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Productivity Shocks, National Saving, and the Current Account in a Two-sector Model with Non-tradable Goods*

Kunhong Kim**

Abstracts

This paper considers a dynamic general equilibrium model of a small open two-period endowment economy with tradable and non-tradable goods. Under some mild assumptions regarding the form of the representative agent's inter-temporal utility function, it is shown analytically that a temporary positive output shock in the tradable good sector increases national saving and improves the current account balance whereas a similar positive output shock in the non-tradable good sector reduces national saving and deteriorates the current account balance if the inter-temporal elasticity of substitution between the current and future consumption is greater than the intra-temporal elasticity of substitution between the tradable and non-tradable good consumption.

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Ⅰ. Introduction

Early real business cycle models (e.g. Kydland and Prescott (1982),

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** Associate professor, Department of Economics, Hallym University, e-mail: kimku@hallym.ac.kr

King, Plosser, and Rebelo (1988)) used a stochastic version of the general equilibrium one-sector optimal growth model and explained the dynamics of macroeconomic fluctuations as reflecting the interaction of real shocks — particularly shocks to production possibilities - and the economic agents' inter-temporal adjustment. In those models, shocks are propagated over time due to the preference of economic agents for smoothing consumption over time. Even though models relied solely on shocks to production possibilities as the underlying source of economic fluctuations, it was shown, for example by Plosser (1989), that they could replicate the movement of real variables along actual post- World War II U.S. business cycles well.

Early real business cycle approach relied on closed economy models with a single good. As a result, models were not able to provide any implications of real business cycles on the current account balance or the terms of trade. In order to address this, models were later extended to include international trade. General equilibrium inter-temporal optimization models were used to explain international economic phenomena. One of the stylized international economic phenomena pointed out by the literature was the counter-cyclical movement of current account balance along business cycle. Current account balance was interpreted as a result of economic agents' inter-temporal optimization decision. Well known real business cycle models of open economy with tradable and non-tradable goods include Stockman and Tesar (1995) and Mendoza (1995). Stockman and Tesar (1995) set up a two country model and analyzed international risk sharing. Mendoza (1995) devised a small open economy model and analyzed the relative importance of terms of trade shocks versus productivity shocks along business cycles. With complicated dynamics in those models and the analyses based on simulations with calibrated parameters, it is not so transparent how the productivity shocks to the tradable sector vs. non-tradable sector affect current account balance differently. Furthermore, Mendoza (1995) assumed that the sectoral productivity shocks are perfectly correlated.

In order to highlight the main dynamics that involve productivity shocks in tradable vs. non-tradable sectors and the current account balance, we use a simple two-period small open endowment economy model and analytically show that if the temporary positive output shock (which can be interpreted as the positive productivity shock) happens to the non-tradable goods sector instead of the tradable goods sector, the current account balance can deteriorate. The intuition is the following. In a small open economy, the world rate of interest is fixed in terms of tradable goods. But the real interest rate relevant for the inter-temporal consumption decision is one measured in terms of consumption baskets. If there is a temporary increase in the output of non-tradable goods, the price of current period non-tradables would fall relative to the price of future period non-tradables. This makes the domestic real interest rate lower than the world real interest rate. As a result, consumption is tilted toward the current period and the current account balance is adversely affected.

Ⅱ. The Model

Consider a small open economy populated by a large number of consumers. There is no uncertainty and consumers have perfect foresight. Each period, consumers receive an exogenous endowment (output) of tradable goods and non-tradable goods. We denote tradable goods by x and non-tradable goods by n.

Consumers also have the ability to borrow or lend at the world real interest rate, r_x, which is fixed in terms of tradable goods x. There is no government. In order to highlight the relative roles of intra-temporal and inter-temporal elasticity of substitutions in consumption in a clear and intuitive way, we use a two-period model without any investment. The current period is denoted by 0 and the future period is denoted by 1. Representative consumer's endowment (output) of tradable and non-tradable goods at period $t(t=0,1)$ are denoted by Y_{xt} and Y_{nt} respectively. Representative consumer's consumption of tradable and non-tradable goods at period $t(t=0,1)$ are respectively denoted by c_{xt} and c_{nt}. p_{nt} is the relative price of non-tradable goods in terms of tradable goods.

The inter-temporal budget constraint faced by the representative consumer is

$$\begin{aligned}&(c_{x0}+p_{n0}c_{n0})+\alpha_{x1}(c_{x1}+p_{n1}c_{n1})=(Y_{x0}+p_{n0}Y_{n0})\\&+\alpha_{x1}(Y_{x1}+p_{n1}Y_{n1})-(1+r_{x,-1})B_{-1}=W_0\end{aligned} \tag{1}$$

where $\alpha_{x1}=1/(1+r_{x0})$ is the discount factor. W_0 denotes wealth and B_{-1} denotes the existing level of debt. The values of wealth, the rates of interest, and the units of debt are measured in terms of tradable goods which serve as the numeraire.

The representative consumer maximizes lifetime utility subject to the inter-temporal budget constraint. The utility function is defined over four goods $(c_{x0}, c_{n0}, c_{x1}, c_{n1})$. We assume that the utility function is homothetic. We also assume that utility can be expressed as a function of two components, C_0 and C_1, which are, in turn, linearly homogeneous subutility functions[1] of the

1) For example, these could be Cobb-Douglas utility functions with the

consumption of goods in period zero (c_{x0}, c_{n0}) and in period one (c_{x1}, c_{n1}) respectively. The maximization problem can be formally specified as

$$\underset{\{c_{x0},c_{n0},c_{x1},c_{n1}\}}{Maximize} \quad U[C_0(c_{x0},c_{n0}),C_1(c_{x1},c_{n1})] \tag{2}$$
$$subject\ to\ (1)$$

The solution to the maximization problem can be decomposed into two parts. The first component involves the intra-temporal allocation of spending, $z_t = c_{xt} + p_{nt}c_{nt}$, between the two goods so as to maximize the subutility $C_t(c_{xt},c_{nt})$, and the second involves the inter-temporal allocation of lifetime spending ($z_0 + \alpha_{x1}z_1 = W_0$) so as to maximize the lifetime utility $U(C_0,C_1)$. In the first stage of the intra-temporal maximization problem the consumer may be viewed as minimizing the cost, z_t, of obtaining a given level of subutility, C_t. The assumption that the subutility functions are linearly homogeneous implies that the cost function is $z_t = P_t(p_{nt})C_t$, where $P_t(p_{nt})$ is the marginal cost of obtaining a unit of C_t (and the marginal cost depends on the relative price, p_{nt}). We refer to P_t, as the price index. It can be shown that the elasticity of the price index, P_t, with respect to the price of non-tradables, p_{nt}, equals the expenditure share of non-tradables in total spending, that is,

$$\frac{p_{nt}}{P_t}\frac{dP_t}{dp_{nt}} = \frac{d[\log P_t]}{d[\log P_{nt}]} = \beta_{nt} \tag{3}$$

where β_{nt} denotes the expenditure share of non-tradables in total

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powers of the adjustments adding to one.

spending at period $t(0<\beta_{nt}<1)$.[2)]

In the second stage the consumer, who has already optimized the intra-temporal allocation of spending, attempts to optimize the inter-temporal allocation. The maximization problem is

$$\underset{\{C_0,\, C_1\}}{Maximize} \quad U(C_0, C_1) \qquad (4)$$
$$subject\ to\ P_0(p_{n0})C_0 + \alpha_{x1}P_1(p_{n1})C_1 = W_0$$

We normalize the budget constraint and express it in real terms by dividing both sides by the price index P_0.

$$C_0 + \alpha_{c1}C_1 = W_{c0} \qquad (5)$$

where

$$\alpha_{c1} = \frac{P_1}{P_0}\alpha_{x1} \text{ and } W_{c0} = \frac{W_0}{P_0}. \qquad (6)$$

We refer to α_{c1} as the real discount factor and to W_{c0} as the real wealth. The real discount factor equals the discount factor expressed in terms of the numeraire, α_{x1}, adjusted by the percentage change in the price index. Corresponding to the concept of the real discount factor, the real interest rate, r_{c0}, is

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2) $z_t = P_t(p_{nt})C_t$ implies $\frac{dz_t}{dp_{nt}} = \frac{dP_t}{dp_{nt}}C_t$.

But from the envelope theorem applied to the intra-temporal cost minimization problem,

$\frac{dz_t}{dp_{nt}} = c_{nt}$.

Therefore, $\frac{p_{nt}}{P_t}\frac{dP_t}{dp_{nt}} = \frac{p_{nt}c_{nt}}{P_tC_t} = \frac{p_{nt}c_{nt}}{z_t} = \beta_{nt}$.

$$r_{c0} = \frac{1}{\alpha_{c1}} - 1 = \frac{1 + r_{x0}}{P_1 / P_0} - 1\,. \tag{7}$$

When the domestic price index rises over time, the domestic real interest rate measured in terms of the consumption bundle is lower than the world interest rate which is fixed in terms of tradable goods. Taking the differential of (7) and applying (3), we get

$$\frac{1}{1 + r_{c0}} dr_{c0} = \frac{1}{1 + r_{x0}} dr_{x0} + \beta_{n0} \hat{p}_{n0} - \beta_{n1} \hat{p}_{n1}\,. \tag{8}$$

A circumflex denotes a logarithmic differential.[3] Equation (8) shows that, *ceteris paribus*, a temporary decline in the current price of non-tradables ($\hat{p}_{n0} < 0$) lowers the real rate of interest measured in terms of the consumption basket while an expected future decline in the price of non-tradables ($\hat{p}_{n1} < 0$) increases the real rate of interest measured in terms of the consumption basket. The effect of a permanent decline in the price of non-tradables (so that $\hat{p}_{n0} = \hat{p}_{n1} < 0$) depends on the inter-temporal changes in the expenditure shares β_{nt}. If these shares do not vary over time, a permanent decline in the non-tradable goods price is neutral in its effect on the real rate of interest.

The maximization of the utility function subject to the normalized budget constraint yields conventional demand functions for C_0 and C_1. As usual, these functions depend on the relevant relative prices α_{c1}, and on the real wealth W_{c0}. Thus, the periodic demand functions (for $t = 0, 1$) are $C_t = C_t(\alpha_{c1}, W_{c0})$. Demand functions for tradables and non-tradables at period $t (t = 0, 1)$ depend on the relative price, p_{nt}, and the total expenditure during

3) For example, $\hat{p}_{n0} = d[\log p_{n0}] = dp_{n0}/p_{n0}$.

the period, $z_t = P_t(p_{nt})C_t$. Therefore, $c_{nt} = c_{nt}[p_{nt}, P_t C_t(\alpha_{c1}, W_{c0})]$ and $c_{xt} = c_{xt}[p_{nt}, P_t C_t(\alpha_{c1}, W_{c0})]$ for $t = 0,1$.

Since non-tradable goods must be produced at home, the market for non-tradable goods must clear during each period. The market clearing condition for the domestic non-tradable goods are

$$c_{n0} = c_{n0}[p_{n0}, P_0 C_0(\alpha_{c1}, W_{c0})] = Y_{n0}, \tag{9}$$

$$c_{n1} = c_{n1}[p_{n1}, P_1 C_1(\alpha_{c1}, W_{c0})] = Y_{n1}. \tag{10}$$

With general functional forms for utility and subutility functions, it is not possible to derive endogenous variables p_{n0}, p_{n1}, c_{x0}, and c_{x1}, as explicit functions of exogenous variables, Y_{n0}, Y_{x0}, Y_{n1}, and Y_{x1}. However, by taking logarithmic differential of equilibrium conditions, we get linear relationships among logarithmic differentials of endogenous and endogenous variables. These linear relationships enable us to represent $\hat{p}_{n0}$, $\hat{p}_{n1}$, $\hat{c}_{x0}$, and $\hat{c}_{x1}$ as explicit functions of $\widehat{Y}_{x0}$, $\widehat{Y}_{x1}$, $\widehat{Y}_{n0}$, and $\widehat{Y}_{n1}$.

Market clearing requires that in each period changes in the demand for non-tradables goods (induced by various shocks) are equal to changes in the supply. Accordingly, differentiating equations (9) and (10) yields

$$(\eta_{n_0 p_{n0}} + \eta_{P_0 p_{n0}})\hat{p}_{n0} + \eta_{C_0 \alpha}\hat{\alpha}_{c1} + \widehat{W}_{c0} = \widehat{Y}_{n0}, \tag{11}$$

$$(\eta_{n_1 p_{n1}} + \eta_{P_1 p_{n1}})\hat{p}_{n1} + \eta_{C_1 \alpha}\hat{\alpha}_{c1} + \widehat{W}_{c0} = \widehat{Y}_{n1}, \tag{12}$$

where η denotes the elasticity of the variable indicated by the first subscript with respect to the variable indicated by the second subscript.[4)]

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4) Because the utility function U and the subutility functions C_0 and C_1 are

We assume that the trade account is balanced initially. Then from the definition of W_0,[5]

$$\widehat{W}_0 = (1-\gamma_s)\beta_{n0}\hat{p}_{n0} + \gamma_s\beta_{n1}\hat{p}_{n1} + (1-\gamma_s)(1-\beta_{n0})\widehat{Y}_{x0} + (1-\gamma_s)\beta_{n0}\widehat{Y}_{n0} + \gamma_s(1-\beta_{n1})\widehat{Y}_{x1} + \gamma_s\beta_{n1}\widehat{Y}_{n1}, \qquad (13)$$

where $\gamma_s = \alpha_{c1}C_1/W_{c0}$ is the share of saving in wealth $(0 < \gamma_s < 1)$. Since $W_{c0} = W_0/P_0$,

$$\widehat{W}_{c0} = \widehat{W}_0 - \beta_{n0}\hat{p}_{n0} \qquad (14)$$

Therefore,

$$\widehat{W}_{c0} = -\gamma_s(\beta_{n0}\hat{p}_{n0} - \beta_{n1}\hat{p}_{n1}) + \Psi. \qquad (15)$$

where

$$\Psi = [(1-\gamma_s)(1-\beta_n)\widehat{Y}_{x0} + (1-\gamma_s)\beta_n\widehat{Y}_{n0} + \gamma_s(1-\beta_n)\widehat{Y}_{x1} + \gamma_s\beta_n\widehat{Y}_{n1}]. \qquad (16)$$

Also, $\alpha_{c1} = \dfrac{P_1}{P_0}\alpha_{x1}$ implies

$$\hat{\alpha}_{c1} = \beta_{n1}\hat{p}_{n1} - \beta_{n0}\hat{p}_{n0}.^{6)} \qquad (17)$$

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homothetic, $\eta_{n_t z_t} = \eta_{x_t z_t} = \eta_{C_t W_{c0}} = 1$ for $t = 0,1$. Also note that, as shown by equation (3), $\eta_{P_t p_{nt}} = \beta_{nt}$ for $t = 0,1$.

5) Note that $d\alpha_{x1} = 0$ because r_{x0} is fixed in a small open economy.

6) Note that $d\alpha_{x1} = 0$ because r_{x0} is fixed.

Using the Slutsky decomposition, it can be shown that[7)]

$$\eta_{n_t p_{nt}} = -[(1-\beta_{nt})\sigma_{nx} + \beta_{nt}], \quad \eta_{x_t p_{nt}} = [\beta_{nt}(\sigma_{nx} - 1)], \tag{18}$$
$$\eta_{C_1 \alpha} = \gamma_s(\sigma - 1), \qquad \eta_{C_1 \alpha} = -[(1-\gamma_s)\sigma + \gamma_s]$$

where σ is an inter-temporal elasticity of substitution between C_0 and C_1, and σ_{nx} is a intra-temporal elasticity of substitution between c_n and c_x, that is,

$$\sigma = \left(\frac{d\log(C_0/C_1)}{d\log\left[\frac{\partial U}{\partial C_1} / \frac{\partial U}{\partial C_0}\right]} \right)_{dU=0}, \quad \sigma_{nx} = \left(\frac{d\log(c_{xt}/c_{nt})}{d\log\left[\frac{\partial C_t}{\partial c_{nt}} / \frac{\partial C_t}{\partial c_{xt}}\right]} \right)_{dC_t=0}.$$

If we substitute equations (15), (17) and (18) into equation (11) and (12), we get

$$-[\gamma_s\beta_{n0}\sigma + (1-\beta_{n0})\sigma_{nx}]\hat{p}_{n0} + \gamma_s\beta_{n1}\sigma\hat{p}_{n1} = -\Psi + \hat{Y}_{n0} \tag{19}$$

$$(1-\gamma_s)\beta_{n0}\sigma\hat{p}_{n0} - [(1-\gamma_s)\beta_{n1}\sigma + (1-\beta_{n1})\sigma_{nx}]\hat{p}_{n1} = -\Psi + \hat{Y}_{n1} \tag{20}$$

Subtracting (20) from (19) yields

$$-[\beta_{n0}\sigma + (1-\beta_{n0})\sigma_{nx}]\hat{p}_{n0} + [\beta_{n1}\sigma + (1-\beta_{n1})\sigma_{nx}]\hat{p}_{n1} = \hat{Y}_{n0} - \hat{Y}_{n1}. \tag{21}$$

If we assume that the expenditure shares on non-tradable goods do not vary over time[8)] (that is, $\beta_{n0} = \beta_{n1} = \beta_n$),

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7) See Appendix at the end of our paper.
8) This is satisfied, for example, in the case of a Cobb-Douglas intra-temporal utility function.

$$\left(\frac{\hat{p}_{n0}}{p_{n1}}\right) = \hat{p}_{n0} - \hat{p}_{n1} = \frac{\hat{Y}_{n1} - \hat{Y}_{n0}}{\beta_n \sigma + (1-\beta_n)\sigma_{nx}}. \tag{22}$$

Ceteris paribus, a temporary increase in the current output of non-tradables ($\hat{Y}_{n0} > 0$) lowers the current price of non-tradables relative to the future price of non-tradables while an expected future increase in output ($\hat{Y}_{n1} > 0$) increases the current price of non-tradables relative to the future price of non-tradables. A permanent increase in the output of non-tradables ($\hat{Y}_{n0} = \hat{Y}_{n1} > 0$) does not affect the relative price. Any change in the output of tradables, as long as it is not accompanied by a change in the output of non-tradables, does not affect the current price of non-tradables relative to the future price of non-tradables.

With the assumption of $\beta_{n0} = \beta_{n1} = \beta_n$, system of equations (19) and (20) become

$$-[\gamma_s \beta_n \sigma + (1-\beta_n)\sigma_{nx}]\hat{p}_{n0} + \gamma_s \beta_n \sigma \hat{p}_{n1} = -\Psi + \hat{Y}_{n0} \tag{19$'$}$$

$$\begin{aligned}(1-\gamma_s)\beta_n \sigma \hat{p}_{n0} - [(1-\gamma_s)\beta_n \sigma \\ + (1-\beta_n)\sigma_{nx}]\hat{p}_{n1} = -\Psi + \hat{Y}_{n1}\end{aligned} \tag{20$'$}$$

In matrix form,

$$\begin{bmatrix} -[\gamma_s \beta_n \sigma + (1-\beta_n)\sigma_{nx}] & \gamma_s \beta_n \sigma \\ (1-\gamma_s)\beta_n \sigma & -[(1-\gamma_s)\beta_n \sigma + (1-\beta_n)\sigma_{nx}] \end{bmatrix} \begin{bmatrix} \hat{p}_{n0} \\ \hat{p}_{n1} \end{bmatrix}$$
$$= \begin{bmatrix} -\Psi + \hat{Y}_{n0} \\ -\Psi + \hat{Y}_{n1} \end{bmatrix}.$$

Applying Cramer's rule, we get

$$\hat{p}_{n0} = \frac{\begin{vmatrix} -\Psi + \hat{Y}_{n0} & \gamma_s \beta_n \sigma \\ -\Psi + \hat{Y}_{n1} & -[(1-\gamma_s)\beta_n \sigma + (1-\beta_n)\sigma_{nx}] \end{vmatrix}}{\begin{vmatrix} -[\gamma_s \beta_n \sigma + (1-\beta_n)\sigma_{nx}] & \gamma_s \beta_n \sigma \\ (1-\gamma_s)\beta_n \sigma & -[(1-\gamma_s)\beta_n \sigma + (1-\beta_n)\sigma_{nx}] \end{vmatrix}} \tag{23}$$

$$= \frac{\Psi[\beta_n \sigma + (1-\beta_n)\sigma_{nx}] - \hat{Y}_{n0}[(1-\gamma_s)\beta_n \sigma + (1-\beta_n)\sigma_{nx}] - \hat{Y}_{n1}\gamma_s \beta_n \sigma}{(1-\beta_n)\sigma_{nx}[\beta_n \sigma + (1-\beta_n)\sigma_{nx}]},$$

$$\hat{p}_{n1} = \frac{\begin{vmatrix} -[\gamma_s \beta_n \sigma + (1-\beta_n)\sigma_{nx}] & -\Psi + \hat{Y}_{n0} \\ (1-\gamma_s)\beta_n \sigma & -\Psi + \hat{Y}_{n1} \end{vmatrix}}{\begin{vmatrix} -[\gamma_s \beta_n \sigma + (1-\beta_n)\sigma_{nx}] & \gamma_s \beta_n \sigma \\ (1-\gamma_s)\beta_n \sigma & -[(1-\gamma_s)\beta_n \sigma + (1-\beta_n)\sigma_{nx}] \end{vmatrix}} \tag{24}$$

$$= \frac{\Psi[\beta_n \sigma + (1-\beta_n)\sigma_{nx}] - \hat{Y}_{n1}[\gamma_s \beta_n \sigma + (1-\beta_n)\sigma_{nx}] - \hat{Y}_{n0}(1-\gamma_s)\beta_n \sigma}{(1-\beta_n)\sigma_{nx}[\beta_n \sigma + (1-\beta_n)\sigma_{nx}]}.$$

Differentiating the demand functions for current and future tradable goods

$$c_{x0} = c_{x0}[p_{n0}, P_0 C_0(\alpha_{c1}, W_{c0})] \text{ and } c_{x1} = c_{x1}[p_{n1}, P_1 C_1(\alpha_{c1}, W_{c0})],$$

we get

$$\hat{c}_{x0} = (\eta_{x_0 p_{n0}} + \eta_{P_0 p_{n0}})\hat{p}_{n0} + \eta_{C_0 \alpha}\hat{\alpha}_{c1} + \widehat{W}_{c0},$$

$$\hat{c}_{x1} = (\eta_{x_1 p_{n1}} + \eta_{P_1 p_{n1}})\hat{p}_{n1} + \eta_{C_1 \alpha}\hat{\alpha}_{c1} + \widehat{W}_{c0}.$$

Substituting equations (15), (17), (18) and assuming $\beta_{n0} = \beta_{n1} = \beta_n$, above two equations become

$$\hat{c}_{x0} = \beta_n \sigma_{nx}\hat{p}_{n0} - \gamma_s \sigma \beta_n(\hat{p}_{n0} - \hat{p}_{n1}) + \Psi, \tag{25}$$

$$\hat{c}_{x1} = \beta_n \sigma_{nx}\hat{p}_{n1} + (1-\gamma_s)\sigma \beta_n(\hat{p}_{n0} - \hat{p}_{n1}) + \Psi. \tag{26}$$

Substituting the solutions for $\hat{p}_{n0}$ and $\hat{p}_{n1}$ given in (23) and (24) into equations (25) and (26), and noting Ψ given by (16), we can represent $\hat{c}_{x0}$ and $\hat{c}_{x1}$ in terms of $\hat{Y}_{x0}$, $\hat{Y}_{x1}$, $\hat{Y}_{n0}$, and $\hat{Y}_{n1}$.

Ⅲ. Productivity Shocks, National Saving, and the Current Account

Using the solution to our model, we can derive impacts of various shocks to output on $Y_{x0} - c_{x0}$.[9] Since non-tradable goods market clears domestically each period, $c_{n0} = Y_{n0}$. Therefore $Y_{x0} - c_{x0}$ is the representative agent's private saving level under equilibrium. With the assumption of no government, this also corresponds to national saving. Since we are analyzing endowment economy, national saving equals current account balance. In the following, we explain the result under several different scenarios.

<u>A temporary increase in the current period output of tradable goods</u>
$(\hat{Y}_{x0} > 0,\ \hat{Y}_{n0} = \hat{Y}_{x1} = \hat{Y}_{n1} = 0)$

Note that, in this case, Ψ in equation (16) becomes $\Psi = (1-\gamma_s)(1-\beta_n)\hat{Y}_{x0}$. Furthermore, equation (22) shows that $\hat{p}_{n0} - \hat{p}_{n1} = 0$ in this case. Considering these and substituting the solution for $\hat{p}_{n0}$ given in equation (23) into equation (25), we get $\hat{c}_{x0} = (1-\gamma_s)\hat{Y}_{x0}$. Since $0 < \gamma_s < 1$, $\hat{c}_{x0} < \hat{Y}_{x0}$. Hence, the rate of change in consumption of tradable goods is smaller than the rate of change of the output of tradable goods. A temporary positive

9) This experiment raises the methodological issue of how shocks (i.e., unexpected changes) can occur in a model in which there is perfect foresight. Purist would probably reject such an intellectual experiment. We can think of the approach taken here as a short cut in which the shock had such a small probability of occurring that it did not affect the initial stationary equilibrium. Same approach appears in Blanchard and Fischer (1989), pp. 66-69, and Obstfeld and Rogoff (1996), pp. 110-111.

supply shock on tradable goods improves the current period trade account balance.

As $\hat{p}_{n0} - \hat{p}_{n1} = 0$ in this case. the time pattern of the non-tradable goods price is not affected. Therefore, as is manifested by equation (8), there is no effect on the domestic real rate of interest. As a result of the consumption smoothing motive, consumption rises less than the amount of the increase in output. The trade balance improves as a result.

An expected increase in the future period output of tradable goods ($\hat{Y}_{x1} > 0,\ \hat{Y}_{x0} = \hat{Y}_{n0} = \hat{Y}_{n1} = 0$)

Equations (16) and (22) shows that $\Psi = (1-\gamma_s)(1-\beta_n)\hat{Y}_{x0}$ and $\hat{p}_{n0} - \hat{p}_{n1} = 0$ in this case. Considering these and substituting the solution for $\hat{p}_{n0}$ given in equation (23) into equation (25), we get $\hat{c}_{x0} = \gamma_s \hat{Y}_{x1}$. Therefore $\hat{c}_{x0} > \hat{Y}_{x0} = 0$. The rate of change of the consumption of tradable goods exceeds the rate of change of the output of tradable goods. An expected positive supply shock on tradable goods deteriorates the current period trade account balance.

As above, there is no effect on the real rate of interest. Only consumption smoothing plays a role here. Consumption rises even before the actual increase in output happens. Therefore the current period trade balance deteriorates.

A permanent increase in the output of tradable goods ($\hat{Y}_{x0} = \hat{Y}_{x1} = \hat{Y}_x > 0,\ \hat{Y}_{n0} = \hat{Y}_{n1} = 0$)

Equations (16) and (22) shows that $\Psi = (1-\gamma_s)(1-\beta_n)\hat{Y}_x + \gamma_s(1-\beta_n)\hat{Y}_x = (1-\beta_n)\hat{Y}_x$ and $\hat{p}_{n0} - \hat{p}_{n1} = 0$ in this case. Considering these and substituting the solution for $\hat{p}_{n0}$ given in equation (23) into equation (25), we get $\hat{c}_{x0} = \hat{Y}_x$. The rate of change of the consumption of tradable goods is equal to the rate of change of the output of tradable goods. A permanent positive supply shock on tradable goods does not affect the current period trade account balance.

Again there is no effect on the real interest rates. Also, there is no consumption smoothing because the increase in output is permanent. Therefore there is no effect on the trade balance.

<u>A temporary increase in the current period output of non-tradable goods</u>
$(\hat{Y}_{n0} > 0,\ \hat{Y}_{x0} = \hat{Y}_{x1} = \hat{Y}_{n1} = 0)$

In this case, Ψ in equation (16) becomes $\Psi = (1-\gamma_s)\beta_n \hat{Y}_{n0}$. Furthermore, equation (22) shows that $\hat{p}_{n0} - \hat{p}_{n1} = -\hat{Y}_{n0}/(\beta_n\sigma + (1-\beta_n)\sigma_{nx})$ in this case. If we substitute the relevant value of Ψ into equation (23), we get

$$\hat{p}_{n0} = \frac{(1-\gamma_s)\beta_n[\beta_n\sigma + (1-\beta_n)\sigma_{nx}] - [\beta_n(1-\gamma_s)\sigma + (1-\beta_n)\sigma_{nx}]}{(1-\beta_n)\sigma_{nx}[\beta_n\sigma + (1-\beta_n)\sigma_{nx}]}\hat{Y}_{n0}.$$

Substituting these into equation (25), we get

$$\begin{aligned}\hat{c}_{x0} &= \beta_n\sigma_{nx}\hat{p}_{n0} + \gamma_s\beta_n\sigma(\hat{p}_{n1} - \hat{p}_{n0}) + (1-\gamma_s)\beta_n\hat{Y}_{n0} \\ &= \frac{\gamma_s\beta_n(\sigma - \sigma_{nx})}{\beta_n\sigma + (1-\beta_n)\sigma_{nx}}.\end{aligned}$$

We see that $\hat{c}_{x0} > 0$ if and only if $\sigma > \sigma_{nx}$. As long as the inter-temporal elasticity of substitution between the current and future period consumption is greater than the intra-temporal elasticity of substitution between the tradable and non-tradable goods, the rate of change of the consumption of tradable goods exceeds the rate of change of the output of tradable goods (which is zero in this case). A temporary positive supply shock on non-tradable goods deteriorates the current period trade account balance.

The current price of non-tradables falls relative to the future price of non-tradables. As a result, the domestic real rate of interest which is measured in terms of consumption basket falls. This makes consumption (of tradables as well as non-tradables) tilted toward the current period. This is an inter-temporal substitution effect. The magnitude of this effect depends on the inter-temporal elasticity of substitution, σ. But the price of tradables relative to the price of non-tradables is now higher as a result of an increase in the current output of non-tradables. This causes consumption to shift away from tradables toward non-tradables. This is a intra-temporal substitution effect of which the magnitude depends on the intra-temporal elasticity of substitution, σ_{nx}. Whether the current consumption of tradables rises or falls depends on the relative magnitudes of inter-temporal and intra-temporal substitution effects. If the inter-temporal substitution effect dominates the intra-temporal substitution effect, $\sigma > \sigma_{nx}$, current consumption of tradables rises and the trade balance worsens.

The result is in stark contrast with the case of a temporary increase in the output of tradable goods where only a consumption smoothing effect takes place.

<u>An expected increase in the future period output of non-tradable goods</u>
$(\widehat{Y}_{n1} > 0,\ \widehat{Y}_{n0} = \widehat{Y}_{x0} = \widehat{Y}_{x1} = 0)$

In this case, Ψ in equation (16) becomes $\Psi = \gamma_s \beta_n \widehat{Y}_{n1}$.

Furthermore, equation (22) shows that $\hat{p}_{n0} - \hat{p}_{n1} = \hat{Y}_{n1} / (\beta_n \sigma + (1 - \beta_n) \sigma_{nx})$ in this case. If we substitute the relevant value of Ψ into equation (23), we get

$$\hat{p}_{n0} = \frac{\gamma_s \beta_n [\beta_n \sigma + (1 - \beta_n) \sigma_{nx}] - \beta_n \gamma_s \sigma}{(1 - \beta_n) \sigma_{nx} [\beta_n \sigma + (1 - \beta_n) \sigma_{nx}]} \hat{Y}_{n1}.$$

Substituting these into equation (25), we get

$$\begin{aligned} \hat{c}_{x0} &= \beta_n \sigma_{nx} \hat{p}_{n0} + \gamma_s \beta_n \sigma (\hat{p}_{n1} - \hat{p}_{n0}) + \gamma_s \beta_n \hat{Y}_{n1} \\ &= \frac{- \gamma_s \beta_n (\sigma - \sigma_{nx})}{\beta_n \sigma + (1 - \beta_n) \sigma_{nx}}. \end{aligned}$$

Therefore $\hat{c}_{x0} < 0$ if and only if $\sigma > \sigma_{nx}$. As long as the inter-temporal elasticity of substitution between the current and future period consumption is bigger than the intra-temporal elasticity of substitution between the tradable and non-tradable goods, the rate of change of the consumption of tradable goods is smaller than the rate of change of the output of tradable goods (which is zero in this case). An expected positive supply shock on non-tradable goods improves the current period trade account balance.

The inter-temporal substitution effect causes current consumption of tradables to fall (because the real rate of interest is now higher) and the intra-temporal substitution effect causes current consumption of tradables to rise. If the inter-temporal substitution effect dominates the intra-temporal substitution effect, $\sigma > \sigma_{nx}$, current consumption of tradables falls and the trade balance improves.

<u>A permanent increase in the output of non-tradable goods</u>

$(\hat{Y}_{n0} = \hat{Y}_{n1} = \hat{Y}_n > 0,\ \hat{Y}_{x0} = \hat{Y}_{x1} = 0)$

Ψ in equation (16) becomes $\Psi = (1-\gamma_s)\beta_n \hat{Y}_n + \gamma_s \beta_n \hat{Y}_n = \beta_n \hat{Y}_n$. Equation (22) shows that $\hat{p}_{n0} - \hat{p}_{n1} = 0$. If we substitute the relevant value of Ψ into equation (23), we get

$$\hat{p}_{n0} = -\frac{1}{\sigma_{nx}} \hat{Y}_n .$$

Substituting these into equation (25), we get $\hat{c}_{x0} = 0$.[10)]

A permanent positive supply shock on non-tradable goods does not affect the current period trade account balance. Since the change in output is uniform over the periods, the real rate of interest is not affected. Therefore there is no inter-temporal substitution. Also, consumption smoothing plays no role when the change in output is uniform over the periods.

Ⅳ. Comparison with the Discussion in Vegh (2012)

After I finished writing the earlier draft of this paper, I was surprised to be informed about the existence of a not yet published manuscript by Carlos Vegh (2012). (See Chapter 4, particularly pp.10-13.) Even though some of the key results presented in this paper overlap with those in Vegh (2012), there are some novel contribution by this paper.

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10) This result can be acquired more quickly by the following argument. $\hat{c}_{x0}$ from the permanent increase in the output of nontradabes is the sum of $\hat{c}_{x0}$ from current increase in the output of non-tradables and $\hat{c}_{x0}$ from the future increase in the output of non-tradables. If we add the results from the previous two scenarios, we get $\hat{c}_{x0} = 0$.

Vegh set up an infinite horizon endowment economy with tradable and non-tradable goods. His argument is based on a necessary condition for representative agent's optimization. Key equation for his argument is equation (22) in p. 11 of the manuscript, which is reproduced below. Along a prefect foresight equilibrium path (i.e., for a given Lagrangian multiplier λ),

$$\frac{dc_t^T}{dy_t^N} = \frac{u_{c^T c^N}(c_t^T, y_t^N)}{u_{c^T c^T}(c_t^T, y_t^N)} \begin{cases} = 0, & \text{if } u_{c^T c^N} = 0, \\ < 0, & \text{if } u_{c^T c^N} < 0, \\ > 0, & \text{if } u_{c^T c^N} > 0. \end{cases}$$

c^T is the tradable goods consumption, y^N is the endowment of non-tradable goods, u is the period utility function, and $u_{c^T c^N}$ etc. denotes cross partial derivative. Whether consumption of tradable goods rises (i.e., trade balance deteriorates) or falls (i.e., trade balance improves) when the endowment of non-tradable goods rises, depends on the cross partial derivate of the utility function. However, readers do not get any idea how the cross partial derivates relate to intra-temporal and inter-temporal elasticity of substitutions. At the end of the chapter, Vegh provides a specific form of the utility function and asks a reader to show that the sign of the cross partial derivate relates specifically to the relative size of intra- and inter-temporal elasticity of substitutions. But the result is only with the specific form of the utility function. Furthermore, Vegh's discussion does not include different effects from temporary versus permanent changes or current versus expected changes.

In comparison, we derived the result with much more general utility function (that is, we only assumed homothetic and linearly homogeneous functions), and the derivation clearly showed how the intra- and inter-temporal elasticity of substitution interact in the dynamic optimization. Furthermore, we showed different outcome from temporary versus permanent changes and the current versus expected changes.

Ⅴ. Concluding Remarks

In this paper we set up a general equilibrium model of a small open endowment economy with tradable and non-tradable goods. It is shown analytically that, as long as the inter-temporal elasticity of substitution in consumption is greater than the intra-temporal elasticity of substitution in consumption, a temporary increase in the productivity of the non-tradable goods sector deteriorates the current account balance whereas an expected increase in the productivity of the non-tradable goods sector improves the current account balance. This is in stark contrast with the case of an improvement in the productivity of the tradable goods sector.[11)]

Surely, our result is conditional on our assumption of endowment economy. In a model that features endogenous capital accumulation and output, current account balance is no longer equal to the national saving. It becomes equal to the national saving minus domestic investment. Therefore there is possibility that the current period increase in the productivity of the tradable goods sector leads to a deterioration of the current account balance if the productivity shock is highly persistent and leads to a significant

11) In deriving our result analytically, we made following assumptions: (1) the lifetime utility function U is homothetic; (2) the subutility functions C_0 and C_1 are linearly homogeneous; (3) the expenditure share on non-tradable goods do not vary over time; and (4) initially, the trade account is balanced. Assumptions (1) and (2) are satisfied in most of the real business cycle models in the literature. With short-run business cycle analysis, it is not unreasonable to assume (3).

rise in the domestic investment. In order to highlight the dynamics that involve consumers' inter-temporal consumption decision, we abstracted away from investment decision and assumed endowment economy.

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Appendix

Derivation of equations listed in (18) is explained in this appendix. Consider a situation where a consumer chooses the utililty maximizing combination of two goods c_1 and c_2 with a given budget z. p_1 and p_2 represent prices of c_1 and c_2. That is, we consider the following problem: $\underset{\{c_1, c_2\}}{Maximize}\, U(c_1, c_2)$ *subject to* $p_1 c_1 + p_2 c_2 = z$.

Slutsky decompostion equation shows the decomposition of the effect of a change in the price of good j on the demand for good i into the substitution effect and the income effect.

$$\frac{\partial c_i}{\partial p_j} = \left(\frac{\partial c_i}{\partial p_j}\right)_{dU=0} - c_j \frac{\partial c_i}{\partial z} \text{ for all } i, j = 1, 2$$

If we multiply each term by p_j / c_i and do some simple manipulation, we get Slutsky decomposition in elasticity form,

$$\frac{\partial c_i}{\partial p_j}\frac{p_j}{c_i} = \left(\frac{\partial c_i}{\partial p_j}\frac{p_j}{c_i}\right)_{dU=0} - \frac{\partial c_i}{\partial z}\frac{z}{c_i}\frac{p_j c_j}{z} \text{ for all } i, j = 1, 2$$

We write above equation more compactly as

$$\eta_{c_i p_j} = (\eta_{c_i p_j})_{dU=0} - \frac{p_j c_j}{z}\eta_{c_i z} \text{ for all } i, j = 1, 2 \qquad \text{(a1)}$$

$\eta_{c_i p_j}$ is the price elasticity of demand, $(\eta_{c_i p_j})_{dU=0}$ is the compensated price elasticity of demand, and $\eta_{c_i z}$ is the income elasticity of demand.

Elasticity of substitution between c_1 and c_2 is, by definition,

$$\sigma_{12}=\left(\frac{d\log(c_2/c_1)}{d\log\left[\frac{\partial U}{\partial c_1}/\frac{\partial U}{\partial c_2}\right]}\right)_{dU=0}=\left(\frac{d\log(c_2/c_1)}{d\log(p_1/p_2)}\right)_{dU=0} \qquad \text{(a2)}$$

$$=\left(\frac{d\log(c_2)-d\log(c_1)}{d\log(p_1)-d\log(p_2)}\right)_{dU=0}$$

Along a given indifference curve,

$$\frac{\partial U}{\partial c_i}dc_1+\frac{\partial U}{\partial c_2}dc_2=0.$$

If we combine the above with the relationship

$$\frac{\partial U}{\partial c_1}/\frac{\partial U}{\partial c_2}=\frac{p_1}{p_2},$$

we get

$$\frac{dc_2}{dc_1}=-\frac{p_1}{p_2}.$$

Put it differently, $p_jc_j\frac{dc_j}{c_j}=-p_ic_i\frac{dc_i}{c_i}$ and therefore $p_2c_2d\log(c_2)=-p_1c_1d\log(c_1)$.

If we substitute $d\log(c_2)=-\frac{p_1c_1}{p_2c_2}d\log(c_1)$ into (a2) with $i=1$, $j=2$ and make $d\log(p_1)=0$, we get

$$\left(\frac{\partial c_1}{\partial p_2}\frac{p_2}{c_1}\right)_{dU=0}=(\eta_{c_1p_2})_{dU=0}=\frac{p_2c_2}{z}\sigma_{12}. \qquad \text{(a3)}$$

Similarly, if we substitute $d\log(c_1)=-\frac{p_2c_2}{p_1c_1}d\log(c_2)$ into (a2) with

$i=2$, $j=2$ and make $d\log(p_1)=0$, we get

$$\left(\frac{\partial c_2}{\partial p_2}\frac{p_2}{c_2}\right)_{dU=0}=(\eta_{c_2p_2})_{dU=0}=-\left(1-\frac{p_2c_2}{z}\right)\sigma_{12}. \tag{a4}$$

Combining (a1) and (a3), we get

$$\eta_{c_1p_2}=\frac{p_2c_2}{z}\sigma_{12}-\frac{p_2c_2}{z}\eta_{c_1z}, \tag{a5}$$

and combining (a1) and (a4), we get

$$\eta_{c_2p_2}=-\left(1-\frac{p_2c_2}{z}\right)\sigma_{12}-\frac{p_2c_2}{z}\eta_{c_2z}. \tag{a6}$$

Notations used in this appendix and the notations used in the main part of the paper is summarized by the following table.

Notation used in appendix	Notation used in intra-temporal optimization	Notation used in inter-temporal optimization
c_1	c_{xt}	C_0
p_1	1	1
c_2	c_{nt}	C_1
p_2	p_{nt}	α_{c1}
$\frac{p_2c_2}{z}$	β_{nt}	γ_s
η_{c_1z}, η_{c_2z}	$\eta_{x_tz}=\eta_{n_tz}=1$	$\eta_{C_0W_{c0}}=\eta_{C_1W_{c0}}=1$
σ_{12}	σ_{nx}	σ

If we apply (a6) to the intra-temporal optimization, we get $\eta_{n_tp_{nt}}=-(1-\beta_{nt})\sigma_{nx}-\beta_{nt}$.

If we apply (a5) to the intra-temporal optimization, we get $\eta_{x_t p_{nt}} = \beta_{nt}\sigma_{nx} - \beta_{nt}$.

If we apply (a5) to the inter-temporal optimization, we get $\eta_{C_0 \alpha} = \gamma_s \sigma - \gamma_s$.

If we apply (a6) to the inter-temporal optimization, we get $\eta_{C_1 \alpha} = -(1-\gamma_s)\sigma - \gamma_s$.

교역재 - 비교역재 2부문 모형에서 생산성 충격, 국민저축, 경상수지의 관계

김 건 홍*

논문초록

본 연구는 교역재와 비교역재가 존재하는 2부문 동태일반균형모형을 사용하여 생산성 충격이 국민저축과 경상수지에 미치는 영향을 비교하여 분석하였다. 대표적 경제주체의 기간간 효용함수에 관하여 일반적인 가정을 한 다음 분석의 단순화를 위하여 2기간 부존경제 모형을 사용하여 결과를 분석적으로 도출하였다. 현재소비와 미래소비간의 대체탄력성이 교역재 소비와 비교역재 소비간의 기간내 대체탄력성을 초과한다면 비교역재 부문에서의 임시적이고 긍정적인 생산성 충격은 국민저축을 감소시키고 경상수지를 악화시킴을 보였다. 이는 교역재 부문에서의 임시적이고 긍정적인 생산성 충격이 국민저축을 증가시키고 경상수지를 개선하는 것과 대조됨을 보였다.

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* 한림대학교 경제학과 부교수, e-mail: kimku@hallym.ac.kr