로 제시한다. 논문의 모형에서는 경제주체가 과거의 소비를 바탕으로 기억을 형성하고 현재효용이 현재소비뿐만 아니라 기억에도 의존한다. 기억 형성과정을 몇 가지 논의하며, 특정 기억 형성과정 하에서 과거회고적인 선호가 현재 편향을 일으킬 수 있음을 보인다.

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