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The Profitability of the Second Product Supplied by an Oligopolist

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I investigate the possibility of product proliferation. It has been asserted that under horizontal differentiation whenever firms decide sequentially upon product specifications and prices, supplying a new product is unprofitable for the firm. This paper, however, shows that the result depends on the number of firms in the industry, the discount factor for future profits, and transportation costs. Specifically, in a circular model, when the transportation cost is quadratic in distance, producing more than one product is a subgame perfect equilibrium if there are more than three firms (when franchised) or more than two firms (when branched) in the industry. However, the equilibrium profits with two products are lower than those with a single product. Thus, in a repeated game, supplying two products is not the only equilibrium. Trigger strategy also constitutes an equilibrium if the discount factor is sufficiently large. However, as the number of firms in the industry increases, the possible gain in current profit by supplying another product becomes greater, so that, with a given discount factor, firms are more likely to supply two products. Consequently, for a given discount factor, there exists an n, such that, if the number of firms is greater than n, each firm will choose multiple products in equilibrium.

I. Introduction

It has been asserted that under horizontal differentiation whenever firms decide sequentially upon product specifications and prices, they give up the possibility of product proliferation. This paper, however, shows that the result depends upon the

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number of firms in the industry, the discount factor for future profits, and transportation costs.

Selling several differentiated products in the same market is common practice for large corporations in the real world. Whether or not the production of more than one product is profitable has been analyzed in the recent literature. Brander and Eaton [2] first examined competition between multiproduct firms, which are allowed to sell a pair of products each, and found each firm to produce only one brand. Martinez-Giralt and Neven [13] and Martinez-Giralt [12] analyzed a two-stage game where duopolists first choose locations of their outlets and then prices (location of each outlet might be considered as product specification). They also found that the only subgame perfect equilibrium is the collapse of the two outlets of each firm into a single point. The justification for this result is that the entailed price competition from opening a new outlet is so intense that firms try to relax it by locating their outlets as far apart as possible. In other words, the loss from a lower price dominates the potential gain from a greater market share when a firm opens its second outlet.

However, it is true that, as the number of firms in the industry increases, the positive market share effect of opening an additional outlet becomes more important, relative to the negative price effect. Eventually, when the number of firms exceeds a critical number, each firm can increase total profits by opening another outlet, given that the other firms do not open a new outlet at the same time. Therefore, in the case, sustaining a single outlet is no longer a Nash equilibrium. Furthermore, under the assumption of symmetry of firms, every firm gains from having two outlets, regardless of the other firms' decisions. Hence, in a one-shot competition, the only Nash equilibrium is obtained when every firm has two outlets.

However, the equilibrium profits with two outlets are lower than those with a single outlet (because prices are lower while the market shares are identical). Thus, in a repeated game, opening two outlets may not be the only equilibrium. Consider trigger strategies: each firm has a single outlet in period 0. It furthermore sustains one outlet in period t if in every period preceding t the other firms open only one outlet each; otherwise it opens the second outlet forever. These strategies constitute an equilibrium if the discount factor is sufficiently large. However, as the number of firms in the industry increases, the possible gain in current profit by opening another outlet becomes greater, so that, with a given discount factor, firms are more likely to have two outlets. Consequently, for a given discount factor, there exists an n, such that, if the number of firms is greater than n, each firm will choose multiple outlets in equilibrium.

In Section Ⅱ, I examine a subgame perfect equilibrium in a one-shot game. In Section Ⅲ, I look for an equilibrium in the repeated game. Section Ⅳ summarizes the conclusions.

Ⅱ. Perfect Equilibrium in a One-shot Game

This analysis is based on a one-shot non-cooperative game, which consists of three stages: In the first stage, each firm chooses whether or not to open a second outlet.[1] Having observed the choices, in the second stage, each firm decides the optimal locations of its outlets. Then, having all the information on the decisions, each firm sets the optimal prices in the final stage.

I adopt Subgame Perfect Equilibrium as a solution concept. A subgame perfect equilibrium in this context can be viewed as a Nash equilibrium in a game in which firms make decisions in each stage, aware of the Nash equilibrium that will occur in the subsequent stages for each of their choices.

In Subsection Ⅱ. 1, I derive the subgame perfect equilibrium in the case that each outlet owned by the same firm is operated independently. In Subsection Ⅱ. 2, I consider the case that two outlets owned by the same firm are operated to maximize their joint profit.

1. Equilibrium under Independent Operation

Two-outlet firm may run each outlet independently. An example might be different divisions within a firm with seperate profit centers and little central control. Even though multioutlet firms maximize their joint profit in most cases. I analyze the case of independent operation for the analytical purpose.

In Ⅱ. 1. 1), I find the equilibrium profit of a single-outlet firm, given that the other firms are also restricted to have a single outlet. In Ⅱ. 1. 2), I derive the profit of the two-outlet firm, show that the gain from opening a new outlet changes as the total number of firms in the industry varies, and find that sustaining a single

1) For simplicity, I assume that each firm can have at most two outlets. Furthermore, no fixed cost of opening a new outlet is assumed. However, relaxation of this assumption does not affect the existence of the number of firms, such that, if the number of firms in the industry is greater, each firm will choose two outlets in equilibrium.

outlet is no longer an equilibrium when there are more than three firms in the industry. In Ⅱ. 1. 3), I characterize the subgame perfect equilibrium, in which every firm has two outlets.

1) The Profit of a Single-Outlet Firm

The specification of the model is as follows: Consumers are uniformly distributed over the unit circle with the same reservation price. The reservation price will be arbitrarily large so that each consumer will buy from the outlet for which the delivered price is lowest. Consumers thus bear the transportation cost which is assumed to be quadratic in distance.

Consider an industry with two firms: As in Figure 1, firm A locates at point a and firm B locates at point b, which is apart from point a by the distance of $b-a$. Production cost is assumed to be zero, without loss of generality. We also pick an origin for normalization, i. e., $a=0$. Since the transportation cost is quadratic, the set of consumers, say v and w, who are just indifferent between product a at price p_a and product b at price p_b is obtained from

$$p_a+v^2=p_b+(b-v)^2 \quad \text{and}$$
$$p_b+(w-b)^2=p_a+(1-w)^2.$$

Thus

$$v=\frac{1}{2}\left(\frac{p_b-p_a}{b}+b\right), \quad \text{and} \quad w=\frac{1}{2}\left(\frac{p_a-p_b}{1-b}+1+b\right).$$

Therefore, the profit functions are

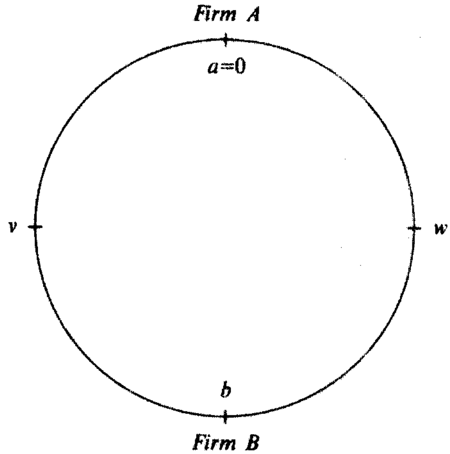
$$\pi_a=p_a(v+1-w) \quad \text{and}$$
$$\pi_b=p_b(w-v).$$

For any location of b decided in the second stage, firms maximize their profits with respect to prices in the third stage of the game. Solving the first-order conditions simultaneously,

$$\frac{\partial\pi_a}{\partial p_a}=\frac{1}{2}\left(\frac{p_b-2p_a}{b}-\frac{2p_a-p_b}{1-b}+1\right)=0, \quad \text{and}$$

$$\frac{\partial\pi_b}{\partial p_b}=\frac{1}{2}\left(\frac{p_a-2p_b}{b}-\frac{2p_b-p_a}{1-b}+1\right)=0,$$

〈Figure 1〉



we obtain the equilibrium prices $p_a^* = p_b^* = b(1-b)$, which are the functions of b.

We now turn to the second stage of the game. Aware of the Nash equilibrium choice that will occur in the subsequent stages for each of their choices, each firm chooses its optimal location. Since the location of firm A is fixed at a=0, to find b, we substitute the optimal prices into the profit function and maximize with respect to the location b. Thus, from

$$\pi_b^* = \frac{b(1-b)}{2}$$

〈Table 1〉

number of the firms in the industry	equilibrium locations	equilibrium market share of each firm	equilibrium price of each firm	equilibrium profit of each firm
2	0, 1/2	1/2	1/4	1/8
3	0, 1/3, 2/3	1/3	1/9	1/27
4	0, 1/4, 2/4, 3/4	1/4	1/16	1/64
5	0, 1/5, 2/5, 3/5, 4/5	1/5	1/25	1/125
⋮	⋮	⋮	⋮	⋮
n	0, $1/n, \cdots, (n-1)/n$	$1/n$	$1/n^2$	$1/n^3$

we have

$$\frac{\partial \pi_b^*}{\partial b} = \frac{1-2b}{2} = 0.$$

Then, we find that b^* is 1/2, p_a^* and p_b^* are 1/4, and π_a^* and π_b^* are 1/8.

It is possible to derive the equilibrium locations, prices, and profits for the cases of more than two firms in the industry. The results are summarized in Table 1.

2) The Profitability of the Second Outlet

In this subsection, I analyze the profit change when a firm opens the second outlet in a distinct location, while the rest of the firms are restricted to have one outlet. When there are n firms, one of which has two outlets, the equilibrium prices and market shares are the same as in $(n+1)$-firm-single-outlet equilibrium. Thus the two-outlet firm obtains a profit which is twice the profit of any other firm. Table 2 shows how the profit of the two-outlet firm changes as the number of the other firms increases.

By comparing Table 1 and Table 2, we can see whether or not opening the second outlet increases profits. That is, when there are two or three firms in the industry, the profit decreases from opening the second outlet. However, when there are four or more firms, the profit increases. To decompose the profit change, we define

〈**Table 2**〉

number of firms in the industry	equilibrium market share of the two-outlet firm	equilibrium price of each outlet	equilibrium profit of the two-outlet firm
2	2/3	1/9	2/27
3	2/4	1/16	2/64
4	2/5	1/25	2/125
5	2/6	1/36	2/216
⋮	⋮	⋮	⋮
n	$2/(n+1)$	$1/(n+1)^2$	$2/(n+1)^3$

$$\hat{p}(n) = \frac{\tilde{p}(n)}{p(n)} = \frac{1/(n+1)^2}{1/n^2} = \frac{n^2}{(n+1)^2}, \quad \text{and}$$

$$\hat{d}(n) = \frac{\tilde{d}(n)}{d(n)} = \frac{2/(n+1)}{1/n} = \frac{2n}{n+1},$$

where $p(n)$ = the equilibrium price when there are n firms with a single outlet; $\tilde{p}(n)$ = the equilibrium price when there are n firms and one of them has two outlets; $d(n)$ = the equilibrium market share when there are n firms having only one outlet; and $\tilde{d}(n)$ = the market share of the two-outlet firm when there are $n-1$ other sin-

(Table 3)

n	$\hat{p}(n)$	$\hat{d}(n)$	$\hat{\pi}(n)$
2	0.4444	1.3333	0.5926
3	0.5625	1.5000	0.8438
4	0.6400	1.6000	1.0240
5	0.6944	1.6667	1.1574
6	0.7347	1.7143	1.2595
⋮	⋮	⋮	⋮
$\to\infty$	$\to 1$	$\to 2$	$\to 2$

gle-outlet firms. Table 3 shows how $\hat{p}(n)$ and $\hat{d}(n)$ change when n increases. From the table, we find that the price decreases at a decreasing rate. As the number of firms becomes very large, the decrease in price becomes almost insignificant. On the other hand, the market share increases at an increasing rate. Finally, as n increases, the two-outlet firm can almost double its market share. Therefore, the effect of a decrease in price is eventually dominated by the effect of an increase in market share.

If we define the rate of profit change as

$$\hat{\pi}(n)=\frac{\tilde{\pi}(n)}{\pi(n)}=\frac{\tilde{p}(n)\tilde{d}(n)}{p(n)d(n)}=\hat{p}(n)\hat{d}(n)$$

(where $\pi(n)$ = the profit when there are n firms with a single outlet, and $\tilde{\pi}(n)$ = the profit of the two-outlet firm when there are $n-1$ other single-outlet firms), we find, in Table 3, that it exceeds one when $n \geq 4$. That is, when there are two or three firms in the industry, opening the second outlet is unprofitable to any firm, due to the dominance of the negative price effect. Hence, sustaining the single-outlet is a Nash equilibrium in this case. However, when there are four or more firms, it is profitable to have two outlets, due to the dominance of the market-share effect. Hence, having one outlet is not a Nash equilibrium.

3) Equilibrium

Each firm chooses its number of outlets, m^1, depending on the decisions of the other firms, (m^2, m^3, m^4). By symmetry of firms, we know that, for example, (1,2,2) describes the same situation as (2,1,2) and (2,2,1). Thus, according to Table 4, in any situation, the representative firm can achieve a higher profit by opening its second outlet. Since the other firms make the same choice, $(m^1, m^2, m^3, m^4) = (2,2,2,2)$ will be a unique Nash equilibrium. Furthermore, for any $n > 4$, every firm's

⟨Table 4⟩

		other firm's decision (m^2, m^3, m^4)			
		(1, 1, 1)	(1, 1, 2)	(1, 2, 2)	(2, 2, 2)
the representative	$m^1=1$	1/64	1/125	1/216	1/343
firm's choice	$m^2=2$	2/125	2/216	2/343	2/512

having two outlets is also a equilibrium (shown in Appendix A1).

2. Equilibrium Under Joint Operation

In this subsection, I will examine the case where the two-outlet firm takes into consideration the interaction between its outlets, and maximizes the joint profit. Comparing the independent running case, the two-outlet firm can achieve a higher level of the total profit.

In Ⅱ. 2. 1), I examine the profitability of the second outlet, and find that sustaining a single outlet is no longer a Nash equilibrium when there are three or more firms in the industry. In Ⅱ. 2. 2), I characterize the subgame perfect equilibrium, in which each firm has two outlets.

1) The Profitability of the Second Outlet

When there are two firms in the industry and one of them has two outlets, as in Figure 2, the marginal consumers indifferent between two neighboring products are defined as:

$$v=\frac{1}{2}\left(\frac{p_{b1}-p_a}{b_1}+b_1\right)$$

$$w=\frac{1}{2}\left(\frac{P_{b2}-P_{b1}}{b_2-b_1}+b_1+b_2\right)$$

$$x=\frac{1}{2}\left(\frac{P_a-P_{b2}}{1-b_2}+1+b_2\right)$$

While firm A maximizes its profit

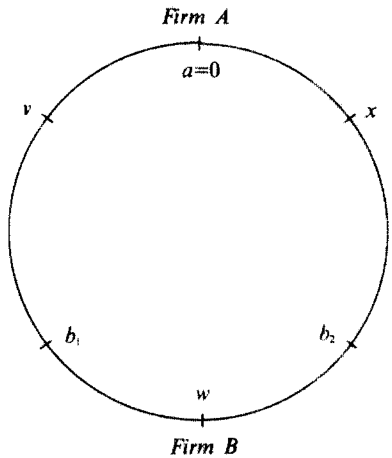
$$\pi_a=p_a(v+1-x),$$

firm B maximizes the joint profit from its outlets b_1 and b_2,

$$\pi_b=\pi_{b1}+\pi_{b2}=p_{b1}(w-v)+p_{b2}(x-w).$$

From maximizing the profit functions with respect to price, we can find the equi-

〈**Figure 2**〉



librium prices, p^*_a, $p^*_{b_1}$, $p^*_{b_2}$. Turning to the second stage of the game, we substitute the equilibrium prices into the profit functions to find π^*_a and π^*_a. We may then find the equilibrium locations by setting

$$\frac{\partial \pi^*_b}{\partial b_1} = 0, \quad \text{and} \quad \frac{\partial \pi^*_b}{\partial b_2} = 0.$$

Our assumption of symmetry gives us $b^*_2 = 1 - b^*_1$, which we can use to find b^*_1

$$\frac{\partial \pi^*_b}{\partial b_1} = (b_1 - 2)(3b_1 - 2)/18 = 0.$$

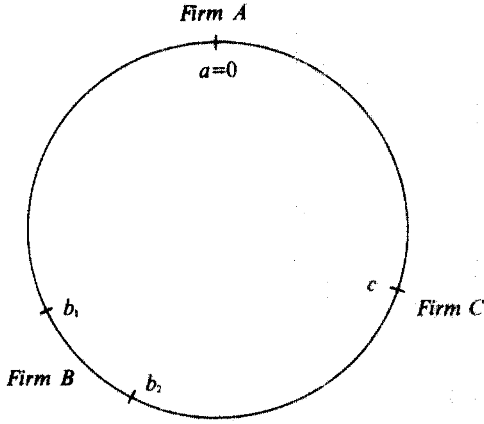
However, $\partial\pi_b{}^*/\partial b_1$ is positive over the entire interval [0, 0.5], which indicates that firm B will maximize profits by collapsing the outlets b_1 and b_2 into a single point at 0.5. That is, when there are two firms in the industry, neither firm has an incentive to open two outlets.

On the other hand, the picture changes when there are more than two firms in the industry. For instance, if firm B has two outlets, b_1 and b_2, and the other two firms, A and C, have a single outlet each, as in Figure 3,[2] the profit function will be:

$$\pi_a = p_a(v + 1 - y)$$
$$\pi_b = p_{b1}(w - v) + p_{b2}(x - w)$$
$$\pi_c = p_c(y - x)$$

2) As it is shown in the appendix A3, firms prefer neighboring positions for their outlets to separated positions.

〈Figure 3〉



By the same way, we obtain p_a^*, p_{a1}^*, p_{b2}^*, and p_c^*. Substituting the equilibrium prices into the profit functions, we get π_a^*, π_b^*, and π_c^*. Then, from

$$\frac{\partial \pi^*_b}{\partial b_1} = 0, \ \frac{\partial \pi^*_b}{\partial b_2} = 0, \quad \text{and} \ \frac{\partial \pi^*_c}{\partial b_3} = 0,$$

we find the equilibrium locations, prices, market shares, and profits. These are summarized in Table 5.[3]

As we see in Table 5, π_b^* is 0.03926, which is greater than the profit in the case where firm B has only one outlet(=0.03704). Thus, we find that, if there are three firms in the industry, each firm can achieve a higher profit by opening its second outlet and managing them jointly. Hence, sustaining a single outlet is not a Nash equilibrium in this case.

〈Table 5〉

outlet	location	price	market share	profit
a	0	0.09313	0.31781	0.02960
b_1	0.29575	0.10776	0.18219	0.01963
b_2	0.41385	0.10776	0.18219	0.01963
c	0.70960	0.09313	0.31781	0.02960

3) I used Mathematica 2.0 for MS/DOS 386/7 to obtain all the numerical results shown in this paper.

〈Table 6〉

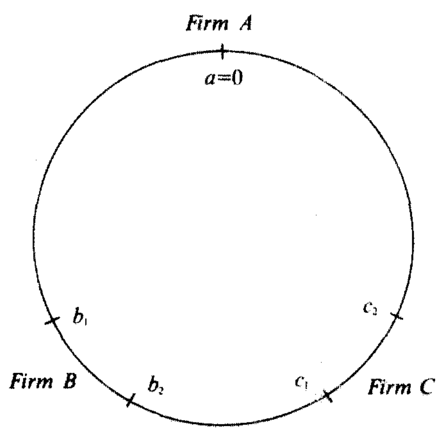
outlet	location	price	market share	profit
a	0	0.08400	0.30976	0.02602
b_1	0.27120	0.09446	0.16747	0.01582
b_2	0.36013	0.09565	0.17765	0.01699
c_1	0.63987	0.09565	0.17765	0.01699
c_2	0.72880	0.09446	0.16747	0.01582

2) Equilibrium

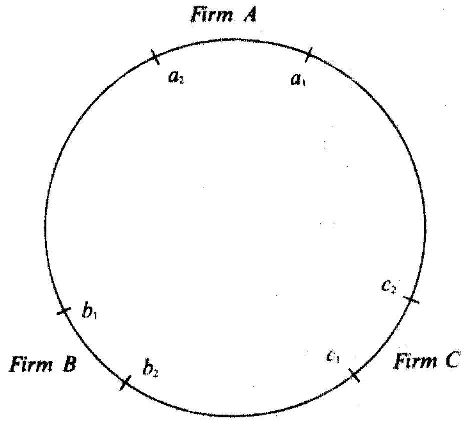
Now, to see the response of the other firms in the industry, we check whether firm C can profitably open its second outlet. Each outlet will locate as in Figure 4. The equilibrium locations, prices, and profits are summarized in Table 6. According to the table, π_c^* is 0.03281. The profit is greater than that in the case where firm C has only one outlet, which is 0.02959. Likewise, firm A can increase its profits from 0.02602 to 0.02945, by opening a new outlet, as we see in Figure 5 and Table 7. Consequently, we find that, when there are three firms in the industry, having two outlets is a Nash equilibrium. This is demonstrated in Table 8.

Similarly, we can show that, when there are four firms in the industry, each firm can increase its profit by opening its second outlet. Unlike the situations(1,1,1) and (2,2,2), when there is only one two-outlet firm in the industry, that is $(m^2, m^3, m^4)=$

〈Figure 4〉



〈**Figure 5**〉



〈**Table 7**〉

outlet	location	price	market share	profit
a_2	0.03411	0.08837	0.16667	0.01473
b_1	0.29922	0.08837	0.16667	0.01473
b_2	0.36744	0.08837	0.16667	0.01473
c_1	0.63255	0.08837	0.16667	0.01473
c_2	0.70077	0.08837	0.16667	0.01473
a_1	0.96589	0.08837	0.16667	0.01473

〈**Table 8**〉

		other firm's decision (m^2, m^3)		
		(1, 1)	(1, 2)	(2, 2)
the representative firm's decision	$m^1=1$	0.03703	0.02960	0.02602
	$m^1=2$	0.03281	0.03281	0.02946

〈**Table 9**〉

		other firm's decision (m^2, m^3, m^4)			
		(1, 1, 1)	(1, 1, 2)	(1, 2, 2)	(2, 2, 2)
the representative firm's decision	$m^1=1$	0.01563	0.01096(i)/0.01215(ii)	0.00985(iii)/0.00971(iv)	0.00854
	$m^1=2$	0.01898	0.01376	0.01230	0.01097

(1,1,2), the profit of the representative firm depends on its position, whether (i) adjacent to or (ii) apart from the two-outlet firm. In addition, when $(m^2, m^3, m^4) =$ (1,2,2), there are two potential profit scenarios, (iii) the firm is between the two two-outlet firms or (iv) between another one-outlet firm and a two-outlet firm. We find from Table 9 that, in all cases, the firm can profitably open a second outlet. Since the other firms make the same choice, $(m^1, m^2, m^3, m^4) = (2,2,2,2)$ will be a dominant strategy equilibrium.

Ⅲ. Supergame

Now, I will look for a subgame perfect equilibrium in a repeated game when there are more than three firms in the independent running case, and two firms in the joint profit maximizing case. According to the preceding result, each firm gains from proliferation, regardless of the other firms' decisions. Hence, in the one-shot competition, the only subgame perfect equilibrium is obtained when every firm has two outlets. However, the equilibrium profits are lower than those with a single outlet. Thus, if we allow long-run interactions between firms, when deciding whether or not to open a new outlet, each firm takes into account not only the possible increase in current profits but also long-run losses.

Firms recognize their interdependence and, thus, have the potential to sustain a single outlet without explicit collusion. The threat of the other firms' opening a new outlet would be sufficient to deter each firm from opening a second outlet. Hence, it is possible for oligopolists to collude in a purely noncooperative manner. Thus, in a repeated game, opening two outlets may not be the only equilibrium. Trigger strategies may constitute an equilibrium, depending on the discount factor for the future profit.

In Ⅲ. 1, I look for the equilibrium of a finite horizon game. In Ⅲ. 2, I examine a game with an infinite horizon.

1. Game with a Finite Horizon

In each period, there is a three stage game, which we described in Section Ⅱ. In the first stage of the game, each firm's choice is whether or not to open a second outlet. Once the number of outlets are determined, the subgame perfect equilibrium for locations and prices is obtained. These decisions are made $T+1$ times, where T is finite. Let $\pi_t^i(m_t^i, m_t^j)$ be firm i's profit at date t ($t=0, \cdots, T$), where m_t^i is the number of outlets opened by firm i, $j \neq i$. Each firm seeks to maximize the present discounted value of its profits, that is,

$$PV^i = \sum_{t=0}^{T} \delta^t \pi_t^i(m_t^i; m_t^j),$$

where δ is the discount factor. In each period t, the firms choose their number of outlets simultaneously. There is no link between the periods. Nevertheless, we will allow the choices at date t to depend on the history of previous m's. Thus m_t^i depends on the history

$$H_t = (m_0^1, \cdots, m_0^n; \cdots; m_{t-1}^1, \cdots, m_{t-1}^n).$$

We require strategies to form a subgame perfect equilibrium. That is, for any history H_t, firm i's strategy from date t onwards must maximize the present discounted value of profits, given the other firms' sequential strategies.

Since the horizon is finite: $T < +\infty$. We need to proceed by backward induction to obtain a subgame perfect equilibrium. Because past outlet histories do not affect the profits in the last period T, each firm maximizes its static profit $\pi_T^i(m_T^i; m_T^j)$, given its rivals' choices. Hence, the equilibrium is, for any history:

$$m_T^i = 2 \text{ for all } i\text{'s}.$$

Since outlet choices in period T are independent of events in the preceding period, decisions are made as though $T-1$ was the last period. Therefore, the firms also choose the two outlets at $T-1$, regardless of the history up to this period. For any H_{T-1},

$$m_{T-1}^i = 2 \text{ for all } i\text{'s}.$$

And so forth by backward induction. Therefore, opening two outlets is the unique equilibrium.

2. Game with an Infinite Horizon

The picture changes when the horizon is infinite ($T=\infty$). It is easy to verify that the two outlet equilibrium repeated infinitely is an equilibrium of this game. To see this, consider the following strategy: each firm opens two outlets, regardless of the history of the game up to t. Given that the rival firms also open two outlets in this manner, each firm can do no better than to open two outlets. On the other hand, the repeated two-outlet equilibrium is no longer the only equilibrium. Let us consider trigger strategies: each firm has a single outlet in period 0. It furthermore sustains one outlet in period t if in every period preceding t the other firms open only one outlet; otherwise it opens the second outlet forever. Each firm thus compares the present value of the sum of its future profits, in both the single and double outlet cases. If there are four firms in the industry and each of them sustains only one outlet, the present value of its profits will be

$$\frac{1}{4^3}+\frac{\delta}{4^3}+\frac{\delta^2}{4^3}=\frac{1}{4^3}(\frac{1}{1-\delta})$$

On the other hand, if a firm opens two outlets with foresight about its rivals' response and run the outlets independently, it can earn

$$\frac{2}{5^3}+\frac{2\delta^2}{8^3}+\frac{2\delta^2}{8^3}+\frac{2\delta^2}{8^3}=\frac{2}{5^3}+\frac{2\delta^2}{8^3}\,\frac{1}{(1-\delta)}$$

Thus, when $\delta > .0310$, sustaining a single outlet is more profitable, so that trigger strategies constitute an equilibrium. In constrast, when $\delta < .0310$, firms will open a second outlet. Therefore, the repeated two-outlet equilibrium is the only equilibrium.

We find, thus, that there exists a critical discount factor, δ^c such that, if the discount factor is higher than δ^c, firms may maintain a single outlet, while, if it is lower than δ^c, they open a second outlet. The critical discount factor increases as n increases, as we see in Table 10.[4)] Therefore, for a given value of δ, an increase in n leads to an increase in the possibility of a multi-outlet equilibrium.

Now, if a two-outlet firm maximizes its joint profits and all other firms respond by opening a second, jointly run, outlet, the critical discount factor will be much greater. Therefore, for a given value of δ at a smaller number of firms, the repeated two-outlet equilibrium can be the only equilibrium. If, in an industry with three firms,

4) As n increases, the potential gain from opening a second outlet becomes greater so that, given a discount factor, a firm is more likely to open a new outlet.

$$0.03927 + 0.02946\delta + 0.02946\delta^2 + \cdots > 0.03704 + 0.03704\delta + 0.03704\delta^2 + \cdots$$

$$0.03927 + 0.02946\delta\left(\frac{1}{1-\delta}\right) > 0.03704\left(\frac{1}{1-\delta}\right)$$

⟨Table 10⟩

n	4	5	6	7	8	9	10	20
δ^c	0.0310	0.1929	0.2619	0.3118	0.3505	0.3791	0.4013	0.4274

ie., $\delta < .2273$, a firm will open its second outlet, followed by all other firms. Similarly, when there are four firms in the industry, if $\delta < .4182$, a firm will open two outlets. Therefore, we see that, for a given δ, an increase in n increases the possibility of a two-outlet equilibrium.

Ⅳ. Concluding Remarks

In this paper, we have developed a circular model, allowing for competition between multiple outlets, and have found a two-outlet Nash equilibrium may occur, depending on the number of firms in the industry. In particular, as the number of firms increases, the incentive to relax price competition becomes dominated by the incentive to segment the market. Under the assumption of quadratic transportation costs, when there are more than three firms, each will decide to open two outlets, regardless of the other firms' decisions. Consequently, in the subgame perfect equilibrium in the three-stage game, every firm has two outlets.

However, since the two-outlet equilibrium is not Pareto-efficient, tacit collusion is possible. When the discount factor for future profits is sufficiently high, a single-outlet equilibrium is likely to be obtained. For any given discount factor, an increase in the number of firms in the industry reduces the likelihood of the single outlet equilibrium.

Furthermore, with the relaxation of the assumption of a quadratic transportation cost, which generates most intense price competition (see d'Aspremont et al. (1979), or Neven [14]), the possibility of a multi-outlet equilibrium might be greater.

◈ *REFERENCES* ◈

1. Bonnano, G., "Location Choice, Product Proliferation and Entry Deterrence," *Review of Economic Studies*, Vol. 54, 1987, pp. 37~45.

2. Brander, J. and J. Eaton, "Product Line Rivalry," *American Economic Review*, Vol. 74, 1984 pp. 323~34.

3. Campbell, T., "Predation and Competition in Antitrust: The Case of Non-fungible Goods," *Columbia Law Review*, Vol. 87, 1987, pp. 1625~45.

4. Capozza, D. R. and R. Van Order, "A Generalized Model of Spatial Competition," *American Economic Review*, Vol. 68, 1979, pp. 896~908.

5. de Palma, A., V. Ginsburgh, Y. Y. Papageorgiou and J.-F. Thisse, "The Principle of Minimum Differentiation Holds Under Sufficient Heterogeneity," *Econometrica*, Vol. 53, 1985, pp. 767~81.

6. de Palma, A., V. Ginsburgh and J.-F. Thisse, "On Existence of Location Equilibria in the 3-Firm Hotelling Problem," *The Journal of Industrial Economics*, Vol. 36, 1987, pp. 245~52.

7. Dixit, A. and J. Stiglitz, "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, Vol. 67, 1977, pp. 297~308.

8. Forbes, K. F., "Product Differentiation, Oligopolistic Interdependence, Market share, and Profits," *Economics Letters*, Vol. 26, 1988, pp. 381~6.

9. Gabszewicz J. J. and J. F. Thisse, "On the Nature of Competition with Differentiated Products," *The Economic Journal*, Vol. 96, 1986, pp. 160~72.

10. Hotelling, H., "Stability in Competition," *Economic Journal*, Vol. 39. 1929, pp. 49~7.

11. Katz, M., "Firm-Specific Differentiation and Competition among Multiproduct Firms," *Journal of Business*, Vol. 57S, 1984, pp. 149~66.

12. Martinez-Giralt, X., "On Brand Proliferation with Vertical Differentiation," *Economic Letters*, Vol. 30, 1989, pp. 279~86.

13. Martinez-Giralt, X. and D. J. Neven, "Can Price Competition Dominate Market Segmentation?," *The Journal of Industrial Economics*, Vol. 36, 1988, pp. 431~42.

14. Neven, D. J., "Two-Stage (Perfect) Equilibrium in Hotelling's Model," *The Journal of Industrial Economics*, Vol. 33, 1985, pp. 317~25.

15. Novshek, W., "Equilibrium in Simple Spatial (or Differentiated Product) Mod-

els," *Journal of Economic Theory*, Vol. 22, 1980, pp. 313~26.

16. Osborne, M. J. and C. Pitchik, "Equilibrium in Hotelling's Model of Spatial Competition," *Econometrica*, Vol. 55, 1987, pp. 911~22.
17. Perloff, J. and S. C. Salop, "Equilibrium with Product Differentiation," *Review of Economic Studies*, Vol. 52, 1985, pp. 107~20.
18. Phlips, L. and J.-F. Thisse, "Spatial Competition and the Theory of Differentiated Markets: An Introduction," *The Journal of Industrial Economics*, Vol. 31, 1982, pp. 1~9,
19. Selton, R., "Re-examination of the Perfectness Concept for Equilibrium Points in Extensive Game," *International Journal of Game Theory*, Vol. 4, 1975, pp. 22~55.
20. Shaked, A. and J. Sutton, "Natural Oligopolies," *Econometrica*, Vol. 51, 1983, pp. 1469~84.
21. Waterson, M., "Models of Product Differentiation," *Bulletin of Economic Research*, Vol. 41, 1989, pp. 1~27.

〈Appendices〉

A1.

When $n = 5$,

〈**Table A1**〉

		other firms' decision (m^2, m^3, m^4, m^5)				
		(1,1,1,1)	(1,1,1,2)	(1,1,2,2)	(1,2,2,2)	(2,2,2,2)
the representative firm's decision	$m^1 = 1$	$1/5^3$	$1/6^3$	$1/7^3$	$1/8^3$	$1/9^3$
	$m^1 = 2$	$2/6^3$	$2/7^3$	$2/8^3$	$2/9^3$	$2/10^3$

When $n = 6$,

〈**Table A2**〉

		other firms' decision(m^2, m^3, m^4, m^5, m^5)					
		(1,1,1,1,1)	(1,1,1,1,2)	(1,1,1,2,2)	(1,1,2,2,2)	(1,2,2,2,2)	(2,2,2,2,2)
the representative firm's decision	$m^1 = 1$	$1/6^3$	$1/7^3$	$1/8^3$	$1/9^3$	$1/10^3$	$1/11^3$
	$m^1 = 2$	$2/7^3$	$2/8^3$	$2/9^3$	$2/10^3$	$2/11^3$	$2/12^3$

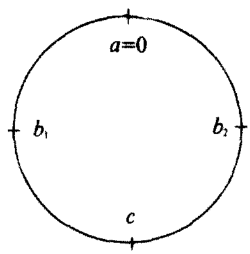
Therefore, in any situation, firms will choose to have two outlets.

A2.

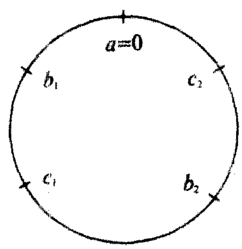
When there are three firms in the industry, if a firm, say firm B, wants to open another outlet, it will compare the profits from neighboring positions(as Figure 3) and separating positions(as Figure A1). Since the profit from the neighboring position is 0.03926 and the profit from the interspersed position is 0.03125(even lower than the profit in the case where firm B has only one outlet), the firm will open its second outlet next to the existing outlet.

When there are three firms and one of them (firm B) has already opened its second outlet as in Figure 3, the other firm, say firm C, will also prefer a neighboring position for its second outlet because it is more profitable (=0.03281) than the separated positions, 0.01700 (when located as in Figure A2) and 0.02430 (when located as in Figure A3). When there are three firms and all other firms except firm A have two outlets, firm A also prefers the neighboring position because it earns higher profits (=0.02946), when compared with the position in Figure A4(=0.02017) and the position in Figure A5(=0.01137).

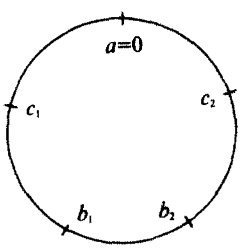
〈Figure A1〉



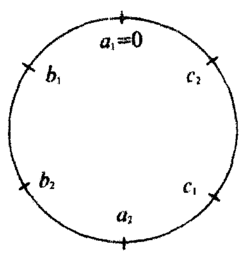
〈Figure A2〉



〈Figure A3〉



〈Figure A4〉



〈Figure A5〉