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Household Behavior : Allocation of Time Uses

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This paper compares two static models for the time allocation of the husband and wife. The first model is an extension of the traditional work-leisure model to the household, and the second one is the household production model. The comparative static results reveal that the household production model uncover much more insightful implications for the time allocation decisions than the traditional work-leisure model. In addition, the efficiency parameters of the household production function play important roles in the household decisions.

I. Introduction

The economic inquiry into the nonmarket activities and intrafamily time allocation of household members was mainly initiated by the Becker's [2] seminal paper. In his paper, Becker criticized that the traditional work-leisure model is substantially limited for the analysis of the various nonmarket activities, and proposed so-called "household production model" in which such nonmarket activities and time uses were systematically incorporated.[1]

This paper compared two theoretical models for the time allocation of individual household members. The first model is an extension of the traditional work-leisure model to the household and the second one is the household production model. In particular, quite rigorous microeconomic comparative static results are drawn from the second model.

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1) For the critical survey of household production, see Gronau [3] and Kim [5].

I. Intrafamily Allocation of Time: Work-Leisure Model

A static model is developed in this section for analyzing the determinants of the supply of labor by the husband and wife to the market. The model assumes that the arguments of the utility function of the family are market goods (X) and leisure of the husband (L_1) and wife (L_2):

$$U = U(X, L_1, L_2). \qquad (1-1)$$

The family is subject to two constraints. In the time constraint, the husband and wife are assumed to allocate all of their time (T_0) between leisure and work (N_i):

$$T_0 = L_1 + N_2 = L_2 + N_2 \qquad (1-2)$$

In the income constraint, the expenditures on market goods cannot exceed the family's money income (I). Money income consists of non-earnings income (Y_N) and earnings income of both the husband and wife:

$$I = X = Y_N + W_1 N_1 + W_2 N_2 \qquad (1-3)$$ [2]

The time and income constraints can be combined into a single constraint, called full income:

$$F = X + L_1 W_1 + L_2 W_2 = Y_N + T_0 (W_1 + W_2). \qquad (1-4)$$

The family maximizes its utility subject to the full income constraint:[3]

$$L = U(X, L_1, L_2) + \lambda (F - X - L_1 W_1 - L_2 W_2) \qquad (1-5)$$

and the necessary conditions for an interior optimum are:

$$\frac{\partial L}{\partial X} = U_X - \lambda = 0 \qquad (1-5.1)$$

$$\frac{\partial L}{\partial L_1} = U_{L_1} - \lambda W_1 = 0 \qquad (1-5.2)$$

$$\frac{\partial L}{\partial L_2} = U_{L_2} - \lambda W_2 = 0 \qquad (1-5.3)$$

$$\frac{\partial L}{\partial \lambda} = Y_N - T_0(W_1 + W_2) - X - L_1 W_1 - L_2 W_2 \qquad (1-5.4)$$

2) Note that income and wages are relative to the price of market goods.

3) More detail mathematical analysis utilized in this paper can be found in Allen [1], Intriligator [4], Silberberg [6], etc.

where $U_z = \frac{\partial U}{\partial Z}(Z: X, L_1, L_2)$ and λ is the Lagrange multiplier. To obtain the various wage and income effects one must differentiate equations(1 – 5.1)~(1 – 5.4) totally

$$\begin{bmatrix} 0 & -W_1 & -W_2 & -1 \\ -W_1 & U_{L_1L_1} & U_{L_1L_2} & U_{L_1X} \\ -W_2 & U_{L_2L_1} & U_{L_2L_2} & U_{L_2X} \\ -1 & U_{XL_1} & U_{XL_2} & U_{XX} \end{bmatrix} \begin{bmatrix} d\lambda \\ -dN_1 \\ -dN_2 \\ dX \end{bmatrix} = \begin{bmatrix} -dY_N \ -N_1dW_1 \ -N_2dW_2 \\ \lambda dW_1 \\ \lambda dW_2 \\ 0 \end{bmatrix} \qquad (1-6)$$

where $dY_N + T_0\,(dW_1 + dW_2) - dX - L_1dW_2 - L_2dW_2 - W_1dL_1 - W_2dL_2$
$= dY_N - dX + N_1dW_1 + N_2dW_2 + W_1dN_1 + W_2dN_2,\ dN_i = -dL_i,$

and $U_{z_iz_j} = \frac{\partial^2 U}{\partial Z_i \partial Z_j}$. Applying Cramer's rule to equation (1 – 6), the equations for the variation in the hours of work by the husband and wife are[4]

$$dN_1 = (S_{11} + N_1\frac{\partial N_1}{\partial Y_N})\,dW_1 + (S_{12} + N_2\frac{\partial N_1}{\partial Y_N})\,dW_2 + \frac{\partial N_1}{\partial Y_N}\,dY_N \qquad (1-6.1)$$

and $$dN_2 = (S_{21} + N_1\frac{\partial N_1}{\partial Y_N})\,dW_1 + (S_{22} + N_2\frac{\partial N_2}{\partial Y_N})\,dW_2 + \frac{\partial N_2}{\partial Y_N}\,dY_N \qquad (1-6.2)$$

where $S_{ii} = \frac{\partial N_i}{\partial W_i}\Big|_{u=u_0} > 0,\ i = 1,\ 2$ and $S_{ij} = \frac{\partial N_i}{\partial W_j}\Big|_{u=u_0} < 0,\ i = 1, 2.$

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4) $dN_1 = [D_{01}(-dY_N - N_1dW_1 - N_2\,dW_2) + D_{11}\lambda dW_1 + D_{21}\,dW_2] \,/\, D$

$$= -\{(\lambda\frac{D_{11}}{D} - N_1\frac{D_{01}}{D})\,dW_1 + (\lambda\frac{D_{21}}{D} - N_2\frac{D_{01}}{D})\,dW_2 - \frac{D_{01}}{D}\,dY_N\}$$

$$= -(L_1 k_{L_1}\sigma_{L_1L_1} + N_1k_{N_1}\eta_{L_1F})dW_1 + (L_1k_{L_2}\sigma_{L_1L2} + N_1k_{N_2}\eta_{L_1F})\,dW_2$$

$$+(L_1 - \frac{Y_N}{F})\,\eta_{L_1F}\,\frac{dY_N}{Y_N} \qquad \text{or}$$

$$\frac{dN_1}{N_1} = -(\frac{L_1}{N_1}\,k_{L_1}\sigma_{L_1L_1} + k_{N_1}\eta_{L_1F})\,\frac{dW_1}{W_1} - (\frac{L_1}{N_1}\,k_{L_2}\sigma_{L_1L_1} + k_{N_2}\eta_{L_1F})\,\frac{dW_2}{W_2}$$

$$-(\frac{L_1}{N_1}\,\frac{Y_N}{F})\,\eta_{L_1F}\,\frac{dY_N}{Y_N} \qquad (1-6.1')$$

$$dN_2 = -[D_{02}(-dY_N - N_1dW_1 - N_2dW_2) + D_{12}\lambda dW_1 + D_{22}\,\lambda dW_2] \,/\, D$$

$$= -\{(\lambda\frac{D_{12}}{D} - N_1\frac{D_{02}}{D})\,dW_1 + (\lambda\frac{D_{22}}{D} - N_2\frac{D_{02}}{D})\,dW_2 - \frac{D_{02}}{D}\,dY_N\}$$

$$= -(L_2k_{L_1}\sigma_{L_2L_1} + N_2k_{N_1}\eta_{L_2F})dW_1 + (L_2k_{L_2}\sigma_{L_2L_2} + N_2k_{N_2}\eta_{L_2F})\,dW_2$$

$$+ (L_2\,\frac{Y_N}{F})\,\eta_{LF}\,\frac{dY_N}{Y_N}$$

Thus, a change in the hours of work of both the husband and wife is induced by a change in the husband's wage rate, the wife's wage rate, and in non-earnings income. If we assume that leisure is a normal good ($\partial N_i / \partial Y_N < 0$), then equation (1−6.2) shows that an increase in the wage of the wife induces substitution and income effects that are of opposite sign. Increasing the wife's wage will increase the price of leisure and lead to a larger number of hours of work. If leisure is a normal good, then the income effect of her wage increase will reduce the number of hours of work. Hours of work will increase if the substitution effect is larger than the income effect. The income effect of an increase in her husband's wage will reduce her hours of work. If leisure of the husband and wife are substitutes ($S_{ij} < 0$), then the negative income effect is reinforced.

Ⅲ. Intrafamily Allocation of Time: Household Production Model

The new theory of consumption provides a model for analyzing the supply of market time of family members. This model overcomes some of the weaknesses of the classical work-leisure model of time allocation. Most of nonmarket time is not leisure. It can best be expressed as work at home. This is especially true for the wife who spends many hours per week in food preparation, child care, cleaning, etc. The new approach to consumption treats the household as a miniature firm that combines purchased market goods and time of the family members to produce the commodities ultimately consumed. Thus, all time is allocated to production activities, either in the market or in the household.

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or
$$\frac{dN_2}{N_2} = -\left(\frac{L_2}{N_2} k_{L_1} \sigma_{L_2 L_1} + k_{N_1} \eta_{L_2 F}\right) \frac{dW_1}{W_1} - \left(\frac{L_2}{N_2} k_{L_2} \sigma_{L_2 L_2} + k_{N_2} \eta_{L_2 F}\right) \frac{dW_2}{W_2} - \left(\frac{L_2}{N_2} \frac{Y_N}{F}\right) \eta_{L_2 F} \frac{dY_N}{Y_N} \qquad (1-6.2')$$

where
$$D = \begin{bmatrix} 0 & -W_1 & -W_2 & -1 \\ -W_1 & U_{L_1 L_1} & & \\ -W_2 & & \ddots & \\ -1 & & & U_{XX} \end{bmatrix}$$

D_{ij} is the co−factor for the i-th row and j-th column of D, $\sigma_{Z_i Z_j}$ is Allen's partial elasticity of substitution, and $\eta_{Z_i F}$ is the elasticity of Z_i with respect to full income, $k_{L_i} = W_i L_i / F$ and $k_{N_i} = N_i W_i / F$.

The household receives utility from the consumption of Z_1 and Z_2:

$$U = U(Z_1, Z_2) \qquad (2-1)$$

which are produced by the combination of market goods (X_i), consumption time of the husband (t_i) and of the wife (x_i'):

$$Z_i = f_i(X_i, t_i, t_i'; \beta_i, r_i), \quad i = 1, 2 \qquad (2-2)$$

where f_i is homogeneous of degree one and β_i and δ_i are efficiency parameters of the husband and wife, respectively.

The household faces both a time and money income constraint. The husband and wife allocate all their time between work in the market (t_w) and production in the household(t_c):

$$T_0 = t_c + t_w = t'_c + t'_w \qquad (2-3)$$

Total money income, the summation of nonearnings income (V) and market earnings of the husband and wife, is equal to expenditures on market goods:

$$I = V + Wt_w + W'\,t'_w = \sum X_i P_i \qquad (2-4)$$

where W and W' are the wage rates of the husband and wife respectively and P_i is the price of market good X_i.

The time and money income constraints can be combined into a single constraint called "full income":

$$F = \sum_{\pi i} Z_i = V + T_0 (W + W') = \sum P_i X_i + W\sum t_i + W'\sum t_i' \qquad (2-5)$$

where π_i is the price (marginal cost) of Z_i.

For the household production function (equation 2-2), marginal cost and average cost are equal and are independent of the level of output. Full income is the summation of the value of the total time of the husband and wife and nonearnings income, and it is "spent" on the Z's by purchasing market goods and forgoing earnings.

The family maximizes its utility subject to the product constraint (2-2) and the full income constraint (2-5). At a utility maximum the following conditions must hold:

$$U_i - \lambda\pi_i = 0, \quad i = 1, 2 \qquad (2-6.1)$$

$$F - \sum \pi_i Z_i = 0 \qquad (2-6.2)$$

$$U_i f'_{t_i} - \lambda W = 0, \quad i = 1, 2 \qquad (2-6.3)$$

$$U_i f'_{t_i} - \lambda W' = 0, \quad i = 1, 2 \qquad (2-6.4)$$

$$U_i f'_{x_i} - \lambda P_i = 0, \quad i = 1, 2 \qquad (2-6.5)$$

where λ is the Lagrange multiplier (marginal utility of full income), $U_i = \partial U / \partial Z_i$ ($U_{ij} = \partial^2 U / \partial U_i \partial U_j$), and f'_j is the marginal product of j.

To derive the price and income effects for the Z's, we differentiate equations (2−6.1) and (2−6.2) totally to obtain:

$$\begin{bmatrix} 0 & -\pi_1 & -\pi_2 \\ -\pi_1 & U_{11} & U_{12} \\ -\pi_2 & U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} d\lambda \\ d_{Z_1} \\ d_{Z_2} \end{bmatrix} = \begin{bmatrix} -dF + Z_1 d\pi_1 + N2 d\pi_2 \\ \lambda d\pi_1 \\ \lambda d\pi_2 \end{bmatrix} \qquad (2-7)$$

solve for dZ_1 and dZ_2, and convert to percentages:

$$EZ_1 = -\{(1-k_{Z_1})\sigma_{Z_1Z_2} + k_{Z_1}\eta_{Z_1F}\} E\pi_1 + \{(1-k_{Z_1})(\sigma_{Z_1Z_2} - \eta_{Z_1F})\} E\pi_2 + \eta_{Z_1F} EF \qquad (2-8.1)$$

and $$EZ_2 = k_{Z_1}(\sigma_{Z_1Z_2} - \eta_{Z_2F}) E\pi_1 - \{k_{Z_1}\sigma_{Z_1Z_2} + (1 - k_{Z_1})\eta_{Z_2F}\} E\pi_2 + \eta_{Z_2F} EF \qquad (2-8.2)$$

where[5] k_{Z_i} = share of full income spent on Z_i, i. e., $Z_i\pi_i/F$

$\sigma_{Z_iZ_j}$ = the Allen partial elasticity of substitution, $\sigma_{ij} = \sigma_{ji}$

η_{Z_iF} = the full income elasticity of Z_i

E = the natural logarithmic differential operator $d(\ln)$, i.e., percentage change.

Thus, a change in the demand for Z_1 is induced by a change in th prices of Z_1 and Z_2 and in full income. Both a substitution effect and an income effect are associated with these price changes.

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5) $EZ_1 = k_{Z_1}(\sigma_{Z_1Z_1} - \eta_{Z_1F})E\pi_1 + k_{Z_2}(\sigma_{Z_1Z_2} - \eta_{Z_1F})E\pi_2 + \eta_{Z_1F}EF$ (2−8.1′)
and
$EZ_2 = k_{Z_1}(\sigma_{Z_1Z_2} - \eta_{Z_2F})E\pi_1 + k_{Z_2}(\sigma_{Z_2Z_2} - \eta_{Z_2F})E\pi_2 + \eta_{Z_2F}EF$, (2−8.2′)
but for two commodities $k_{Z_1}\sigma_{Z_1Z_1} = -k_{Z_2}\sigma_{Z_1Z_2} < 0$ and $k_{Z_2} = 1 - k_{Z_1}$; therefore, when the above substitutions are made in equations (2−8.1′) and (2−8.2′), we obtain (2−8.1) and (2−8.2). Also $E\pi\sum k_{Z_i}\sigma_{Z_iZ_j} = 0$ and $-E\pi\eta_{Z_iF}\sum k_{Z_i} = -\eta_{Z_iF}E\pi$ so $EZ_i = k_{Z}(\sigma_{Z_1Z_i} - \eta_{Z_iF})E\pi^*_1 + k_{Z_1}(\sigma_{Z_iZ_2} - \eta_{Z_iF})E\pi^*_2 + \eta_{Z_iF}EF^*$
where $\pi^*_i = \pi_i/\pi$, $F^* = F/\pi$, and $\pi = \sum v_i\pi_i$, an aggregate price index.

The production of Z_i meets the conditions of cost minimization. Upon rearranging equations (2−6.3)~(2−6.5), we obtain the familiar condition for cost minimization:

$$\frac{MPx_i}{P_i} = \frac{MP_{t_i}}{W} = \frac{MP_{t'_i}}{W'} = \frac{1}{MC_i} = \frac{1}{\pi_i} \qquad (2-9)$$

For a linear homogenous production function, the percentage change in marginal cost (and price) of Z_i is a weighted average of the percentage changes in the factor prices:

$$EMC_i = \alpha_{t_i}EW + \alpha_{t'_i}EW' + \alpha_{x_i}EP_i = E\pi_i, \quad i = 1,\ 2 \qquad (2-10)$$

where[6] α_{t_i} = the share of expenditures on husband's time in the total cost of Z_i, i.e. $=(t_iW/Z_i\pi_i)$

$\alpha_{t'_i}$ = share of expenditures on wife's time in the total cost of Z_i

α_{x_i} = share of the expenditures on market goods in the total cost of Z_i

Upon substituting equation (2−10) into equation (2−8.1) for $i=1$ and into (2−8.2) for $i=2$, the percentage change in the demand for Z_1 and Z_2 can be expressed in terms of the percentage change in the prices of the factors of production.

$$\begin{aligned} EZ_1 = & \{(1-k_{Z_1})(\alpha_{t_2} - \alpha_{t_1})\sigma_{Z_1Z_2} + k_{t_1}\eta_{Z_1F}\} EW \\ & + \{(1-k_{Z_1})(\alpha_{t'_2} - \alpha_{t'_1})\sigma_{Z_1Z_2} + k_{t'_1}\eta_{Z_1F}\} EW' \\ & -\alpha_{x_1}\{(1-k_{Z_1})\sigma_{Z_1Z_2} + k_{Z_1}\eta_{Z_1F}\} EP_1 \\ & -\alpha_{x_2}(1-k_{Z_1})\sigma_{Z_1Z_2} + \eta_{Z_1F})EP_2 + \frac{V}{F}\eta_{Z_1F}\, EV \end{aligned} \qquad (2-11.1)$$

and

$$\begin{aligned} EZ_2 = & \{-k_{Z_1}(\alpha_{t_2} - \alpha_{t_1})\sigma_{Z_1Z_2} + k_{t_1}\eta_{Z_1F}\} EW \\ & + \{-k_{Z_1}(\alpha_{t'_2} - \alpha_{t'_1})\sigma_{Z_1Z_2} - k_{t'}\eta_{Z_2F}\} EW' \\ & +\alpha_{x_1}k_{Z_1}(\sigma_{Z_1Z_2} - \eta_{Z_1F})\, EP_1 - \alpha_{x_2}\{k_{Z_1}\sigma_{Z_1Z_2} + (1-k_{Z_1})\eta_{Z_1F}\} EP_2 \\ & + \frac{V}{F}\eta_{Z_1F}\, EV \end{aligned} \qquad (2-11.2)$$

where[7] k_{t_w} = share of full income from husband's market earnings,

6) Initially the change in the efficiency parameters is ignored. Their effects will be incorporated late.

7) $$\begin{aligned} EZ_1 = & \{\alpha_{t_1}k_{z_1}(\sigma_{z_1z_1} - \eta_{z_1F}) + \alpha_{t_2}f_{z_2}(\sigma_{z_1z_1} - \eta_{z_1F})\alpha_{t_2} + \eta_{z_1F}\} EW \\ & + \{\alpha_{t'}k_{z_1}(\sigma_{z_1z_2} - \eta_{z_1F}) + \alpha_{t}k_{z_1}(\sigma_{z_1z_2} - \eta_{z_1F}) + \eta_{z_1F}\} EW' \\ & +\alpha_{x}k_{z_1}(\sigma_{z_1z_1} - \eta_{z_1F})EP_1 + \alpha_{x}k_{z_1}(\sigma_{z_1z_2} - \eta_{z_1F})EP_2 + \eta_{z_1F}EF \end{aligned} \qquad (2-11.1)$$

$k_{z_1}\sigma_{z_1z_2} = -k_{z_1}\sigma_{z_2z_1}$, $k_{z_2} = 1-k_{z_1}$, and from equation (2−5),

$$\frac{WT}{F} - \frac{t_c W}{F} = \frac{t_w W}{F}$$

$k_{t_w'}$ = share of full income from wife's market eamings.

Also $k_{t_c} = k_{z_1}\alpha_{t_1} + k_{z_2}\alpha_{t_2} = \frac{\pi_1 Z_1}{F} \cdot \frac{t_1 W}{\pi_1 Z_1} + \frac{\pi_2 Z_2}{F} \cdot \frac{t_2 W}{\pi_2 Z_2}$

$= \frac{t_1 W}{F} + \frac{t_2 W}{F} = \frac{t_c W}{F}$

= share of full income "spent" on husband's nonmarket time;

and $k_{t_c'} = k_{z_1}\alpha_{t_1'} + k_{z_2}\alpha_{t_2'} = \frac{t_c' W'}{F}$

= share of full income "spent" on husband's nonmarket time.

A similar derivation is followed for equation (2−11.2). If we make the assumption that Z_1 and Z_2 are normal commodities, then the signs of only two elasticity coefficients for each equation are known. For Z_1 (Z_2), the elasticity coefficient of P_1 (P_2) is negative and of F is positive. If in addition we assume that Z_2 has a larger share of t_1 and t_1' in total cost than Z_1 and restrict ourselves to the compensated elasticities, then for Z_1 the compensated elasticities of W, W', and P_2 are positive and of P_1 is negative; and for Z_2 the compensated elasticities of W_1, W_2, and P_1 are positive and of P_2 is negative.

The demand for husband's and wife's time and for market goods is a derived demand. Drawing upon a familiar relationship for the derived demand for factors of production at cost minimization (Allen [1], pp. 503~508), we have the demand for husband's and wife's time in the production of Z_1 and Z_2:

$EF = \frac{V}{F}EV + \frac{WT}{F}EW + \frac{W'T}{F}EW'$, so

$EZ_1 = (1-k_{z_1})(\alpha_{t_2}-\alpha_{t_1})\sigma_{z_1z_2} - \{k_{z_1}\alpha_{t_1} + (1-k_{z_1})\alpha_{t_2}\}\eta_{z_1F} + \frac{WT}{F}\eta_{z_1F}EW$

$+(1-k_{z_1})(\alpha_{t_2'}-\alpha_{t_1'})\sigma_{z_1z_2} - \{k_{z_1}\alpha_{t_1'} + (1-k_{z_1})\alpha_{t_2'}\}\eta_{z_1F} + \frac{W'T}{F}\eta_{z_1F}EW'$

$-\alpha_{x_1}\{(1-k_{z_1})\sigma_{z_1z_2} + k_{z_1}\eta_{z_1F}\}EP_1 + \frac{V}{F}\eta_{z_1F}EV$

$+ \alpha_{x_1}(1-k_{z_1})(\sigma_{z_1z_2} - \eta_{z_1F})EP_2$

Also $k_{t_c} = k_{z_1}\alpha_{t_1} + k_{z_2}\alpha_{t_2} = \frac{\pi_1 Z_1}{F} \cdot \frac{t_1 W}{\pi_1 Z_1} + \frac{\pi_2 Z_2}{F} \cdot \frac{t_2 W}{\pi_2 Z_2}$

$= \frac{t_1 W}{F} + \frac{t_2 W}{F} + \frac{t_c W}{F}$

=share of full income "spent" on husband's nonmarket time;

and $k_{t_c'} = k_{z_1}\alpha_{t_1'} + k_{z_2}\alpha_{t_2'} = \frac{t_c' W'}{F}$

= share of full income "spent" on wife's nonmarket time.

A similar derivation is followed for equation(2−11.2).

$$Et_1 = EZ_1 + \alpha_{t_1}\sigma_{t_1 t_1} EW + \alpha_{t_1'}\sigma_{t_1 t_1'} EW' + \alpha_{x_1}\sigma_{t_1 x_1} EP_1 \qquad (2-12.1)$$

$$Et_1' = EZ_1 + \alpha_{t_1}\sigma_{t_1' t_1} EW + \alpha_{t_1'}\sigma_{t_1' t_1'} EW' + \alpha_{x_1}\sigma_{t_1' x_1} EP_1 \qquad (2-12.2)$$

$$Et_2 = EZ_2 + \alpha_{t_2}\sigma_{t_2 t_2} EW + \alpha_{t_2'}\sigma_{t_2 t_2'} EW' + \alpha_{x_2}\sigma_{t_2 x_2} EP_2 \qquad (2-13.1)$$

and
$$Et_2' = EZ_2 + \alpha_{t_2}\sigma_{t_2' t_2} EW + \alpha_{t_2'}\sigma_{t_2' t_2'} EW' + \alpha_{x_2}\sigma_{t_2' x_2} EP_2 \qquad (2-13.2)$$

The demand for husband's and wife's nonmarket time is the summation of the amount of time allocated to the production of Z_1 and Z_2. Consequently, the percentage change in the demand for husband's and wife's nonmarket time is:

$$Et_c = \tau_1 Et_1 + (1-\tau_1)Et_2 = \frac{\alpha_{t_1} k_{z_1}}{k_{t_c}} Et_1 + \frac{\alpha_{t_2}(1-k_{z_1})}{k_{t_c}} Et_2 \qquad (2-14.1)$$

$$E_{t_c'} = \tau_1' E_{t_1'} + (1-\tau_1')Et_2' = \frac{\alpha_{t_1'} k_{z_1}}{k_{t_c'}} Et_1' + \frac{\alpha_{t_2'}(1-k_{z_1})}{k_{t_c'}} Et_2' \qquad (2-14.2)$$

where τ_1=share of husband's nonmarket time allocated to the production of Z_1, i.e. $= \frac{t_1}{t_c}$, and

$$\frac{t_1}{t_c} = \frac{t_1 W}{\pi_1 Z_1} \cdot \frac{\pi_1 Z_1}{t_c W} = \frac{t_1 W}{\pi_1 Z_1} \cdot \frac{\pi_1 Z_1 / F}{t_c W / F} = \frac{\alpha_t k_{z_1}}{k_t}$$

τ_1' = share of wife's nonmarket time allocated to the produtcion of Z_1

k_{t_c} = share of full income spent on husband's nonmarket time, i. e., $t_c W/F$

$k_{t_c'}$ = share of full income spent on wife's nonmarket time, i. e., $t_c' W'/F$.

Upon combining equations (2−14.1), (2−13.1), (2−12.1), (2−11.2), and (2−11.1), the demand for husband's nonmarket time can be expressed as a function of the percentage change of V, W, W', P_1, and P_2:

$$\begin{aligned} Et_c = & \left(\frac{V}{F}\right)\eta_{tF} EV \\ & + \left\{ -\frac{k_{z_1}}{k_{t_c}}(1-k_{z_1})(\alpha_{t_1} - \alpha_{t_2})^2 \sigma_{z_1 z_2} + k_{t_c}\eta_{tF} + \tau_1 \alpha_{t_1}\sigma_{t_1 t_1} + (1-\tau_1)\alpha_{t_2}\sigma_{t_2 t_2} \right\} EW \\ & + \left\{ -\frac{k_{z_1}}{k_{t_c}}(1-k_{z_1})(\alpha_{t_1} - \alpha_{t_2})(\alpha_{t_1'} - \alpha_{t_2'})\sigma_{z_1 z_2} + k_{t_c}\eta_{tF} + \tau_1 \alpha_{t_1'}\sigma_{t_1 t_1'} \right. \\ & \left. + (1-\tau_1)\alpha_{t_2'}\sigma_{t_2 t_2'} \right\} EW' \\ & + \alpha_{x_1}\{k_{z_1}(\sigma_{z_1 z_2} - \eta_{tF}) + \tau_1(\sigma_{t_1 x_1} - \sigma_{z_1 z_2})\} EP_1 \\ & + \alpha_{x_2}\{(1 - k_{z_1})(\sigma_{z_1 z_2} - \eta_{tF}) + (1 - \tau_1)(\sigma_{t_2 x_2} - \sigma_{z_1 z_2})\} EP_2 \qquad (2-15.1) \end{aligned}$$

where[8] $\eta_{tF} = \tau_1 \eta_{z_1 F} + \tau_2 \eta_{z_2 F}$

= the "derived" full income elasticity of husband's nonmarket time.

Equation (2−15.1) is a formidable expression; however, it contains only substitution (in production and consumption) and income effects associated with a change in the four price variables (W, W', P_1, and P_2) and nonearnings income (V). If we assume that the "derived" full income elasticity of husband's time ($\eta_{t,F}$) is positive, then the sign of only one elasticity coefficient in equation(2−15.1) is certain. Increasing nonearnings income (V) increases the demand for husband's nonmarket time. Increasing the husband's wage rate has an ambiguous effect on the demand for his nonmarket time. The compensated substitution effect between Z_1 and Z_2 is negative; the substitution effects between husband's time and other factors (wife's time and market goods) in the production of each Z is negative, but the income effect is positive. (The positive effect of an increase in W on F directly exceeds the negative effect of W on the cost of t_c. Increasing the wife's wage has an ambiguous effect on the demand for husband's nonmarket time. If $(\alpha_{t_2} - \alpha_{t_1})(\alpha'_2 - \alpha'_1) <$ 0 and if husband's and wife's time are "substitutes" in the production of both Z's ($\sigma_{t_i t'_i} < 0$), then increasing the wife's wage unambiguously increases the demand for husband's nonmarkte time. If $(\alpha_{t_2} - \alpha_{t_1})(\alpha'_2 - \alpha'_1) < 0$ and/or husband's and wife's time are complements in the production of one or both $Z(s)$, then the effect on the demand for husband's nonmarket time depends upon the net effects. Increasing the

8) $ET_c = \tau_1(EZ_1 + \alpha_{t_1}\sigma_{t_1t_1}EW + \alpha_{t_1}\sigma_{t_1t_1}EW' + \alpha_{x_1}\sigma_{t_1x_1}EP_1)$

$$+ \tau_2(EZ_2 + \alpha_{t_2}\sigma_{t_2t_2}EW + \alpha_{t_2}\sigma_{t_2t_2}EW' + \alpha_{x_2}\sigma_{t_2x_2}EP_2)$$

$$= [\frac{\alpha_{t_1}k_{z_1}}{k_t}\{(1-k_{z_1})(\alpha_{t_1} - \alpha_{t_1})\sigma_{z_1z_1}\} - \frac{\alpha_{t_2}(1-k_{z_1})}{k_t}\{k_{z_1}(\alpha_{t_2} - \alpha_{t_1})\sigma_{z_1z_1}\}$$

$$+ k_{t_c}\{\tau_1\eta_{z_1F}\tau_1\eta_{z_2F}\} + \tau_1\alpha_{t_1}\sigma_{t_1t_1} + \tau_2\alpha_{t_2}\sigma_{t_2t_2}]EW$$

$$+ [\frac{\alpha_{t_1}k_{z_1}}{k_t}\{(1-k_{z_1})(\alpha_{t_1} - \alpha_{t_1})\sigma_{z_1z_1}\} - \frac{\alpha_{t_2}(1-k_{z_1})}{k_t}\{k_{z_1}(\alpha_{t_2} - \alpha_{t_1})\sigma_{z_1z_2}\}$$

$$+ k_{t_c}(\tau_1\eta_{z_1F}) + \tau_1\alpha_{t_1}\sigma_{t_1t_1} + \tau_2\alpha_{t_2}\sigma_{t_2t_2}]EW'$$

$$+ [\alpha_{x_1}\{-\tau_1(1-k_{z_1}) + \tau_2k_{z_1}\}\sigma_{z_1z_1} + \alpha_{x_1}k_{z_1}(\tau_1\eta_{z_1F} + \tau_2\eta_{z_2F})$$

$$+ \tau_1\alpha_{x_1}\sigma_{t_1x_1}]EP_1$$

$$+ \alpha_{x_2}\{\tau_1(1-k_{z_1}) - \tau_2k_{z_1}\}\sigma_{z_1z_2} - \alpha_{x_2}(1-k_{z_1})(\tau_1\eta_{z_1F} + \tau_2\eta_{z_2F})$$

$$+ \tau_2\alpha_{x_2}\sigma_{t_2x_2}]EP_2$$

$$+ (\frac{V}{F})(\tau_1\eta_{z_1F} + \tau_2\eta_{z_2F})EV$$

where $k_{t_c} = k_{z_1}\alpha_{t_1} + k_{z_2}\alpha_{t_2}$

$$= \frac{\pi_1Z_1}{F}\cdot\frac{t_1W}{\pi_1Z_1} + \frac{\pi_2Z_2}{F}\cdot\frac{t_2W}{\pi_2Z_2} = \frac{t_1W}{F} + \frac{t_2W}{F} + \frac{t_cW}{F}$$

price of either market good yields ambiguous results. They depend upon the sign and size of the difference $\sigma_{z_1 z_2} - \eta_{t'F}$ and $\sigma_{t_1} X_i - \sigma_{z_1 z_2}$.

Similarly for the wife, the demand for her nonmarket time can be expressed as a function of the percentage change of V, W, W', P_1 and P_2 by combining equations (2−14.2), (2−13.2), (2−12.2), (2−11.2) and (2−11.1):

$$
\begin{aligned}
Et_c' = {} & (\frac{V}{F})\eta_{t'F}\, EV + \{-\frac{k_{z_1}}{k_{t_c}}(1-k_{z_1})(\alpha_{t_2} - \alpha_{t_1})(\alpha_{t_2} - \alpha_{t_1})\sigma_{z_1 z_2} \\
& + k_{t_c}\eta_{t'F} + \tau_1'\alpha_{t_1}\sigma_{t_1 t_2} + (1-\tau_1')\alpha_{t_2}\sigma_{t_1 t_2}\}\, EW \\
& + \{-\frac{k_{z_1}}{k_{t_c}}(1-k_{z_1})(\alpha_{t_2} - \alpha_{t_1})^2\sigma_{z_1 z_2} + k_{t_c}\eta_{t'F} + \tau_1'\alpha_{t_1}\sigma_{t_1 t_2} \\
& \quad + (1-\tau_1')\alpha_{t_2}\sigma_{t_1 t_2}\}EW' \\
& + \alpha_{x_1}\{k_{z_1}(\sigma_{z_1 z_2} - \eta_{t'F}) + \tau_1'(\sigma_{t_1 x_1} - \sigma_{z_1 z_2}\}EP_1 \\
& + \alpha_{x_2}\{(1-k_{z_1})(\sigma_{z_1 z_2} - \eta_{t'F}) + (1-\tau_1')(\sigma_{t_1 x_2} - \sigma_{z_1 z_2})\}EP_2 \qquad (2-15.2)
\end{aligned}
$$

where $\eta_{t'F}$ = the "derived" full income elasticity for nonmarket time of the wife, i.e.

$$= \frac{t_1'}{t_c'}\eta_{z_1 F} + \frac{t_2'}{t_c'}\eta_{z_2 F}$$

The introduction of the effect of a change in the efficiency parameters of the husband (β_i) and wife (γ_i) alters equations (2−15.1) and (2−15.2) slightly. If the effects of a change in the efficiency parameters are introduced as Hicks neutral technical change and if the rates of efficiency change associated with husband (wife) are the same ($d\beta_1/\beta_1 = d\beta_2/\beta_2 = d\beta/\beta$ and $d\gamma_1/\gamma_1 = d\gamma_2/\gamma_2 = d\gamma/\gamma$) in the production of both Z_1 and Z_2, then two terms are added to the demand for nonmarket time of the husband and wife. With these assumptions the relative price of Z_1 and Z_2 is unchanged, so the two terms reflect the net effect on the demand for nonmarket time of an increase in real full income because the marginal cost of Z_1 and Z_2 is reduced and of the reduction in demand for nonmarket time because any given number of units of nonmarket time will produce a larger nunber of units of Z_1 and Z_2. The percentage change in the demand for nonmarket time of the husband and wife is now a function of the percentage change of nonearnings income, of the wage of the husband and wife, of the prices of market goods, and of the efficiency parameters associated with the husband and wife:

$$Et_c = (\frac{V}{F})\,\eta_{tF}\,EV$$
$$+\{-\frac{k_{z_1}}{k_{t_1}}(1-k_{z_1})(\alpha_{t_2}-\alpha_{t_1})^2\sigma_{z_1z_1}+\tau_1\alpha_{t_1}\sigma_{t_1t_1}+(1-\tau_1)\alpha_{t_1}\sigma_{t_1t_2}+k_{t_1}\eta_{tF}\}\,EW$$
$$+\{-\frac{k_{z_1}}{k_{t_1}}(1-k_{z_1})(\alpha_{t_2}-\alpha_{t_1})\sigma_{z_1z_1}+\tau_1\alpha_{t_1}\sigma_{t_1t_1}+(1-\tau_1)\alpha_{t_1}\sigma_{t_1t_2}+k_{t_1}\eta_{tF}\}\,EW'$$
$$+\alpha_{x_1}\{k_{z_1}(\sigma_{z_1z_1}-\eta_{tF})+\tau_1(\sigma_{t_1x_1}-\sigma_{z_1z_1})\}\,EP_1$$
$$+\alpha_{x_2}\{(1-k_{z_1})(\sigma_{z_1z_1}-\eta_{tF})+(1-\tau_1)(\sigma_{t_2x_1}-\sigma_{z_1z_1})\}\,EP_2$$
$$+(\eta_{tF}-1)\,E\beta+(\eta_{tF}-1)\,E\Upsilon, \qquad (2-15.1')$$

and
$$Et'_c = (\frac{V}{F})\,\eta_{t'F}\,EV$$
$$+\{-\frac{k_{z_1}}{k_{t_1}}(1-k_{z_1})(\alpha_{t_2}-\alpha_{t_1})(\alpha_{t_2}-\alpha_{t_1})\sigma_{z_1z_1}+\tau'_1\alpha_{t_2}\sigma_{t_1t_2}$$
$$+(1-\tau'_1)\alpha_{t_2}\sigma_{t_2t_2}+k_{t_1}\eta_{t'F}\}\,EW$$
$$+\{-\frac{k_{z_1}}{k_{t_1}}(1-k_{z_1})(\alpha_{t_2}-\alpha_{t_1})^2\sigma_{z_1z_1}+\tau'_1\alpha_{t_2}\sigma_{t_1t_2}+(1-\tau'_1)\alpha_{t_2}\sigma_{t_2t_2}$$
$$+k_{t_1}\eta_{t'F}\}\,EW'$$
$$+\alpha_{x_1}\{k_{z_1}(\sigma_{z_1z_1}-\eta_{t'F})+\tau'_1(\sigma_{t_2x_1}-\sigma_{z_1z_1})\}\,EP_1$$
$$+\alpha_{x_2}\{(1-k_{z_1})(\sigma_{z_1z_1}-\eta_{t'F})+(1-\tau'_1)(\sigma_{t_2x_2}-\sigma_{z_1z_1})\}\,EP_2$$
$$+(\eta_{t'F}-1)\,E\beta+(\eta_{t'F}-1)\,E\Upsilon. \qquad (2-15.2')$$

Equation (2−15.1′) and (2−15.2′) imply that the percentage change in the hours of market work by the husband and wife is also a function of the percentage change of nonearnings income, of the wages of the husband and wife, of the prices of market goods, and of the efficiency parameters associated with the husband and wife. Each family member is assumed to allocate all of his (her) time between work in the market and home production (equation 2−3), so any increase (decrease) in nonmarket time results in a decrease (increase) in hours of work in the market:

$$Et_w = -\frac{t_c}{t_w}\,Et_c \qquad (2-16.1),$$

and
$$Et'_w = -\frac{t'_c}{t'_w}\,Et'_c \qquad (2-16.2).$$

An increase in nonearnings income will reduce the hours of market work of the husband and wife if the "derived" income elasticity of demand for husband's and wife's time is positive. However, unless one is willing to speculate about the size of other parameters, the signs of the elasticity coefficients of all of the other variables (W, W', P_1, P_2, β, Υ) are unknown.

Ⅳ. Concluding Remarks

This paper comparatively explored two theoretical models for the time allocation of individual household members. The first model was an extension of the traditional work-leisure model to the household, and the second one was the household production model for hours of work of individual family members.

According to the results of comparative static analysis, the household production model uncovers much more insightful implications for the time uses than the traditional work-leisure model. In paticular, although introducing the efficiency parameters of the household production function did not alter the results much they have some important roles in the household members' time allocation decisions.

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