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Expectational Stability and Determinacy in Multivariate Linear Rational Expectations Models*

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Abstracts

This paper reexamines the aspect of expectational stability as an alternative economic refinement scheme of rational expectations (RE) solutions to determinacy. While expectational stability (ES) has been a powerful economic guidance for understanding RE models, it is not the sole alternative to determinacy since it is well-known that more than one solution can be expectationally stable (E-stable), particularly in the case of indeterminacy. In this paper, we also show that the concept of Expectational stability depends inherently on the representation of a RE model in a multivariate linear RE framework. The reason is that the extent of an overparameterization of a perceived law of motion (PLM) formed by economic agents to learn about the REE can vary across alternative representations of a RE model. This implies that one may draw different conclusions on E-stability of a REE to a model when represented alternatively. This aspect of overparameterization differs from what is known as the distinction of weak and strong E-stability where different functional forms of PLM are studied. We examine how robust are the E-stability results across different representations and emphasize that the information contents of the learning agents matter in study of E-stability for linear RE models.

KRF Classification : C2102

Keywords : E-stability, Determinacy, Rational Expectations Equilibrium, Overparameterization, Multivariate Model

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Ⅰ. Introduction

The concept of Expectational stability (E-stability hereafter) proposed and developed by George Evans and Seppo Honkapohja in a series of papers has been one of the major contributions to the literature on convergence to a Rational Expectations equilibrium (REE) under adaptive learning. Based on the results by Marcet and Sargent (1989), Evans and Honkapohja (1998, 1999, 2001) have extensively analyzed the relation between E-stability and least-squares learnability of REEs. It is now well-known that there is a tight relation between them, known as the E-stability Principle. E-stability has been popular in the literature because it is much easier to implement E-stability than to implement least-squares learnability. Evans and Honkapohja (2001) provide a general treatment of E-stability for multivariate models and several authors have applied E-Stability in this framework. A selected list of papers includes Bullard and Mitra (2002), Honkapohja and Mitra (2004), Gauthier (2002), Adam (2003) and Evans and Honkapohja (2003b).

Recently, the relation between determinacy, learnability and E-stability has also been explored by Woodford (2003a,b), Giannitsarou (2005), McCallum (2007) and Bullard and Eusepi (2008). While determinacy has been mostly accepted as a economically desirable property of linear RE models, it is rather a mathematical property so that this stand of research examine whether E-stability can provide a economic interpretation of determinacy. Indeed, McCallum (2007) shows that these two concepts are tightly related even though they are not exactly the same. (See Cho and McCallum (2009) for instance.) In the case of indeterminacy, the relation becomes less tight because more than one solution can actually be E-stable.

In this paper, we provide a more specific account of the cases where E-stability may not be a robust and consistent solution refinement for REEs to linear RE models particularly in the case of indeterminacy. Specifically, we show that the concept of E-stability in a multivariate framework is inherently model-dependent, implying that the E-stability property is not directly comparable across models or different representations of a given model. An immediate consequence is that one may draw different conclusions on E-stability of a REE to a model at hand under alternative representations of the same model and the same REE. Nevertheless, this result should not be treated as a weak point of E-stability, but as a consequence of the lack of clear specification of the information contents of the learning agents with bounded rationality.

The reason can be understood in terms of overparameterization of the perceived law of motion (PLM) relative to a REE of interest. To build up some intuition, it is instructive to first recall the implications of the well-known overparameterization associated with different PLM classes in a univariate framework. "Weak" E-stability applies when a REE (solution) and the PLM have the same functional form. For each coefficient of a state variable in a REE, an unrestricted PLM parameter is assigned to that variable. This implies that the number of PLM parameters is the same as that of the REE. When a more general functional form of the PLM relative to the REE is postulated, the PLM is overparameterized relative to the REE because the PLM has more state variables, and thus more parameters than the REE. In this case, a different concept, "strong" E-stability, applies. As such, weak and strong E-stability are associated with different learning rules. Intuitively, when economic agents postulate different types of PLMs, their implications on the REE may well be different and it is not surprising that they can lead to different conclusions on E-stability for the same solution. For future reference, we dene this type of overparameterization as the between-PLM overparameterization.

In this paper, we show that the concept of E-stability in a multivariate framework is in general also subject to a very different type of overparameterization and that the extents of this kind of overparameterization are model-specific. For ease of exposition, let the fundamental (non-fundamental) PLM denote the PLM that has the same functional form as the class of fundamental (non-fundamental) solutions.[1)] For instance, consider a fundamental solution to a multivariate model and suppose that the fundamental PLM is postulated. Conceptually, E-stability in this case would be analogous to weak E-stability in a univariate framework because the PLM and the REE are of the same functional form. Indeed, the E-stability conditions described in chapter 10 of Evans and Honkapohja (2001) nest those of the univariate cases so that they are direct generalizations of the weak E-stability conditions from a technical point of view. However, it turns out that the concept of E-stability in multivariate models differs from weak E-stability in univariate models, just as weak and strong E-stability are different.

The reason is that virtually every macroeconomic model imposes model-specific restrictions on the parameters of the REE, and thus not all the coefficients of the state variables in a REE are free in general. In contrast, a PLM is postulated a priori without such restrictions and, as Evans and Honkapohja (2001) show, an unrestricted PLM is the most natural benchmark because agents are not likely to know

1) By fundamental solutions, we mean the REEs that depend on the minimal set of state variables. Non-fundamental (bubble or sunspot) solutions are the REEs that typically depend on additional variables to the minimal set of state variables, plus some other variables outside the model at hand. The fundamental solutions are also known as the minimal state variable (MSV) solutions in the literature. However, the solution obtained via the MSV criterion of McCallum (1983) is also often called the MSV solution. To avoid confusion throughout the paper, we use the term fundamental solution to denote the solution that depends on the minimal set of state variables and do not use the term MSV solution.

the exact restrictions implied by the model. Hence, the PLM is in general overparameterized relative to the solution even within the same class of PLMs as the REEs. We call this type of overparameterization the within-PLM overparameterization.[2] Since different models impose different restrictions on their REEs, the extents of the within-PLM overparameterization vary across models. Moreover, they also vary across different representations of the same model and the same solution. Consequently, the concept of E-stability of the solution depends on each model and its representation.

For the purpose of this paper, it is sufficient to show that the concept of E-stability is model-dependent in the context of the fundamental class of solutions and the fundamental PLMs. When a class of solutions and a broader class of PLMs are considered, analogously to strong E-stability in univariate models, then the PLM would be subject to both the within-PLM and between-PLM overparameterization and hence, E-stability would again be model-dependent. While we do not discuss the issue of underparameterization, E-stability associated with underparameterized PLMs would also be model-dependent in multivariate models.

Our finding that E-stability depends on the model and its representation is independent of the model determinacy, the dates at which expectations are formed or the stability of the REEs. In particular, the inclusion of constants in the model can lead to more stringent results for E-stability. For instance, the unique stationary fundamental REE to a multivariate model can be E-unstable. We provide a numerical example of this kind using a standard New-

2) Evans and Honkapohja (2003a) and Evans and McGough (2005) also examine different representations of sunspot equilibria and show that the stability properties depend on the solution representations. However, they postulate different classes of REEs and PLMs to a given representation of the model, rather than the same PLM to different representations of the model. Therefore, they study the implications of the between-PLM overparameterization, rather the within-PLM overparameterization.

Keynesian macro model analyzed by Honkapohja and Mitra(2004), which has become a workhorse for the monetary policy analysis.

This paper is organized as follows. In section 2, we show that a modified version of the Dornbusch model considered by Evans and Honkapohja (1994, 2001) can be represented differently and that the E-stability results are different across model representations. Section 3 derives the E-stability conditions in general linear RE models and show that E-stability is subject to the within-PLM overparameterization in a multivariate framework. Section 4 provides several examples where different representations lead to different conclusions on E-stability. Section 5 concludes.

Ⅱ. The Dornbusch (1976) Model

Evans and Honkapohja (1994) and Evans and Honkapohja (2001) (EH hereafter) examine E-stability of fundamental solutions to the Dornbusch (1976) model under a univariate representation in terms of the log of the price level. The Dornbusch model considered by EH consists of a Phillips curve, an open economy IS curve, an LM curve and the open-economy parity condition. The model is reproduced as follows:

$$p_t = \alpha_p + p_{t-1} + \pi E^*_{t-1} d_t \tag{1a}$$

$$d_t = \alpha_d - \gamma(r_t - E^*_{t-1} p_{t+1} + p_t) + \eta(e_t - p_t) \tag{1b}$$

$$r_t = \alpha_r + \lambda^{-1}(p_t - \vartheta p_{t-1}) \tag{1c}$$

$$e_t = \alpha_e + E^*_{t-1} e_{t+1} - r_t \tag{1d}$$

where p_t is the (log) price level, d_t is (log) aggregate demand, r_t is the nominal interest rate and e_t is the (log) nominal exchange rate.

$E^*_{t-1}(\bullet)$ is the subjective expectations operator formed at time $t-1$. While EH use contemporaneous expectations in equations (1b) and (1d), we use lagged expectations in order to avoid complications regarding mixed dating of expectations. In addition, we add arbitrary constants in the model.

Whereas most modern macroeconomic models are derived using Rational Expectations, the primary concern in the learning literature is whether the subjective forecasting rule of the economic agents in the model converge to a REE when economic agents are not fully rational. Therefore, the expectations operator in the model is not equivalent to the Rational Expectations operator. However, it would be reasonable to assume that the degree of bounded rationality possessed by agents in the formation of the forecasting rule is the same as that in the model. To this end, we adopt two assumptions that Adam (2003) uses in his OLG model with sticky prices in order to make his model internally consistent with the forecasting rule.

$A1$: *Economic agents in a model hold the same subjective expectations provided that they are given the same information set.*

$A2$: *Subjective expectations obey the law of iterative expectations:* $E^*_{t-1}[E^*_t(\bullet)] = E^*_{t-1}(\bullet)$

In this paper, we do not analyze the model of heterogenous agents, so that $A1$ naturally holds. As Adam (2003) stated, $A2$ implies that agents act as econometricians and their expectations forecast is unbiased.

Under the two assumptions, the Dornbusch model can be represented in several forms as:

$$x_t = \alpha_p + \beta E^*_{t-1} x_t + \beta E^*_{t-1} x_{t+1} + \beta E^*_{t-1} x_{t+2} + \delta x_{t-1} \quad (2)$$

where x_t is defined in table 1 for the 5 representations of the model. For instance, $R4$ is the original model itself and $R1$ is the univariate representation considered by EH derived under the two assumptions above. The definitions of $\beta_0, \beta_1, \beta_2$ and for each representation are given in Appendix A.

【Table 1】 Five Representations of the Dornbusch Model

Representation	$R1$	$R2$	$R3$	$R4$	$R2'$
x_t	p_t	$(p_t\, e_t)'$	$(p_t\, d_t\, r_t)'$	$(p_t\, d_t\, r_t\, e_t)'$	$(p_t\, d_t)'$

Under Rational Expectations, i.e., $E^*_{t-1}(\bullet)$ of the model is replaced by the RE operator, $E_{t-1}(\bullet)$. We consider a class of fundamental RE solutions as:

$$x_t = \bar{a} + \bar{b} x_{t-1} \tag{3}$$

where $\bar{b}$ must satisfy the following restriction imposed by the model:3)

$$\beta_0 \bar{b} + \beta_1 \overline{b^2} + \beta_2 \overline{b^3} + \delta = \bar{b}. \tag{4}$$

For $R1$, $x_t = p_t$ and the solution to this equation is given by $p_t = \bar{a}_p + \bar{b}_p p_{t-1}$ where $\bar{a}_p$ and $\bar{b}_p$ are scalars. The remaining variables are solved for as $d_t = \bar{a}_d - (1-\bar{b}_p)/\pi\, p_{t-1}$, $r_t = \bar{a}_r + [(\bar{b}_p - \vartheta)/\lambda)]\, p_{t-1}$ and $e_t = \bar{a}_r - [(\bar{b}_p - \vartheta)/(\lambda(1-\bar{b}_p))]\, p_{t-1}$. Therefore, they are completely characterized by a single solution parameter, $\bar{b}_p$. For

3) It is straightforward to compute $\bar{a}$ once $\bar{b}$ is solved for. However, it is not necessary to do so as the value of $\bar{a}$ does not enter the E-stability condition.

the other representations, $\bar{b}$ can also be defined corresponding to $\bar{b}_p$ as we show in Appendix A. Consequently, while different researchers may analyze different representations of the model and a solution, and there is no "right" or "wrong" representation, they in fact analyze an identical model and solution.

Since we consider a class of fundamental PLMs, this has the same functional form as (3):

$$x_t = a + bx_{t-1} \tag{5}$$

where b is unrestricted for each representation. Therefore, E-stability of a fundamental REE with respect to the fundamental PLM should be conceptually equivalent across different representations. In $R1$, E-stability of a fundamental solution is defined as "weak" E-stability because the same PLM class is postulated. E-stability in multivariate models shown in chapter 10 of EH may also be analogously interpreted as "weak" E-stability precisely because of the same reason. Furthermore, the conditions of E-stability in multivariate models nest those in univariate models. That is, the conditions in multivariate models are direct generalizations of those in univariate models.

Consequently, it is natural to expect that the E-stability results of the REEs to the model would be invariant across different representations of the model and the solutions. However, it turns out that different representations lead to different conclusions on E-stability. The numerical parameter values considered by EH are $\pi = 1.5$, $\gamma = 1.5$, $\lambda = 10$, $\vartheta = 1.1$, and $\eta = -0.1$. In this case, there are three stationary fundamental solutions for $\bar{b}$. All the technical details can be found in the following section where we generalize the E-stability conditions outlined in chapter 10 of EH. Table 2 summarizes the E-stability results (table 8 in Appendix A contains all the detailed results). The results for R1 (the first and the third solution are E-stable) are those reported in Evans and Honkapohja (1994) and EH. However, none of the solutions is E-stable for $R4$, the original model and for $R2$.[4)5)] From the table, it is clear that the concept of E-stability must be in fact different across different model representations of the same model and REE.

【Table 2】 E-stability of REEs to Five representations of the Dornbusch Model

Representation	$\bar{b}=0.716$	$\bar{b}=0.772$	$\bar{b}=0.990$
$R1$	Yes	No	Yes
$R2$	No	No	No
$R3$	Yes	No	Yes
$R4$	No	No	No
$R2'$	Yes	No	Yes

What leads to different conclusions on E-stability across different representations of the model and REE? The reason can be understood in terms of the within-PLM overparameterization. Whereas $\bar{b}$ in (3) as a fundamental solution is subject to (4), b in (5) as the fundamental PLM is postulated without restrictions. Specifically, the solution can be completely characterized by a single solution parameter $\bar{b}_p$ as shown in table 3. Across all representations, the solution (3) has only one free parameter while there are 4, 9, 16 and 4 free PLM parameters in $R2$, $R3$, $R4$ and $R2'$, respectively. Consequently, the PLMs are overparameterized relative to the

4) When constant terms are excluded as EH did, the first solution is E-stable for all representation but the third solution is not for $R2$ and $R4$.

5) When $\vartheta=0.5$ and $\eta=0.2$, the model is determinate and the unique stationary solution, $\bar{b}=0.384$, is E-stable in all representations. However, as Bullard and Mitra (2002) and McCallum (2008) show, a determinate but E-unstable REE can exist, so that the REE under determinacy may not always be E-stable across different representations. Indeed, we provide such an example using the standard New-Keynesian model later in this paper.

respective REEs in multivariate representations, and the concept of E-stability precisely reflects these representation- dependent extents of the within-PLM overparameterization. In addition, the within-PLM overparameterization does not just depend on the dimension of the model representation, but also on the variables with which the model is represented, as E-stability results for $R2$ and $R2'$ are also different. Furthermore, E-stability of the REE in a larger dimensional representation is not "strong" relative to that in a smaller dimensional representation. For instance, E-stability of the solution $\bar{b}$ associated with $\bar{b}_p = 0.99$ in $R3$ does not imply E-stability of the same solution in $R2$.

【Table 3】 $\bar{b}$ in Five Representations of the Dornbusch Model

Representation	$R1$	$R2$	$R3$	$R4$	$R2'$
$\bar{b}$	$\bar{b}_p$	$\begin{bmatrix} \bar{b}_p 0 \\ \bar{b}_e 0 \end{bmatrix}$	$\begin{bmatrix} \bar{b}_p 0\,0 \\ \bar{b}_d 0\,0 \\ \bar{b}_r 0\,0 \end{bmatrix}$	$\begin{bmatrix} \bar{b}_p 0\,0\,0 \\ \bar{b}_d 0\,0\,0 \\ \bar{b}_r 0\,0\,0 \\ \bar{b}_e 0\,0\,0 \end{bmatrix}$	$\begin{bmatrix} \bar{b}_p 0 \\ \bar{b}_d 0 \end{bmatrix}$

More importantly, regardless of the E-stability results, the extents of the PLM overparameterization depend on model representations, implying that the concept of E-stability should be distinguished across model representations, just as we distinguish between weak and strong E-stability across the different PLMs. For instance, while $R2$ and $R4$ yield the same E-stability results, the results in fact reflect different concepts of E-stability.

Our results here critically hinge on the assumption $A2$.[6] It is this assumption that enables us to derive the different model representations. When agents form a particular PLM, they derive the mapping

6) However, it should be noted that E-stability is representation-dependent even without this assumption, as we show in an example in section 4.

from this given PLM to the ALM using the law of iterative expectations even under subjective expectations. Since the subjective expectations in the model reflect the same extents of bounded rationality by those agents who form the PLM, it is natural to assume that $A2$ applies to the expectations formation in both the model and the PLM. If this assumption is not allowed, then any representation, including $R1$ considered by EH, cannot be derived from the original Dornbusch model (the $R4$ representation) and it simply becomes a different model. In this instance, E-stability results for each representation cannot be interpreted as those of the Dornbusch model. Therefore, under $A2$, our results indicate that it matters for learning who is learning which variables.

In the following section, we present E-stability conditions for general multivariate linear macro models and show that the type of E-stability varies not just across different representations of a given model, but also across different models. We also point out critical differences in the economic implications associated with the within and between-PLM overparameterization.

Ⅲ. Characterizing E-Stability in a General Framework

We present two classes of models under lagged and contemporaneous expectations that nest most of the models considered by EH and their series of papers, and derive E-stability conditions for the fundamental class of REEs. Then we show that the concept of E-stability differs across alternative representations of the same model.[7)]

7) Some models may include mixed dating of expectations as in Adam, Evans, and Honkapohja (2006) and Evans, Honkapohja, and Marimon (2007). While

3.1 Lagged Expectations Models

Consider a linear model:

$$x_t = \alpha + \beta_0 E^*_{t-1} x_t + \beta_1 E^*_{t-1} x_{t+1} + \beta_2 E^*_{t-1} x_{t+2} + \delta x_{t-1} + \epsilon_t \tag{6}$$

where x_t is an $n \times 1$ vector of variables observed at time t for a natural number n including 1. α is an $n \times 1$ vector of constants and $\beta_0, \beta_1, \beta_2$ and δ are $n \times n$ matrices of parameters. $E^*_t(\bullet)$ is the subjective expectation operator conditional on information available at time t, which obeys $A1$ and $A2$. ϵ_t is an error term such that $E^*_t(\epsilon_{t+1}) = 0$ In order to solve for the REE, we replace the subjective expectations with Rational Expectations, $E_t(\bullet)$, in equation (6). The class of fundamental RE solutions is given by:

$$x_t = \bar{a} + \bar{b} x_{t-1} + \epsilon_t \tag{7}$$

where the $n \times n$ matrix $\bar{b}$ must solve the following restrictions implied by the model:

$$\beta_2 \overline{b^3} + \beta_1 \overline{b^2} + \beta_0 \bar{b} + \delta = \bar{b}. \tag{8}$$

In order to study learnability of the REE of the form (7) in terms of E-stability, a particular functional form of the PLM must be specified. In this paper, we restrict our interest to the fundamental PLM and it is given by:

$$x_t = a + b x_{t-1} + \epsilon_t \tag{9}$$

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it is straightforward to derive E-stability conditions for such class of models, we do not consider them here for simplicity.

where a and b are free and not subject to the parameter restrictions in (8). By evaluating the model (6) with the PLM (9), we can derive the actual law of motion (ALM). The mapping from the PLM to the ALM is given by:

$$T(a,b) = (\alpha + (\beta_0 + \beta_1(I+b) + \beta_2(I+b+b^2))a,\ \beta_2\overline{b^3} + \beta_1\overline{b^2} + \beta_0\overline{b} + \delta). \tag{10}$$

It should be noted that $A2$ is repeatedly used to derive this mapping. The derivatives of the T-map are taken with respect to the unrestricted PLM parameters, and are given by:[8)]

$$DT_a(a,b) = (\beta_0 + \beta_1(I+b) + \beta_2(I+b+b^2)) \tag{11}$$

$$DT_b(a,b) \equiv \frac{\partial vec(T(b))}{\partial(vec(b))'} = I \otimes (\beta_0 + \beta_1 b + \beta_2 b^2) + b' \otimes (\beta_1 + \beta_2 b) + (b^2)' \otimes \beta_2. \tag{12}$$

Notice that the derivative of the T-map with respect to a, $DT_a(a,b)$ does not depend on α. To examine whether a particular REE $(\overline{a}, \overline{b})$ is E-stable, we evaluate these derivatives with the chosen solution. Specifically, a fundamental solution (7) is said to be E-stable if all the eigenvalues of $DT_a(a,b)$ and $DT_b(a,b)$ have real parts less than 1.

It is crucial to understand that the PLM coefficient matrix b in (9) is unrestricted whereas the REE coefficient matrix $\overline{b}$ is restricted to satisfy (8). The number of parameters in $\overline{b}$ is at most n^2 for the identification of the model while b has exactly n^2. Throughout this

8) It is straightforward to compute $DT(b)$ using the simple formula, $d(X,Y) = X(dY) + d(X)Y$, where $X = b$ and $Y = b^2$.

paper, we assume that $\bar{b}$ has strictly less free parameters than n^2 in multivariate models, as virtually every structural macro model has parameter restrictions on its REE. Then, the PLM is overparameterized relative to the REE in multivariate (representations of these) models. In addition, the coefficient matrices $(\beta_0, \beta_1, \beta_2, \delta)$ are model-specific and $\bar{b}$ is restricted by them. The same is true for the constants in PLM and the REE. Therefore, while the PLM is not model-dependent by itself, the extents of overparameterization of the PLM relative to a REE are model-dependent. Furthermore, the extents of overparameterization differ across representations of a given model and its REE, as we showed in the previous section. This type of overparameterization is what we call the within-PLM overparameterization. Therefore, E-stability must be defined with respect to a model, its representation and the class of PLM considered. Consequently, E-stability is not comparable across different models as well as different representations of a given model.

In the literature, however, E-stability is defined with respect to a particular PLM form, without an explicit reference to a model and its representation. For ease of exposition, let us classify RE models depending on the dimension of y_t and the values of β_2 as in table 4. E-stability conditions of fundamental solutions for the $LU1$ and $LM1$ models with respect to the fundamental PLM are given in pages 196 and 231 of EH, respectively. E-stability of $LU2$ is also discussed on page 215 of EH. Although E-stability in $LM2$ is not discussed in their book, it is straightforward to derive the E-stability condition as in (12).

【Table 4】 Classes of RE Models under Lagged Expectation

	$\beta_2 = 0$		$\beta_2 \neq 0$	
	$n=1$	$n>1$	$n=1$	$n>1$
Class	$LU1$	$LM1$	$LU2$	$LM2$

Since $LM2$ nests $LU2$, $LM1$ and $LU1$ as special cases, it seems natural to interpret E-stability of a fundamental REE with the corresponding fundamental PLM as the same kind for all classes of models as "weak" E-stability for univariate models. However, the concept of E-stability differs across multivariate models because it is defined with respect to the model-dependent within-PLM overparameterization. An immediate consequence is that one may draw different conclusions on E-stability of REEs to a given model when researchers use different representations of the same model. An example of this kind is given in the previous section: The representations $R1$ through $R4$ and $R2'$ of the Dornbusch model belong to $LU2$, $LM1$, $LM2$, $LM1$ and $LM2$, respectively.

We now compare the implications of the within-PLM and between-PLM overparameterizations on E-stability. When a more general functional form of the PLM relative to the solution of interest is postulated, E-stability is subject to the between-PLM overparameterization, as different classes of PLMs represent different ways in which agents forecast the economic variables at hand. Consequently, it is natural for E-stability to have different economic implications on the REE across different PLMs. An example of this kind is strong E-stability of REEs to univariate models. For a given PLM, strong E-stability is model-independent.[9] In contrast, E-stability associated with the within-PLM overparameterization in multivariate models is model-specific in spite of the fact that the PLM and the solution have the same functional form. Weak E-stability in multivariate models is such an example. Another example is strong E-stability in multivariate models, which is subject to both the within-PLM and between-PLM overparameterizations. As EH argue,

9) Strictly speaking, however, strong E-stability must also be defined with a particular PLM because different general PLMs imply different extents of overparameterization, leading to different concepts of E-stability.

unrestricted PLMs are the most natural ones because agents with imperfect information are unlikely to know the existence of these equilibrium restrictions. Unfortunately, the specification of the unrestricted PLMs is precisely the source of the E-stability mismatch across representations in multivariate models.

As a result, a concept of model-independent E-stability in a multivariate framework is called for, comparable across models and yielding the same E-stability results regardless of the model representations of a given model. To do so, one may impose the model specific restrictions on the PLM parameters. However, imposing such restrictions directly on the PLM is not so natural in the bounded rationality framework as we discussed above. Instead, note that only E-stability in a univariate framework, such as $LU1$ and $LU2$, is in general comparable across models because they are not subject to the within-PLM overparameterization. Therefore, if a given multivariate model can be reduced into a univariate representation, then E-stability would be model-independent in general. However, this type of system reduction into a univariate framework would involve different dates at which expectations are formed as well as autoregressive and moving average terms, as EH emphasize. Furthermore, it is not clear whether such a system reduction is robust against the order of the variables with which the model is reduced.

3.2 Contemporaneous Expectations Models

Consider a linear model where expectations are taken contemporaneously:

$$x_t = \alpha + \beta_1 E_t^* x_{t+1} + \beta_2 E_t^* x_{t+2} + \delta x_{t-1} + \epsilon_t \tag{13}$$

$E_t^*(\bullet)$ also obeys $A1$ and $A1$. The class of fundamental solutions and the restrictions on the REEs are given by:

$$x_t = \overline{a} + \overline{b} x_{t-1} + c\epsilon_t \tag{14}$$

$$\beta_2 \overline{b^3} + \beta_1 \overline{b^2} + \delta = \overline{b}. \tag{15}$$

[10]The fundamental PLM has the same functional form as (14) but without the parameter restriction (15):

$$x_t = a + b x_{t-1} + c\epsilon_t \tag{16}$$

The T-mapping from the PLM to the ALM is given by:

$$T(a,b) = (F(b)^{-1}(\alpha + \beta_1 (I+b) + \beta_2 (I+b+b^2))a),\ F(b)^{-1}\delta) \tag{17}$$

where $F(b) = (I - \beta_1 b - \beta_2 b^2)$. The derivatives of the mapping with respect to the unrestricted PLM parameters are given by:

$$DT_a(a,b) = F(b)^{-1}(\beta_1 (I+b) + \beta_2 (I+b+b^2)) \tag{18}$$

$$\begin{aligned} DT_b(a,b) = {} & [F(b)^{-1}\delta]' \otimes F(b)^{-1}(\beta_1 + \beta_2 b) \\ & + (bF(b)^{-1}\delta)' \otimes F(b)^{-1}\beta_2 \end{aligned} \tag{19}$$

The E-stability conditions for the univariate and multivariate models with $\beta_2 = 0$ are given in pages 202 and 238 of EH. The E-stability conditions for the models with $\beta_2 \neq 0$ are not explicitly discussed. However, once again, it is straightforward to derive the E-stability conditions for these models. All the arguments laid out in models with lagged expectations are preserved under contemporaneous expectations.

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10) Since there will not be E-stability conditions for c, because $E_t^*(\epsilon_{t+1}) = 0$, we do not need to discuss about c in what follows.

Ⅳ. Examples

In this section, we present several models that can be represented in two forms and derive the conditions under which a particular REE to a model can be E-stable or E-unstable, depending on the representation. First, we present a bivariate model composed of two independent univariate equations under lagged expectations. Then we show that a solution to the bivariate model consisting of individually E-stable solutions to each univariate model can be E-unstable. We also show that exactly the same results are obtained when a two-variable model has a recursive structure, where the second variable is independent of the first one but the first variable depends on the second one. Second, we present a bivariate model that has no E-stable REE. Then we show that when the model is represented in a univariate form, it has one or more E-stable solutions. By comparing the extents of the PLM overparameterization in the two models, we show that E-stability is not just representation-dependent, but also model-dependent.[11] Finally, we show that when the model and the PLM contain constant terms, the E-stability conditions for the vector of constants are representation-dependent as well. As an economic example, we present a New-Keynesian model analyzed by Honkapohja and Mitra (2004) and show that the fundamental REE can be E-unstable in case of determinacy.

4.1 Model A: Combination of Independent Univariate Equations

We first consider a model that can be represented in $LU1$ and

11) In the previous version of this paper, we performed analogous exercises under these two models with contemporaneous expectations, and the results were essentially similar to those obtained under lagged expectations.

$LM1$ forms.

$LU1$ **Representation:** Consider two completely unrelated univariate equations without constants belonging to $LU1$. The (representation of the) model, the fundamental solutions, the solution restrictions, the fundamental PLM, the T-map and its derivative corresponding to equations (6) through (12) are respectively given by:

$$y_{i,t} = \beta_{0,i}E^*_{t-1}y_{i,t} + \beta_{1,i}E^*_{t-1}y_{i,t+1} + \delta_i y_{i,t-1} + \epsilon_{i,t} \quad (20a)$$

$$y_{i,t} = \overline{b_i}y_{i,t-1} + \epsilon_{i,t} \quad (20b)$$

$$\overline{b_i} = \beta_{1,i}\overline{b_i^2} + \beta_{0,i}\overline{b_i} + \delta_i \quad (20c)$$

$$y_{i,t} = b_i y_{i,t-1} + \epsilon_{i,t} \quad (20d)$$

$$T(b_i) = \beta_{1,i}b_i^2 + \beta_{0,i}b_i + \delta_i \quad (20e)$$

$$DT(b_i) = \beta_{0,i} + 2\beta_{1,i}b_i \quad (20f)$$

for $i = 1, 2$. Suppose that there are two real-valued but not necessarily stationary REEs, with $\bar{b}_i(1) < \bar{b}_i(2)$ (without loss of generality) in each equation.

$LM1$ **Representation:** The $LU1$ representation of the model can be written in a bivariate $LM1$ form with $x_t = (y_{1,t}\ y_{2,t})'$ and $v_t = (\epsilon_{1,t}\ \epsilon_{2,t})'$. The analogous equations to (20) are as follows:

$$x_t = \beta_0 E^*_{t-1}x_t + \beta_1 E^*_{t-1}x_{t+1} + \delta x_{t-1} + v_t \quad (21a)$$

$$x_t = \bar{b}x_{t-1} + v_t \quad (21b)$$

$$\bar{b} = \beta_1\overline{b^2} + \beta_0\bar{b} + \delta \quad (21c)$$

$$x_t = bx_{t-1} + v_t \quad (21d)$$

$$T(b) = \beta_1 b^2 + \beta_0 b + \delta \quad (21e)$$

$$DT(b) = I\otimes(\beta_0 + \beta_1 b) + b'\otimes\beta_1 \quad (21F)$$

where

$$\beta_0 = \begin{bmatrix} \beta_{0,1} & 0 \\ 0 & \beta_{0,2} \end{bmatrix}, \ \beta_1 = \begin{bmatrix} \beta_{1,1} & 0 \\ 0 & \beta_{1,2} \end{bmatrix}, \ \delta = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}$$

(20a)-(20c) and (21a)-(21c) are just different representations of the same model, solution and solution restrictions. Moreover, we did not use the assumption $A2$ in this case. Specifically, the RE solution $\bar{b}$ is given by:

$$\bar{b} = \begin{bmatrix} \bar{b}_1 & 0 \\ 0 & \bar{b}_2 \end{bmatrix}$$

where $\bar{b}_i$ is identical to that in (20b) subject to (20c) for $i = 1,2$. Consequently, it is natural to expect that (20f) and (21f) deliver the same conclusions on E-stability. (21f) is the condition stated by Proposition 10.1 of EH in a multivariate context, which generalizes the E-stability condition in univariate models. Indeed, when the model is univariate, (21f) is identical to (20f). The latter condition is stated in Proposition 8.2 of EH.

However, it turns out that E-stability defined in (21f) differs from that defined in (20f). When evaluated with b in (23), it is straightforward to show that $DT(b)$ is diagonal (so that its eigenvalues are the diagonal elements) and can be analytically expressed as:

$$diag(DT(\bar{b})) = [\beta_{0,1} + 2\beta_{1,1}\overline{b_1} \ \ \beta_{0,2+}\beta_{1,2}(\overline{b_1} + \overline{b_2}) \ \ \beta_{0,1} + \beta_{1,1}(\overline{b_1} + \overline{b_2}) \ \ \beta_{0,2+}2\beta_{1,2}\overline{b_2}]'. \qquad (24)$$

This is where the discrepancy between the E-stability conditions in the $LU1$ and $LM1$ representations arises. The off-diagonal elements of $\bar{b}$ are in fact zeros and thus are not free. However, b is postulated without such restrictions and $DT(b)$ produces non-zero second and third diagonal elements. For example, the second diagonal element contains the parameters of the second equation, $\beta_{0,2}$ and $\beta_{1,2}$, and the completely unrelated parameter of the first equation, $\bar{b}_1$. Note that the first and fourth diagonal elements are just the E-stability conditions of each equation in (20f). Hence, the second and third roots are the additional conditions induced by the overparameterized PLM in the $LM1$ representation. Therefore, (20f) and (21f) are conditions for different types of E-stability, implying that the concept of E-stability of a REE to a given model is representation-dependent.

If the E-stability results were the same across different representations, then the fact that the concept of E-stability is model-dependent would not pose a problem in practice. However, the results on E-stability may actually differ across representations. We now derive a condition under which the solution $\bar{b}$ consisting of the E-stable solutions $\bar{b}_1(1)$ and $\bar{b}_2(1)$ in the $LU1$ form is not E-stable in the $LM1$ representation. Suppose that all the parameter values are positive. Then, one such condition is given by:

$$\bar{b}_1(1) > \bar{b}_2(2) \tag{25}$$

That is, whenever the two solutions of the first equation are larger than those of the second equation in $LU1$ form, the solution consisting of individually E-stable solutions to both equations turns out to be E-unstable in the $LM1$ representation.[12)]

As a numerical example, suppose that $\beta_{1,1} = 0.4, \beta_{0,1} = 0.32,$ $\delta_1 = 0.288,$ $\beta_{1,2} = 0.5,$ $\beta_{0,2} = 0.35,$ and $\delta_2 = 0.21$. Table 5 shows the

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12) To see this, note that $\beta_{0,1} = 1 - \beta_{1,i}(\bar{b}_i(1) + (\bar{b}_i(2)),$ for $i = 1,2$. Thus, the second diagonal element of $DT(\bar{b})$ can be written as $\beta_{0,2} + \beta_{1,2}(\bar{b}_1(1) + (\bar{b}_2(1)) = 1 + \beta_{1,2}(\bar{b}_1(1) - (\bar{b}_2(2))$. Therefore, it is greater than 1 as long as $\bar{b}_1(1) > \bar{b}_2(2)$. By symmetry, the other case is $\bar{b}_2(1) > \bar{b}_1(2)$.

four diagonal elements of $DT(\bar{b})$ for the two solutions of each equation.

【Table 5】 $DT(\bar{b})$ of the $LU1$ and $LM1$

		$LM1$			
		$LU1$			
$\bar{b}_1$	$\bar{b}_2$	DT_{11}	DT_{44}	DT_{22}	DT_{33}
$\bar{b}_1(1)=0.8$	$\bar{b}_2(1)=0.6$	0.96	0.95	1.05	0.88
$\bar{b}_1(1)=0.8$	$\bar{b}_2(2)=0.7$	0.96	1.05	1.1	0.92
$\bar{b}_1(2)=0.9$	$\bar{b}_2(1)=0.6$	1.04	0.95	1.1	0.92
$\bar{b}_1(2)=0.9$	$\bar{b}_2(2)=0.7$	1.04	1.05	1.15	0.96

As can be seen from the table, while $\bar{b}_1(1)$ and $\bar{b}_2(1)$ are E-stable in $LU1$, the solution b corresponding to $\bar{b}_1(1)$ and $\bar{b}_2(1)$ is not E-stable in $LM1$. Note also that the results are independent of the stationarity of the solutions; as long as (25) holds, the same outcome is obtained.

While we provide this example in order to clearly show that the concept of E-stability depends on the representation of a given model, there is no reason to put the two independent equations in a bivariate framework. A less trivial example would be a recursive two-equation-two-variable $(y_{1,t}, y_{2,t})$ model where $y_{2,t}$ is an autonomous process and also influences $y_{1,t}$. Thus, consider the following model:

$$
\begin{aligned}
y_{1,t} &= f(y_{2,t}) + \beta_{0,1}E^*_{t-1}y_{1,t} + \beta_{1,1}E^*_{t-1}y_{1,t+1} + \delta_1 y_{1,t-1} + \epsilon_{1,t} \\
y_{2,t} &= \beta_{0,2}E^*_{t-1}y_{2,t} + \beta_{1,2}E^*_{t-1}y_{2,t+1} + \delta_2 y_{2,t-1} + \epsilon_{2,t}
\end{aligned}
$$

where $f(y_{2,t})$ can adopt any form such as $\kappa y_{2,t}$, $\kappa E^*_{t-1}y_{2,t}$, $\kappa E^*_{t-1}y_{2,t+1}$ and $\kappa y_{2,t-1}$. Then, it can be analytically shown that none of the previous results is altered.[13] This is because the solution $\bar{b}$ would be upper triangular and $DT(\bar{b})$ would be block-recursive (upper triangular) with the same diagonal elements as those in equation (24).

4.2 Model B: Bivariate Model and its Univariate Representation

Consider a model that can be represented in $LU2$ and $LM1$ forms:

$$\begin{aligned} y_t &= \beta_{0,y} E^*_{t-1} y_t + \beta_{1,y} E^*_{t-1} y_{t+1} + E^*_{t-1} z_{t+1} \\ &\quad + \delta_y y_{t-1} + \epsilon_t \qquad (26) \\ z_t &= \beta_{2,y} E^*_{t-1} y_{t+1} \qquad (27) \end{aligned}$$

$LU2$ **Representation:** The model can be represented in a univariate form in terms of y_t by substituting out z_t. This $LU2$ representation of the model, the fundamental solutions, the solution restriction, the fundamental PLM, the T-map and its derivative, corresponding to equations (6) through (12), are respectively given by:[14]

$$\begin{aligned} y_t &= \beta_{0,y} E^*_{t-1} y_t + \beta_{1,y} E^*_{t-1} y_{t+1} + \beta_{2,y} E^*_{t-1} y_{t+2} \\ &\quad + \delta_y y_{t-1} + \epsilon_t \qquad (28a) \\ y_t &= \overline{b_y} y_{t-1} + \epsilon_t \qquad (28b) \\ \overline{b_y} &= \beta_{2,y} \bar{b}^3_y + \beta_{1,y} \bar{b}^2_y + \beta_{0,y} \bar{b}_y + \delta_y \qquad (28c) \end{aligned}$$

---

13) There is however, one additional E-stability condition for the first equation. For example, suppose $f(y_{2,t}) = \kappa y_{2,t}$. Then the PLM of the first equation would be $y_{1,t} = b_1 y_{1,t-1} + c y_{2,t}$. Therefore, E-stability must also be examined with respect to c. In our example, the conclusions on E-stability are not affected by this additional condition.

14) Once the fundamental REE to the first equation is obtained and E-stability is examined, the fundamental solutions to the z_t equation can be obtained. Since this equation does not involve its own expectational term, we do not need to examine E-stability for the solutions to this equation.

$$y_t = b_y y_{t-1} + \epsilon_t \tag{28d}$$

$$T(b_y) = \beta_{2,y} b_y^3 + \beta_{1,y} b_y^2 + \beta_{0,y} b_y + \delta_y \tag{28e}$$

$$DT(b_y) = 3\beta_{2,y} b_y + 2\beta_{1,y} b_y + \beta_{0,y} \tag{28f}$$

$LM1$ **Representation:** In matrix form, the model can also be written as:

$$x_t = \beta_0 E^*_{t-1} x_t + \beta_1 E^*_{t-1} x_{t+1} + \delta x_{t-1} + v_t \tag{29}$$

where $x_t = (y_t, z_t)'$, $v_t = (\epsilon_t, 0)'$, β_0, β_1 and δ are given by:

$$\beta_0 = \begin{bmatrix} \beta_{0,y} & 0 \\ 0 & 0 \end{bmatrix}, \; \beta_1 = \begin{bmatrix} \beta_{1,y} & 1 \\ \beta_{2,y} & 0 \end{bmatrix}, \; \delta = \begin{bmatrix} \delta_y & 0 \\ 0 & 0 \end{bmatrix}. \tag{30}$$

Since the functional form of the $LM1$ representation of this model is identical to (21a), the fundamental solution, the solution restriction, the fundamental PLM, the T-map and its derivative with respect to the unrestricted PLM parameters are exactly of the same form as (21b) through (21f). However, the extents of the restrictions on $\bar{b}$ are of course different from those in the $LM1$ representation of Model A, shown in the previous subsection. This is simply because the definitions of β_0, β_1 and δ are different, and $\bar{b}$ is given by:

$$\bar{b} = \begin{bmatrix} \bar{b}_y & 0 \\ \beta_{2,y} \bar{b}_y^2 & 0 \end{bmatrix}. \tag{31}$$

In Appendix B, we show that the E-stability conditions are given by:

$$3\beta_{2,y}\overline{b_y^2} + 2\beta_{1,y}\bar{b}_y + \beta_{0,y} < 1, \quad \beta_{0,y} + 2\beta_{1,y}\bar{b}_y + \beta_{2,y}\overline{b_y^2} < 2,$$

$$\beta_{0,y} + \beta_{1,y}\bar{b}_y + \beta_{2,y}\overline{b_y^2} < 1. \tag{32}$$

Note that the first condition is the $LU2$ E-stability condition for $\bar{b}_y$. Therefore, one can reject E-stability of b in $LM1$ and accept E-stability of the same solution in $LU2$ if the first condition is met but either the second or the third, or both conditions are violated.15)

We replicate the example in section 9.5.1 of EH in order to show that the finding of representation-dependent E-stability is independent of the uniqueness of a stationary fundamental solution. With $\beta_{0,y} = -0.4$, $\beta_{1,y} = 1.9$, $\beta_{2,y} = -1$ and $\delta_y = 0.45$, there exists a unique stationary fundamental solution, $\bar{b}_y = 0.9$, and a pair of complex conjugates.16)

With $\bar{b}_y = 0.9$, the three values in (32) are $(0.59, 2.21, 0.5)$. Since the first condition holds, the solution must be E-stable in $LU2$, but not in $LM1$ because the second condition is violated. Indeed, when $\bar{b}_y = 0.9$, $DT(\bar{b}_y) = 0.59$ so that by is E-stable, but the eigenvalues of $DT(\bar{b})$ are $1.1050 \pm 0.6316i, 0$ and 0.5, implying a rejection of E-stability in $LM1$.

We also consider a numerical example with three stationary

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15) When a model is given in $LU2$ form (equation (28a)), it is sometimes easy to examine determinacy of the model and solve for the REE by transforming the model into $LM1$ using an auxiliary expectational variable, z_t in equation (27). This kind of model transformation is not uncommon in the literature and in his study of E-stability and determinacy, McCallum (2007) generalizes models by employing such transformation. George Evans and Seppo Honkapohja pointed out to us that representing (26) with (27) into the form of (29) may not be appropriate for the purpose of examining E-stability because z_t is itself a forecasting variable for agents. However, the opposite directional transformation from the $LM1$ into $LU2$ form under the assumption $A1$ would pose no such problem, as we do here.

16) Since the absolute values of the complex roots are less than 1, the model is still indeterminate although the real-valued fundamental solution is unique. We thank Evans and Honkapohja for pointing this out.

solutions taken from page 217 of EH. They showed that a model in the $LU2$ form with $\beta_{0,y}=-3.53968254$, $\beta_{1,y}=6.66666667$, $\beta_{2,y}=-3.17460318$ and $\delta_y=1$ has two E-stable solutions. But when it is represented in $LM1$ form, none of the REEs becomes E-stable, as table 6 shows.

【Table 6】 $DT(\bar{b})$ of the $LU2$ and $LM1$

$\bar{b}_y$	$LU2$ $DT(\bar{b}_y)$	$LM1$ Eigenvalues of $DT(\bar{b})$			
0.5	0.75	$1.17-0.48i$	$1.17+0.48i$	0	-1
0.7	1.13	3.29	0.94	0	-0.43
0.9	0.75	4.82	1.07	0.750	-0.11

Comparison between Model A and Model B

We have shown that E-stability of fundamental REEs to models A and B is representation-dependent. Here we show that E-stability is in general model-dependent as well when models are represented in multivariate form. In the $LM1$ representation of both models A and B, the fundamental PLM is given by $b=\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$. However, the RE solutions are restricted by $\bar{b}_A=\begin{bmatrix} \bar{b}_1 & 0 \\ 0 & \bar{b}_2 \end{bmatrix}$ and $\bar{b}_B=\begin{bmatrix} \bar{b}_y & 0 \\ \beta_2 \bar{b}_y^2 & 0 \end{bmatrix}$ for models A and B, respectively. That is, while the PLM is model-independent, the REEs differ across models. $\bar{b}_A$ has two independent parameters on its diagonal position. $\bar{b}_B$ has two non-zero elements on the first column, but they are not independent. As such, the way the PLM is overparameterized relative to the respective REE is different. This difference is reflected in the E-stability conditions: The E-stability condition for the $LM1$ representation of Model A is that all the elements of (24) be less than one. In contrast, it is given by (32) for Model B. Therefore, the extents of the within-PLM overparameterizations differ across multivariate models in general and, consequently, the concept of E-stability is model-dependent.

Univariate representations of the models are, however, in general not model-dependent. In both the $LU1$ representation of Model A and the $LU2$ representation of Model B, the PLM has one unrestricted parameter b and the REE has also only one solution parameter. Therefore, E-stability is not subject to the within-PLM overparameterization.[17] The concept of E-stability applied in these univariate representations is precisely "weak" E-stability. Consequently, E-stability in the $LU1$ or $LU2$ representations can be interpreted as "weak" E-stability in a multivariate framework in the sense that the functional form of the PLM is identical to that of the REE, and the model-specific restriction (21c) is taken into account.

In models A and B, the E-stability conditions in $LM1$ are "stronger" than those in the univariate representation of each model because the former are sufficient for the latter. However, it is not known whether the concept of E-stability in any arbitrary multivariate representation of a model is stronger than that in the univariate representations in general. Also, E-stability in higher dimensional representations is neither necessary nor sufficient for that in a lower dimensional representation, as the numerical example of the Dornbusch model showed in section 2.

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17) Even in univariate models, the fundamental PLM could potentially be overparameterized relative to the fundamental solutions. For instance, suppose that a univariate model has n state variables so that the REE has n solution parameters. If the number of structural parameters of the model is less than n, then the number of independent solution (reduced-form) parameters would be less than n as well. Then, the PLM would technically be overparameterized as well. We do not investigate this issue in the present paper.

4.3 New-Keynesian Model

We have shown that E-stability in multivariate models is in general representation-dependent. In the previous examples of section 4, constant terms have not been included in the models and the PLM. We now show that E-stability is representation-dependent under contemporaneous expectations, independently of model determinacy when constants are included in the model.

Honkapohja and Mitra (2004) consider a canonical New-Keynesian model consisting of the Phillips curve, an intertemporal IS equation and a monetary policy rule, taken from Clarida, Gali, and Gertler (1999). While they focus on E-stability of non-fundamental solutions in case of indeterminacy, we consider E-stability of fundamental solutions. We reproduce the model as follows:

$$z_t = \psi(i_t - E_t^* \pi_{t+1}) + E_t^* z_{t+1} + g_t \quad (33a)$$

$$\pi_t = \lambda z_t + \beta E_t^* \pi_{t+1} + u_t \quad (33b)$$

$$i_t = \alpha_{MP} + \theta i_{t-1} + (1-\theta)\chi_\pi E_t^* \pi_{t+1} + (1-\theta)\chi_z z_t \quad (33c)$$

where z_t is the output gap, π_t is the inflation rate and i_t is the nominal interest rate. g_t and u_t are exogenous innovations to the IS equation and the Phillips curve, respectively. We introduce a constant α_{MP} in their model so that the fundamental solutions depend on the constant term.[18)]

Let $x_t = (z_t \; \pi_t \; i_t)'$. Then the model can be written in matrix form

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18) Bullard and Mitra (2002) pointed out that there may be a constant to the policy rule to account for the idea that the monetary authority may well wish to stabilize nominal interest rates around a value different from the steady state value consistent with zero inflation and steady state output growth, as argued by Woodford (1999). From a technical standpoint, the analysis in this section is not altered at all as long as an arbitrary constant is included in any equation(s).

as:

$$x_t = \alpha + \beta_1 E_t^* x_{t+1} + \delta x_{t-1} \tag{34}$$

$$B_0 = \begin{bmatrix} 1 & 0 \; \psi \\ -\lambda & 1 \; 0 \\ -(1-\theta)\chi_z & 0 \; 1 \end{bmatrix}, \quad \alpha = B_0^{-1} = \begin{bmatrix} 0 \\ 0 \\ \alpha_{MP} \end{bmatrix}$$

$$\beta_1 = B_0^{-1} \begin{bmatrix} 1 & \psi & 0 \\ 0 & \beta & 0 \\ 0 & (1-\theta)\chi_\pi & 0 \end{bmatrix}, \quad \delta = B_0^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \theta \end{bmatrix}$$

The model can also be written as a bivariate system as:

$$x_t = \alpha + \beta_1 E_t^* x_{t+1} + \beta_2 E_t^* x_{t+2} + \delta x_{t-1} \tag{35}$$

where $x_t = (z_t \;\; i_t)'$ and:[19]

$$B_0 = \begin{bmatrix} 1 & \psi \\ -(1-\theta)(\chi_z + \chi_\pi/\psi) & 1-(1-\theta)\chi_\pi \end{bmatrix},$$

$$\beta_1 = B_0^{-1} = \begin{bmatrix} 1+\beta+\lambda\psi & \beta\psi \\ -(1-\theta)\chi_\pi/\psi & 0 \end{bmatrix},$$

$$\beta_2 = B_0^{-1} \begin{bmatrix} -\beta & 0 \\ 0 & 0 \end{bmatrix}, \quad \delta = B_0^{-1} \begin{bmatrix} 0 & 0 \\ 0 & \theta \end{bmatrix}$$

The class of fundamental REEs to the model has the following form:

$$x_t = \bar{a} + \bar{b}\, x_{t-1} \tag{36}$$

where $\bar{a}$ and $\bar{b}$ can be easily solved numerically. Now we assume

19) It is also possible to represent the model in terms of $(\pi_t \; i_t)'$ or $(z_t \; \pi_t)'$. Note also that this is not the only possible representation of the model even with the same variables, $x_t = (z_t \; i_t)'$. The E-stability results are sensitive to different formulations of the model in terms of the same variables.

that agents postulate the fundamental PLM as:

$$x_t = a + b x_{t-1} \qquad (37)$$

where a is a vector of constants. Then the T-mapping from the PLM to the ALM for a and b are given by equation (17). We use the baseline parameter values of Honkapohja and Mitra (2004) in case of determinacy, which are taken from Clarida, Gali, and Gertler (1999) $(\lambda = 0.3,\ \beta = 0.99,\ \theta = 0.79,\ \chi_z = 0.93,\ \chi_\pi = 2.17.)$ Instead of $\psi = 1$, we set it to 0.2, which is also widely accepted in the literature. With these parameter values, the model is determinate and the unique stationary fundamental REE and the corresponding solution to the bivariate representation are given by:

$$x_t = (z_t\ \pi_t\ i_t)' \qquad x_t = (z_t\ i_t)'$$

$$\bar{b} = \begin{bmatrix} 0 & 0 & -0.4256 \\ 0 & 0 & -0.3272 \\ 0 & 0 & 0.6159 \end{bmatrix} \qquad \bar{b} = \begin{bmatrix} 0 & -0.4256 \\ 0 & 0.6159 \end{bmatrix}$$

Since the model is determinate, it is natural to expect that this solution be E-stable, independently of the model representation. However, whereas the solution to the original three-variable system is E-stable, it is not the case for the bivariate representation as is shown in table 7:

【Table 7】 $DT(\bar{a}, \bar{b})$ of the New-Keynesian Model

	Eigenvalues of $DT_a(\bar{a}, \bar{b})$	Eigenvalues of $DT_b(\bar{a}, \bar{b})$	E-Stability
$x_t = (z_t\ \pi_t\ i_t)'$	$0.8708 \pm 0.1162i,\ 0$	$0.5363 \pm 0.0716i,\ 0(7)$	Yes
$x_t = (z_t\ i_t)'$	$1.0299 \pm 0.1712i$	$0.5967 \pm 0.2397i,\ 0(2)$	No

Note: The number of repeated eigenvalues is in parentheses.

Specifically, while the eigenvalues of $DT_b(\bar{a},\bar{b})$ have real parts less than 1 across representations, $DT_a(\bar{a},\bar{b})$ has a root greater than 1 in the bivariate representation. Therefore, E-stability results are also representation- dependent when constants are present in the PLMs. This again reflects that the extents of the restrictions on $\bar{a}$ and $\bar{b}$ differ across the two representations. This is also the case with no constants in the model (i.e., with a being a 3×1 and a 2×1 vector of zeros, respectively) as long as an unrestricted vector of constants is included in the PLM. The E-stability conditions for a do not hold for quite a large parameter space over which the model is determinate. For $0<\psi<0.234$, the E-stability condition fails to hold in the bivariate representation. The same is true for $0<\psi<0.878$ when there is no lagged interest rate $(\theta=0)$.

It could be argued that this type of representation is not really necessary as one typically analyzes the original model or a two-variable system of inflation and the output gap by substituting out the interest rate (when $\theta=0$). However, we derive this representation using the assumption $A2$, the one under which EH reduce the 5-variable Dornbusch model into a univariate representation in the price level.[20] Moreover, the AS equation (33b) can be derived a la Calvo (1983) utilizing the quasi-differencing technique, which relies heavily on the law of iterative expectations $A2$. As such, it is hard to justify that $A2$ cannot be applied in a model which is derived using $A2$, even under bounded rationality. This example shows that in study of REEs using E-stability, it is critical to clearly specify who are the learning agents and exactly which variables they postulate as state variables.

20) As EH analyzed the IS-LM model, they also wrote on page 228, “Although it is possible to solve out a reduced-form in the price level, the resulting equation would incorporate different dates at which expectations are formed..., It is simpler to look at general multivariate techniques...”.

Our finding is hard to reconcile with the existing property of the determinate solution. First of all, the model considered here is arguably the most popular one in monetary policy analysis. When the model is determinate and the Taylor principle holds, such that there is a unique stationary fundamental solution, it is hard to argue against its validity as an economically relevant solution. While Bullard and Mitra (2002) narrow down the parameter space over which a determinate solution is E-stable, our analysis shows that the solution can be E-unstable over such an economically reasonable parameter space. If one accepts this solution as an economically sensible REE, then the E-stability condition for constants seem too stringent as a REE refinement device. Finally, we also perform analogous analysis with lagged expectations for robustness, and we find that similar results are obtained: E-stability conditions for constants can fail to hold for the two-variable representation of the model.

Ⅴ. Conclusion

This paper shows that the concept of E-stability in a multivariate framework is model-dependent. We also show that the model-specific nature of E-stability surfaces independently of the model determinacy, the uniqueness of the stationary fundamental solution, the stability of the REEs and the information structure. An immediate consequence of our analysis is that it is hard to compare the results of E-stability across models. We show that the source of model-dependent E-stability lies in the fact that a postulated PLM is in general overparameterized relative to the REE, which is subject to the model-specific restrictions. Therefore, developing model-independent E-stability conditions requires that the PLM at hand be not subject to the within-PLM overparameterization.

According to our results, the concept of E-stability would also be model-dependent in studies on the relation between determinacy, learnability and E-stability. For example, under fairly general conditions, E-stability and learnability are shown to be equivalent. Assuming this, Bullard and Mitra (2002) and Bullard and Eusepi (2008) study the relation between determinacy and learnability. One important finding of Bullard and Mitra (2002) is that determinacy does not necessarily imply learnability and indeterminacy does not necessarily imply lack of learnability. Alternatively, Heinemann (2000) and Giannitsarou (2005) show that E-stability and learnability may not be identical in some environments. All of these studies may deal with different types of E-stability if their models are multivariate. We leave the study of the interrelation between determinacy, learnability and model-independent E-stability as a future research topic.

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Appendix

A. Five Representations of the Dornbusch Model

All representations of the model can be written in the following general form:

$$\begin{aligned} Ax_t = \ & c+\beta_0 E_{t-1}^* x_t + \beta_1 E_{t-1}^* x_{t+1} \\ & + \beta_2 E_{t-1}^* x_{t+2} + D x_{t-1.} \end{aligned} \tag{38}$$

By pre-multiplying this equation by A^{-1}, the model can be written as:

$$\begin{aligned} x_t = \ & \alpha+\beta_0 E_{t-1}^* x_t + \beta_1 E_{t-1}^* x_{t+1} \\ & + \beta_2 E_{t-1}^* x_{t+2} + \delta x_{t-1.} \end{aligned} \tag{39}$$

where $\beta_0 = A^{-1}\beta_0$, $\beta_1 = A^{-1}\beta_1$, $\beta_2 = A^{-1}\beta_2$, $\alpha = A^{-1}c$ and $\delta = A^{-1}D$. Since the constant term and x_{t-1} are the state variables, the fundamental solution has the following form:

$$x_t = \ \bar{a} + \bar{b}\, x_{t-1} \tag{40}$$

where $\bar{b}$ is subject to:

$$\beta_0 \bar{b} + \beta_1 \overline{b^2} + \beta_2 \overline{b^3} + \delta = \bar{b} \tag{41}$$

Finally, the fundamental PLM is given by:

$$x_t = \ a + b\, x_{t-1} \tag{42}$$

where a and b are unrestricted. x_t, the parameter matrices A, B_0,

B_1, B_2, D (or $\beta_0, \beta_1, \beta_2, \delta$) and $\bar{a}$ and $\bar{b}$ are representation-specific. The 5 representations are defined as follows ($\bar{a}$ needs not be defined as it is not present in the E-stability conditions):

$R1$. **Univariate Representation with** $x_t = p_t$: The fundamental solution is given by $p_t = \bar{a}_p + \bar{b}_p p_{t-1}$, Let $\mu_0 = -(1+\pi(\gamma+\eta+\gamma/\lambda+\eta/\lambda+\gamma\vartheta/\lambda))$, $\mu_1 = 1+\pi(2\gamma,+\eta+\gamma/\lambda)$, $\mu_2 = -\pi\gamma$ and $\delta_0 = 1+\pi\vartheta(\gamma+\eta)/\lambda$. Then, $\beta_0, \beta_1, \beta_2, \delta$ and $\bar{b}$ are defined as:

$$\beta_0 = \mu_{0,}\ \beta_1 = \mu_{1,}\ \beta_2 = \mu_{2,}\ \delta = \delta_{0,}\ \bar{b} = \bar{b}_p$$

We need to solve for the remaining variables sequentially. They can be characterized in terms of $\bar{b}_p$ as $d_t = \bar{a}_d + \bar{b}_d p_{t-1}, r_t = \bar{a}_r + \bar{b}_r p_{t-1}$, and $e_t = \bar{a}_e + \bar{b}_e p_{t-1}$ where $\bar{b}_d = -(1-\bar{b}_p)/\pi, \bar{b}_r = (\bar{b}_p - \vartheta)/\lambda$ and $\bar{b}_e = -(\bar{b}_p - \vartheta)/(\lambda(1-\bar{b}_p))$.

$R2$. **Bi-variate Representation with** $x_t = (p_t\ e_t)'$:

$$A = \begin{bmatrix} 1 & 0 \\ 1/\lambda & 1 \end{bmatrix},\ B_0 = \begin{bmatrix} -\pi(\gamma+\eta+\gamma/\lambda) & \pi\eta \\ 0 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} \pi\gamma & 0 \\ 0 & 1 \end{bmatrix},$$

$$B_2 = 0_{2\times 2},\ D = \begin{bmatrix} 1+\pi\gamma\vartheta/\gamma & 0 \\ \vartheta/\gamma & 0 \end{bmatrix},\ \bar{b} = \begin{bmatrix} \bar{b}_p & 0 \\ \bar{b}_e & 0 \end{bmatrix}.$$

$R3$. **Tri-variate Representation with** $x_t = (p_t\ d_t\ r_t)'$:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1/\gamma+\eta & 1 & \gamma+\eta \\ -\lambda^{-1} & 0 & 1 \end{bmatrix},\ B_0 = \begin{bmatrix} 0 & \pi & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 & 0 \\ 2\gamma+\eta & 1 & \gamma \\ 0 & 0 & 1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 & 0 \\ -\gamma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ \vartheta/\lambda & 0 & 0 \end{bmatrix},\ \bar{b} = \begin{bmatrix} \bar{b}_p & 0 & 0 \\ \bar{b}_e & 0 & 0 \\ \bar{b}_r & 0 & 0 \end{bmatrix}.$$

$R4$. **Four-variable Representation with** $x_t = (p_t\ d_t\ r_t\ e_t)'$:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \gamma+\eta & 1 & \gamma & -\eta \\ -\lambda^{-1} & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},\ B_0 = \begin{bmatrix} 0 & \pi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \gamma & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$B_2 = 0_{4\times 4},\ D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\vartheta/\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ \bar{b} = \begin{bmatrix} \bar{b}_p & 0 & 0 & 0 \\ \bar{b}_d & 0 & 0 & 0 \\ \bar{b}_r & 0 & 0 & 0 \\ \bar{b}_e & 0 & 0 & 0 \end{bmatrix}$$

$R2'$. **Bi-variate Representation with** $x_t = (p_t\ d_t)'$:

$$A = \begin{bmatrix} 1 & 0 \\ (\lambda+1)(\gamma+\eta)/\lambda & 1 \end{bmatrix},\ B_0 = \begin{bmatrix} 0 & \pi \\ -\gamma\vartheta/\lambda & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 2\gamma+\eta+\gamma/\lambda & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & 0 \\ -\gamma & 1 \end{bmatrix},$$

$$D = \begin{bmatrix} \delta & 0 \\ \vartheta(\gamma+\eta)/\gamma & 0 \end{bmatrix},\ \bar{b} = \begin{bmatrix} \bar{b}_p & 0 \\ \bar{b}_d & 0 \end{bmatrix}.$$

【Table 8】 $DT_a(\bar{a},\bar{b})$ and $DT_b(\bar{a},\bar{b})$ for Five Representations of the Dornbusch Model

Panel A. $R1$ representation

$\bar{b}$	$DT_a(\bar{a},\bar{b})$	$DT_b(\bar{a},\bar{b})$
0.716	0.995	0.996
0.772	0.993	1.028
0.990	0.854	0.866

Panel B. $R2$ Representation

$\bar{b}$	$DT_a(\bar{a},\bar{b})$		$DT_b(\bar{a},\bar{b})$			
0.716	1.541	1.010	0.821	0.807	−0.669	−0.030
0.772	1.667	1.010	1.137	0.800	−0.532	−0.041
0.990	2.025	1.142	1.999	1.134	−0.042	$\pm 0.399i$

Panel C. $R3$ Representation

$\bar{b}$	$DT_a(\bar{a},\bar{b})$			$DT_b(\bar{a},\bar{b})$				
0.716	−2.308	0.998	0	−2.58	0.99	0.56(2)	−2.87(2)	0(3)
0.772	−2.308	0.998	0	−2.54	1.01	0.59(2)	−2.90(2)	0(3)
0.990	−2.265	0.995	0	−2.28	0.96	0.69(2)	−3.00(2)	0(3)

Panel D. $R4$ Representation

$\bar{b}$	$DT_a(\bar{a},\bar{b})$				$DT_b(\bar{a},\bar{b})$					
0.716	−3.433	1.113	1.011	0	−3.29	0.97	0.72	−2.87(3)	0.56(3)	0(7)
0.772	−3.461	1.140	1.011	0	−3.35	1.03	0.78	−2.90(3)	0.59(3)	0(7)
0.990	−3.5419	1.1159	$\pm 0.1365i$	0	−3.53	1.1	$\pm 0.13i$	−3.00(3)	0.69(3)	0(7)

Panel E. $R2'$ representation

$\bar{b}$	$DT_a(\bar{a},\bar{b})$		$DT_b(\bar{a},\bar{b})$			
0.716	0.998	−2.308	0.990	−2.584	0.555	−2.865
0.772	0.998	−2.308	1.008	−2.546	0.591	−2.901
0.990	0.995	−2.265	0.959	−2.279	0.689	−2.999

Note: The number of repeated eigenvalues of $DT(\bar{b})$ in Panels C and D is in parentheses.

All the representations are nested in the $LM2$ class of RE models in section 3. E-stability of a REE of each representation can be examined by computing the derivatives of the T-mapping from the fundamental PLM to the ALM evaluated with a REE, $(\bar{a}, \bar{b})$, in (11) and (12). In each representation of the model, there are three fundamental solutions, $\bar{b}$, corresponding to the three values of $\bar{b}_p$. Table 8 shows the derivatives of the T-mapping computed for the three values of b in each representation.

B. E-stability Conditions for the $LM1$ Representation of Model B in Section 4.2

Here we derive $DT(\bar{b})$ analytically. Let $P(\xi : \bar{b}_y)$ be the characteristic function of $DT(\bar{b})$. Then $P(\xi : \bar{b}_y) = |DT(\bar{b}) - \xi I_2|$

where

$$DT(\bar{b}) = I_2 \otimes \left(\begin{bmatrix} \beta_{0,y} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} \beta_{1,y} & 1 \\ \beta_{2,y} & 0 \end{bmatrix} \begin{bmatrix} \bar{b}_y & 0 \\ \beta_{2,y}\bar{b}_y^2 & 0 \end{bmatrix} \right) + \begin{bmatrix} \bar{b}_y & 0 \\ \beta_{2,y}\bar{b}_y^2 & 0 \end{bmatrix} \otimes \begin{bmatrix} \beta_{1,y} & 1 \\ \beta_{2,y} & 0 \end{bmatrix}.$$

Direct computation yields:

$$P(\xi : \bar{b}_y) = C(\xi : \bar{b}_y)\xi(\xi - \beta_{0,y} - \beta_{1,y}\bar{b}_y - \beta_{2,y}\bar{b}_y)$$
$$C(\xi : \bar{b}_y) = \xi^2 - (\beta_{0,y} + 2\beta_{1,y}\bar{b}_y + \beta_{2,y}\bar{b}^2)\xi - 2\beta_{2,y}\bar{b}_y.$$

In this example, the analytical solution of $\bar{b}_y$ is not available in general. However, we can still characterize the E-stability condition, i.e., the condition under which the real part of all roots of $P(\xi : \bar{b}_y)$ is less than 1, as follows:

$$C(1 : \bar{b}_y) > 0$$
$$\xi_1 + \xi_2 = \beta_{0,y} + 2\beta_{1,y}\bar{b}_y + \beta_{2,y}\bar{b}_y^2 < 2$$
$$\xi_3 = \beta_{0,y} + \beta_{1,y}\bar{b}_y + \beta_{2,y}\bar{b}_y^2 < 1 \Leftrightarrow (\bar{b}_y - \delta_y)/\bar{b}_y < 1$$

where ξ_1 and ξ_2 are the two roots of $C(\xi : \bar{b}_y)$. $C(1 : \bar{b}_y) > 0$ implies that $3\beta_{2,y}\bar{b}_y^2 + 2\beta_{1,y}\bar{b}_y + \beta_{0,y} < 1$, which is precisely the E-stability condition for the $LU2$ representation of the model. The second and the third conditions are the additional conditions associated with the $LM1$ representation of the model.

다변수 선형 합리적 기대모형에서 기대 안정성(Expectational Stability)과 결정성(Determinacy)에 관한 연구*

조 성 훈**

논문초록

본 연구는 선형 합리적 기대 모형에서 해(solution)를 선택할 때 결정성(determinacy)에 대한 대안으로서 잘 알려져 있는 기대 안정성(expectational stability)의 특성을 재검증하고자 한다. 기대 안정성은 합리적 기대 모형을 이해하는데 있어서 매우 중요한 역할을 하지만 기대 안정성을 만족하는 복수의 해가 존재할 수 있기 때문에 결정성에 대한 완전한 대안이 되지 못하는데, 특히 복수의 안정적 해가 존재하는 비결정성(indeterminate) 모형의 경우에 더욱 더 그렇다. 본 연구에서는 동일한 모형에서도 모형의 표현방식에 따라 기대 안정성이 서로 다른 결과를 줄 수 있음을 보인다. 그 이유는 서로 다른 모형의 표현은 경제주체가 인식하는 해의 계수를 학습(learning)하는데 있어서 선택하는 상태변수들에 대한 정보가 암묵적으로 서로 다르기 때문이다. 약(weak) 기대 안정성과 강(strong) 기대 안정성이 다르다는 것은 잘 알려져 있지만 이는 경제주체가 인식하는 해의 형태가 다르기 때문에 일어나는 당연한 현상이다. 그러나 본 연구에서는 경제주체가 인식하는 해의 형태가 동일한 경우에도 기대 안정성의 정도가 다르고 이에 따라 특정 해가 모형의 표현방식에 따라 안정성을 가질 수도 있고 그렇지 않을 수도 있음을 보인다. 본 연구에서는 이러한 특성이 기대 안정성의 문제점이라기보다 경제주체가 인식하는 해에서 쓰이는 정보를 명확하게 나타내지 않는 관행 때문이며 학습관련 연구에서 정보를 명확히 설정하는 것이 필요함을 주장한다.

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