연세경제연구 제 II권, 제1호
Yonsei Economic Studies, Vol. II, No. 1(March 1995)

Reputation in Market Participation Games

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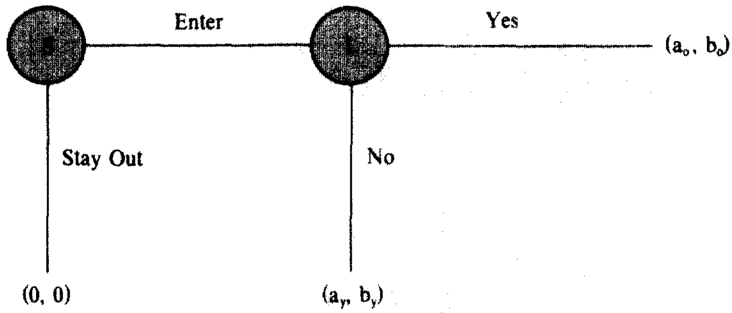
We examine reputation equilibria of what we call participation games, that have many economic examples, such as entry deterrence and product quality game. By perturbing the original game with types, we show that the lower bound of the single long run player's payoff is almost his Stackelberg commitment payoff in the limit as the finite horizon grows. Discontinuity exists between the infinite horizon and the limiting finite horizon problem. Our result is robust to a model modification in which the long run player announces the payoff structure before the whole game begins so that the rational type of long run player has to mimic not only the strategy of the Stackelberg commitment type but also the initial payoff announcement.

I. Introduction

The participation game is described as follows. In each stage game, player 1 decides first whether to choose an outside option or to enter a certain market. With player 1's staying out of the market, the stage game immediately ends. With her decision to participate into the relevant market, a chance is given for player 2 to either say yes or no. The extensive form is drawn in Figure 1. Depending on the payoffs, participation games are divided into two interesting classes, entry-deterrence game and entry-inducement game. Entry-deterrence game has been much studied,

Department of Economics, Queen Mary College, University of London. This is a revised version of the previously titled paper, "Characterization and Properties of Reputation Effects in Finitely Repeated Extensive Form Games." The author thanks Drew Fudenberg, David K. Levine and Wolfgang Pesendorfer for valuable comments. Financial support from University of California Graduate Division is gratefully acknowledged. The usual disclaimer applies.

⟨Figure 1⟩ Market Participation Game



Payoff=(L, S)

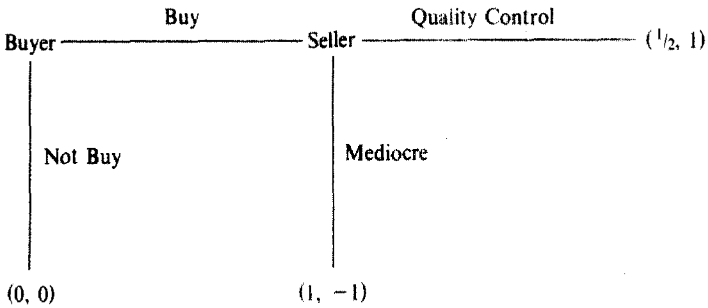
e.g. Selten [15], Kreps and Wilson [12] and milgrom and Roberts [13]. We focus on entry-inducement game, which has many economic applications. Examples are the product quality game between consumers and a monopolistic producer (Fudenberg and Levine [4, 5]), the asset market game between investors or workers and a capitalist (Fudenberg and Levine [6]), the sovereign debt game between foreign banks and a less developed country (Bulow and Rogoff [1]), the corruption and extortion game between private agents and an officer (Tirole [16]), and so forth.

Consider a situation in which a single long run player faces a sequence of short run opponents, each of whom plays only once, but who observes all previous outcomes. In addition, the short run players have slight uncertainties about the long run player's rationality or payoff. Define the "Stackelberg outcome" as the long run player's most preferred pure strategy profile of the stage game under the constraint that the short run player chooses a best response based on her beliefs and the strategies of her opponent. Under a very general set of conditions, Fudenberg and Levine [4, FL hereafter] demonstrate a strikingly strong result that the long run player can always guarantee himself the Stackelberg leader's payoff.[1] The basic intuition is that if the long run player repeatedly chooses the Stackelberg leader's strategy for a long time, he may be able to convince the opponents that he is committed to choose that strategy.

FL go on to note that this intuition, and therefore the above result, can fail when the strategies in the stage game are not observable. This happens in particular when the stage game involves sequential moves, because only the realized actions on the

1) Fudenberg and Levine [5] derived the similar result when mixed strategies are allowed.

〈**Figure 2**〉 **Product Quality Game**



Payoff=(Seller, Buyer)

equilibrium path, not the strategies, are observable in such a case. FL show that their strong result can fail in product quality game depicted in Figure 2. The Stackelberg outcome of this game is that the long run player says yes (i.e. controls quality) and the short run players (or consumers) choose to participate. FL demonstrate, however, that this outcome may not always be sustained. They construct a sequential equilibrium where the short run players always stay out. The reason why their strong result fails in this counter-example is the unobservability: if consumers never purchase, the long run player's strategy, whether to control quality or not, can never be observed, and therefore there is no way to build a reputation.

The main purpose of this paper is to point out that the above counter-example relies on the assumption of infinite horizon. We show that, for any long enough finite time horizon, the same incomplete information entry-inducement game has a unique sequential equilibrium which achieves the Stackelberg outcome for all but a bounded number of periods. This shows that the long run player is able to build a reputation despite the fact that his strategy can potentially be unobservable.

It may be interesting to enumerate other recipes to the failures of reputation building in inducement games. First, precommitment or enforceable preplay contract will simply guarantee the long run player at least the Stackelberg payoff. Second, as Fudenberg and Levine [4, 5] propose, we may solve the transformed game with simultaneous move structure by requiring short run players to choose their best reponse from "observationally equivalent" set of actions. Their idea is based on the known fact that the long run player always can aquire a reputation for some commitment type in any simultaneous move game. Third, the long-lived player may send some credible signals, such as advertising (Klein and Leffler [10]) and private disclosure or warranty in a quality game (Grossman [8]) and collateral in a debt game. Without any of these assumptions, we analyze whether a sequential reputational equilibrium can be constructed only by introducing small perturbations into original participation games.

The remainder of the paper is organized as follows. Section Ⅱ proposes the model. Section Ⅲ analyzes the sequential equilibria in participation games. Section Ⅳ deals with the situation where the long run player determines and announces the payoff structure before the whole game begins. The last section concludes.

Ⅱ. The Model

There is a finite sequence of dates indexed backwards by $t = T, \cdots, 2, 1$. At each date t, there are two players, L and S_t. The single long run player L lives forever and a short run player S_t lives only one period during the date t. At the beginning of each date t, the entire past history of outcomes up to date $t+1$ is public information. We assume no discounting so that player L's time-averaging profit will be $\frac{1}{T}\sum_{t=1}^{T}\Pi_t$. A participation game is defined as in the opening section. Consider the simplest case in which, given player S_t decision to participate into the relevant market, player L must make a binary decision of whether to say yes or no. The general form of the stage game payoffs is depicted in Figure 2. We may assume that $b_y > b_0$ always holds.

I study only two versions of game of great importance: with $b_y > 0 > b_0$ in common, either $0 > a_y > a_0$, or $a_0 > a_y > 0$. All other cases are of little interest, since the unique perfect equilibrium will be trivial, no matter what one sided incomplete information in my sense there might be. I name the game with the first type of payoff structure as an entry-deterrence game (D-game, for short) and the second as an entry-inducement game (I-game, for short). A stage D-game has two Nash equilibria [Out] and [In, Yes], but only the latter one is subgame perfect.[2] On the other hand, a one stage I-game has the unique Nash and perfect equilibrium [Out]. Notice that, under the assumption of complete information, there is no reason why

2) The other Nash equilibrium is subgame imperfect, in the sense that it can be supported only by an incredible off-the-equilibrium threat, i.e. player L's no.

the equilibrium of the T period repeated game becomes anything other than the mere repetitions of the perfect equilibrium of the stage game. Thus, a D-game refers to the situations where, even though the noncooperative equilibrium that naturally arises would be short run players' participations, the long run player wants them to stay out of the market. An I-game refers to the opposite situation. If we define a *Stackelberg payoff* as what the single patient player prefers most as far as short run opponents choose best response to their own beliefs and the long run player's strategies, it would be the value a_0 in any participation game. The question is whether the single patient player can build the reputation in the I-game as nicely as in the D-game. The answer is positive. If and only if we let the original game perturbed by introducing a little incomplete infomation, we can construct a sequential reputational equilibrium in both D-and I-game. Moreover, this has a uniqueness property.

Throughout this paper, one sided incomplete information and perfect recall will be assumed. Also assumed is that any player in the game may implement neither precommitment technology nor signalling device. A single long run player and T short run players will play one of two possible games,[3] each of which involves T repetitions of a particuar stage game. This one-sided informational incompleteness stems from short run players' uncertainty about exactly which type of the long run player they are against. The long run player knows exactly which of these actually obtains. The first possible game is the original game, while the second one is the game in which the long run player behaves as if he committed himself to a particular action. The long run player in the original unperturbed game is called a "rational" type. He is called a "strong" and an "honest" type in the D-game and the I-game, repectively. Every short run player has an identical initial belief that the long run player is likely to be rational with probability $1-\rho$ and to be strong or honest with its complementary probability ρ, where $0 < \rho < 1$.

Without loss of generality, we may normalize the payoffs as follows: let $a_1=0$, $a_0=-1$, $b_1=b$, $b_0=b-1$, and L's payoff with S's Out equals a in the D-game and let $a_1=a$, $a_0=1+a$, $b_1=1-b$, $b_0=-b$ in the I-game, where $a>0$[4] and $0>b>1$.

3) Milgrom and Roberts [13] analyzes a richer model with three types in the framework of D-game. We will consider two type case only at the expense of analytical complications.

4) However, we assume $a>1$ in D-game. The case of $0<a<1$ would result in a qualitatively similar characterization as $a>1$ possibly except in the endgame. For details, refer to K-reps and Wilson [12] p. 265.

Attention ought to be made on the I-game, thus all proofs and explanations will be made with repect to the I-game. For the purpose of comparisons, however, we also put down the results for the D-game in parenthesis. Let $p_T = \rho \in (0, 1)$, and for $t = T-1, \cdots, 1$,

$$p_t = \Pr\{L \text{ is of a honest(strong) type} \mid H^T_{t+1}\},$$

with the recursive definitions as follows:

i) S_{t+1}'s Out conveys no information, thus $p_t = p_{t+1}$
ii) S_{t+1}'s In and L's Yes(No) together with $p_{t+1} > 0$ result in $p_t = \max(b^t, p_{t+1})$
iii) Otherwise, $p_t = 0$

Whereas the honest(strong) L is always to say Yes(No), the strategy of the rational L would be as follows:

For $t=1$, say No(Yes) surely.
For $t > 1$,

i) if $p_t \geq b^{t-1}$ then say Yes(No) surely.
ii) if $p_t < b^{t-1}$ then say Yes(No) with probability $q_t = \dfrac{(1-b^{t-1})p_t}{(1-p_t)b^{t-1}}$

Strategy of S_t is to choose In(Out) with probability:

1 if $p_t > b^t$
$\dfrac{1}{1+a}(\dfrac{1}{a})$ if $p_t = b^t$
0 otherwise.

Proposition 1. *The strategies and belifs given above is a unique sequential equilibrium for the T-repeated I-game(D-game).*

Proof : The rough idea of the proof is as follows. Suppose that the first period short run player S_1 entered the relevant market and that the remaining periods were sufficiently long. Then even the rational L should behave as if he was of the honest type. The reason is that, if player L is somehow given an opportunity to move, to say no brings about an immediate gain of $1+a$ but zero in all subsequent dates since all the subsequent short run players interpreting L's previous saying no as a definite evidence that L is not the honest type will simply stay out, whereas to say yes yields only a at the date T but a sttream of positive expected profits later. Given L's stategy described above, S_t would participate into the market. This is indeed optimal for every S_t, $T \le t \le T^*$, and for the rational L during $T \le t \le T^* + 1$, where $T^* = \inf\{t \mid b^t < \rho\}$. Each player S_t from the date $T^* - 1$ on randomizes optimally, as long as all previous short run players actually came in and player L always responded with yes. It is in player L's interest to start randomization between yes and no from the date T^* on.

Uniqueness is shown as follows. The constuction proceeded by backward induction on the number of periods remaining, subject to the assumptions that No is taken as sure proof that the long run player's type is rational, and the short run player's probability of Enter is a monotone nondecreasing function of the probability of the type being rational. For detailed construction steps, refer to Kreps and Wilson [12] pp. 259~260, or Milgrom and Roberts [13] pp. 306~311.[5] Q. E. D.

Proposition 1 shows that the properties as well as the paths are similar on sequential reputational equilibria for T-repeated games of both D-game and I-game. However, qualitative nature are different in the endgame. Let us look at the D-game at the date $T^* - 1$ where $p_{T^*-1} > 0$, which implies that all the previous S's participations have been met by player L's response of no. Now if S_{T^*-1}'s randomization leads him to staying out of the market, then his immediate successor S_{T^*-2} would certainly enter(since $p_{T^*-2} = p_{T^*-1} = b^{T^*-1} < b^{T^*-2}$) and player L at the date T^*-2 would randomize. If player L happens to say no at the date T^*-2, his reputation for toughness could be restored so that again $p_{T^*-2} = b^{T^*-2}$ attains. The game will evolve in the same manner for any $t = T^*-1, \cdots, 3, 2$. In other words, the long run player can demonstrate that a short run player's decision of entering was mistaken even near the end of the D-game in a weak sense that he actually does this only in the course of optimal randomizations. On the contrary, there is a "double-sided trap" in each date after T^* in the I-game. The first trap refers to the situation where player L loses his reputation for honesty in the event of saying no, which stems from player L's randomization processes. The D-game also has this feature in common. The second type of trap exists only for the I-game. Sup-

5) Nowadays, this style of analysis is mostly replaced with ideas from stability as in Kohlberg and Mertens [11]. In particular, Govindan [7] proved that the good equilibrium is the only stable equilibrium of the D-game.

posed that $p_{T^*-1} > 0$ and that S_{T^*-1}'s randomization leads her to staying out, then every subsequent short run player will simply stay out. This may happen with non-negligible probability although the long run player has been always replied with yeses. Moreover, once this happened, even the honest guy has no way of demonstrating his honesty. In summary, the reputational equilibrium of the I-game is far more fragile, in the sense that a player S's observing not only no by player L but also out by one of her predeccessors makes her simply choose staying out of the market.

Immediate from the results thus far is the following:

Corollary 1. *Fix any participation game. In the limit as the horizon goes to infinity, the lower bound that the long run player obtains is almost his Stackelberg payoff.*

In the infinite horizon participation game, it is easy to construct a situation in which the long run player cannot obtain his Stackelberg payoff.[6] Hence, there is a discrepancy between the limit of the least equilibrium payoff to the long run player as its finite horizon goes to infinity and that when the horizon is infinite.

Ⅲ. Announcement and Commitment

A practical aspect that many examples of the I-game have in common may be that, given a short run player's participation into the relevant market, there is a tradeoff between the long run player's short term profit and the relevant short run player's payoff. Moreover, their payoffs are usually control variables the long run player can determine. In the quality game, given a consumer's decision to purchase one unit of goods the monoplist wants to sell, a negative relationship between the level of product quality and the monopolist's short term profit seems to obviously exist. In the asset market game as in Fudenberg and Levine [6], after some investors or workers provide their assets or labors to the single patient capitalist, a similar conflict may exist between returns to investors wage compensations to the workers and profits to the capitalist.

6) Refer to Section 5 of Fudenberg and Levine [4].

To investigate this situation, we slightly modify the payoff structure. As before, player S_t's choosing an outside option yields nothing to both player L and himself. Player S_t's participation directly brings about -1 to himself and y to player L. Here -1 that player S_t gets can be interpreted as disutility from consuming low quality goods in a quality game and as value of financial assets provided to the capitalist in an asset market game. The long run player decides whether to offer a compensation $1+w$ to the short run player or not at all. We assume that player L determines a level of w and that all the short run players somehow get to know the precise value of w before the whole game begins.[7] Presumably, a condition that $1+w>0$ must hold, since otherwise In is a dominated strategy for player S_t, $\forall_t$, thus every S_t will simply stay out. On the other hand, player L has no incentive to offer the gross compensation greater than y, so that $y>1+w$ also holds. Some reader might guess that player L has no incentive to offer more than $1+\varepsilon$, $\forall\ \varepsilon>0$. This is wrong because a reduction of the compensation by player L brings about not only benefits from directly raising his own share but also costs from losing some customers who would have surely come in before.

As a prelimenary for the main result of this section, the reader can check the following lemma by mimicking proofs of Proposition 1:

Lemma 1. *For w fixed, the beliefs and strategies described below is the unique plausible sequential equilibrium for a perturbed T-repeated game.*

Beliefs of S_t

i) S_{t+1}'s staying out reveals no information, thus $P_t = P_{t+1}$,

ii) S_{t+1}'s In and L's Yes together with $P_{t+1} > 0$ result in $p_t = \max\{(1+w)^{-t}, p_{t+1}\}$,

iii) Otherwise, $p_t = 0$

Strategy of the rational L.

For $t=1$, say No surely

For $t>1$,

i) if $p_t \geq (1+w)^{t-1}$, then say Yes surely,

ii) if $p_t < (1+w)^{-t+1}$, then say Yes with prob $q_t = \dfrac{p_t}{1-p_t}[(1+w)^{-t+1}-1]$.

7) This is not an innocuous assumption. Refer to Hart and Tirole [9] for some results without this restriction.

Strategy of S_t

In surely if $p_t > (1+w)^{-t}$,

In with prob $\frac{1+w}{y}$ if $p_t = (1+w)^{-t}$;
Out surely otherwise.

Let us define $T^* = \inf\{ t \mid (1+w)^{-t} < \rho\}$. On the sequential equilibrium path, every short run player S_t for $t = T, T-1, \cdots, T^*$ participate into the market with probability one, and player L optimally replies with sure yeses to those entries up to $t = T^* + 1$ and then randomizes thereafter. Now suppose that player L can determine w before the whole game begins. Let w^* be the level of net compensation that maximizes player L's time-averaging payoff in a T-repeated game. We should notice that player L may lose some sure customers by raising his own share $\{y - (1+w^*)\}$, thus there is a tradeoff between w^* and T^*. Notwithstanding, it is optimal for player L to reduce the value of w^* as much as he can keep the number of short run players who surely enter the same as before. Therefore, the profit maximization of the rational long run player requires the local condition, which states formally: For any type of player L and for given T^*, profit maximizing w^* must satisfy $(1+w^*)^{-T^*} = \rho$.

First, we calculate the best randomizing strategy on the part of the rational L. Since his time-averaging payoff is $V_R = \frac{1}{T}[(T-T^*)(y-(1+w^*)+y]$ by using the optimality principle of Bellman, the rational player L's objective will be to maximize V_R subject to

$$\text{(LOC)} \quad (1+w^*)^{-T^*} = \rho,$$
$$\text{(ICC)} \quad y > 1 + w^* > 1,$$

given $y > 1$, $\rho > 0$, and T. Define a pair (w_R^*, T_R^*) to be the rational L's maximization solution.

Now we characterize the optimal announcement on the part of the honest type of the long run player. Recall that the sequential equilibrium of I-game suffers from double traps in the endgame. From player S_t's strategy described in Lemma 1 and the local condition for profit maximization, the honest type's expected payoff along the equilibrium path would be

$$V_H = \frac{1}{T}[y-(1+w^*)][T-T^*+1+\Delta+\cdots+\Delta^{T^*-1}],$$

where $\Delta = \frac{1+w^*}{y}$ and $(1+w^*)^{-T^*} = \rho$. Rearrangement yields

$$V_H = \frac{1}{T}[\{y-(1+w^*)\}(T-T^*)+y-\frac{1}{\rho y^{T^*-1}}]$$

$$= V_R - \frac{1}{(T\rho y^{T^*-1})} \tag{1}$$

It is not difficult to check that $(\partial V_H/\partial T^*) > (\partial V_R/\partial T^*) = 0$ and $(\partial^2 V_H/\partial T^{*2}) <$ $(\partial^2 V_H/\partial T^{*2}) < 0$ at $T^* = T^*_R$, unless y is too small. Henceforth, if we denote the honest type's optimal randomizing strategy as (w^*_H, T^*_H), it is true that $w^*_H <$ w^*_R and $T^*_H > T^*_R$. This implies that, in order to conceal his type, the rational L has to propose the same payoff announcement as the honest type, so that he offers a smaller compensation to short run players and has to sacrifice some of the sure customers.

The point is that the rational L must mimic the behavior of the honest counterpart in terms of not only actions but also payoff announcement. Even with this additional constraint, we get the following:

Proposition 2. *In the limit as T goes to infinity, we have i) $T^* \to \infty$, but $(T^*/T) \to 0$; ii) $w^* \to 0$. Moreover,* $(\partial T^*/\partial y) < 0$, $(\partial T^*/\partial \rho) > 0$, $(\partial w^*/\partial y) > 0$, $(\partial w^*/\partial \rho) < 0$.

Proof : I deal with T^* as continuous variable, since doing so loses nothing but calculating complications. Applying the Lagrangian method to the maximization problem together with (LOC) to substitute w^* for T^* and rearranging the resulting equation, we have

$$T = T^* + \frac{\rho^{\frac{1}{T^*}} y - 1 - \frac{\log y}{\rho y^{T^*-1}}}{-\log\rho} T^{*2}$$

Given $y > 1$ and $0 < \rho < 1$, the condition that $T \to \infty$ requires $T^* \to \infty$. Now it is clear that

$$\frac{T^*}{T} = [1 + \frac{\rho^{\frac{1}{T}} y - 1 - \frac{\log y}{\rho y^{T^*-1}}}{-\log\rho} T^*]^{-1}$$

thus $\lim_{T \to \infty} \frac{T^*}{T} = \lim_{T \to \infty} \frac{T^*}{T} = 0$,

together with $\lim_{T \to \infty} T^* = \infty$. we proved i).

On the other hand, taking logarithms to both sides of (LOC) and rearranging yields

$$w^* = \rho^{\frac{-1}{T^*}} - 1, \quad \text{thus}$$

$$\lim_{T \to \infty} w^* = \lim_{T^* \to \infty} w^* = 0$$

The second part ii) is also done. Q.E.D.

The proposition above implies that, as the horizon grows larger, the number of short run players who optimally randomize near the end of the game also should be controlled larger, while its relative proportion gets negligible. In other words, the proportion of sure customers who enter the market with probability one monotonically approaches to unity. In addition, the long run player can optimally reduce the amount of net compensation theat provides to some short run players incentives to participate into the relevant market. The second part shows some comparative statics which states that the optimal net compensation become smaller as the horizon gets larger, as shortrun player's probability assessment that player L is of the honest type gets bigger, and as the total revenue to player L gets smaller. As a consequence, the long run player is able to obtain almost extensive form Stackelberg payoff for sufficiently long horizon T. Moreover, in the limit as the horizon T approaches to linfinity, the ε-first-best is indeed attainable.

IV. Final Remarks

We study sequential equilibria of the finitely repeated perturbed games. The class of game is simple two-stage, called market participation game. An important problem will be to calculate the lower bound that the long run player can obtain on any sequential equilibrium of general extensive form games in the limit as the horizon grows. We have to await further research.[8)]

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8) Cripps, Schmidt and Thomas [14] is pursuing this line of research.

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