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Unemployment and the Asset Market for Jobs

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Abstracts

Search models of unemployment motivate analysis through a matching problem between two distinct groups in the economy: firms and workers. A question which arises is why can't workers circumvent search frictions by creating jobs themselves? Meanwhile, firms are able to supply vacant jobs perfectly elastically: how are vacancies sourced? I address these concerns in an environment where ex ante identical agents can create, buy or sell jobs, and assess the model quantitatively.

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Ⅰ. Introduction

Search models of unemployment motivate analysis through a matching problem between two distinct groups in the economy: firms and workers. A question which arises is why can't workers circumvent search frictions by creating jobs themselves?[1)] Meanwhile, firms are able to supply vacant jobs perfectly elastically: how are

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1) The option to create own employment opportunities is typical in models of occupational choice.

vacancies sourced? This paper attempts to address these concerns in an environment where ex ante identical agents can create, buy or sell jobs.

I extend the canonical search model of unemployment along two dimensions: (i) allowing agents to create jobs, and (ii) allowing vacancies to be re-matched. Only agents who are unmatched with jobs have incentives to create jobs, and jobs are valued not just for their output in current matches, but also their anticipated output in future matches. Then I show that parameter specifications of the extended model conform with empirical dimensions of unemployment dynamics.

Specifically, I construct a model of asset entry and exit, where asset transactions are subject to search frictions. Agents can create new assets in spot markets or search to be matched with existing assets to carry out production. Shocks which are specific to an agent-asset match motivate agents to pursue other options, and motivate a listing of assets for use by other agents. As in the standard search model of unemployment, I interpret a job as an asset, and refer to agents searching to be matched with existing jobs as unemployed.

【Figure 1】 Flow of agents and jobs

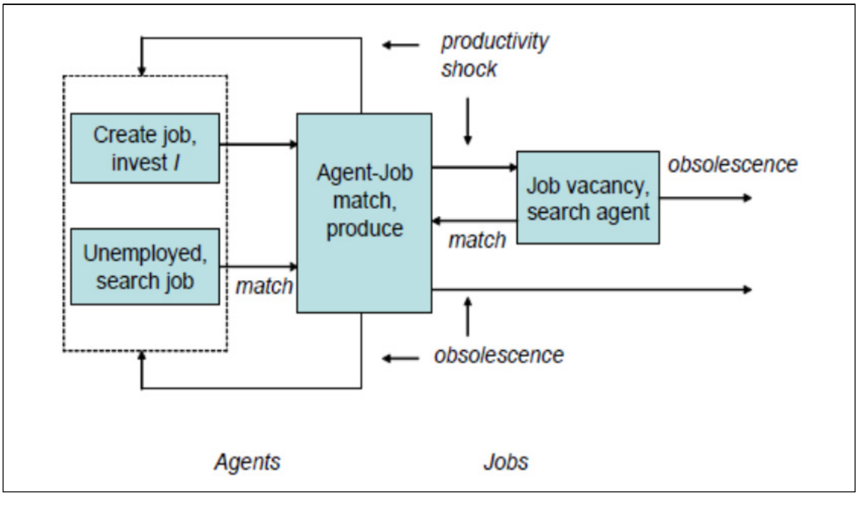

Figure 1 summarizes the flow of agents and jobs through the economy.

Agents who are unmatched with jobs can create jobs on the spot, or search to be matched with existing jobs, in which case they are unemployed. Agents who are matched with jobs are subject to match specific productivity shocks which cause them to re-enter the pool of unmatched agents, and cause the job to be listed as a vacancy. Agents can also re-enter the unemployment pool through the exogenous obsolescence of their job.

The model is assessed quantitatively. When parameters are restricted to match the empirical elasticity of job finding rates with respect to labor productivity, the model generates unemployment rates, investment rates, capital-output ratios, labor shares and wage elasticities which match the data. An active literature has highlighted that the standard model is difficult to reconcile with this elasticity. Notable papers include Costain and Reiter (2008), Shimer (2005), Hornstein, Krusell, Violante (2005), Hall (2007), Mortensen and Nagypal (2007), Hagedorn and Manovskii (2008a) and Pissarides (2007). Thus, extending the standard unemployment model along theoretical dimensions can also help improve its quantitative performance.

This paper combines insights from the search-theoretic literature of unemployment with the search-theoretic literature of asset markets. Pissarides (2000) summarizes the first body of work, while recent contributions to the second body of work include Duffie, Gârleanu and Pedersen (2005), Lagosand Rocheteau (2007), Miao (2006), Rust and Hall (2003), Spulber (1996), and Weill (2007). In a companion a paper, Kim (2008), I analyze the interaction between asset liquidity and selection in the presence of search frictions. In that paper, the asset stock is exogenous, and focus is placed on implications for asset pricing.

The next section introduces the model. Section 3 conducts numerical simulations. The final section concludes.

Ⅱ. Model

All agents are risk-neutral and infinitely lived, with time preferences determined by a constant discount rate $r > 0$. By assuming the agents are risk neutral, I abstract away from risk considerations regarding employment shocks and investment in new jobs in the theoretical and quantitative analysis.

The population of agents is 1. I assume that jobs once created, are freely traded in asset markets and analyze outcomes when agents matched with jobs are distinct from agents owning jobs in the spirit of existing search unemployment models.[2] Jobs will have different prices depending on whether they are currently matched with agents or vacant.

The productivity of an agent-job match is px. $p > 0$ is the aggregate component of the job productivity. $x \in \{0,1\}$ is the component of the productivity specific to the agent-job match. With Poisson arrival rate λ there is a draw of match specific productivity $x = 0$, which motivates the agent to pursue his outside option, and motivates the job owner to list a *vacancy*. With Poisson arrival rate δ assets become permanently obsolete. The obsolescence shock δ is introduced to consider labor activities which get phased out of the economy through new technologies or international trade, and the presence of this will ensure that the measure of jobs in the economy does not grow overtime in steady state.

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2) Interestingly, the analysis is not at all affected under an alternative specification where jobs are always sold to agents matched with the job. Such a specification coincides with a model of business turnover.

Agents who are not matched can create a new job on the spot, or search to be matched with existing vacancies. First consider the latter, in which case agents are *unemployed*. There is a constant returns to scale match function with the stock of unemployed agents and vacancies as arguments. In submarket π, the Poisson arrival rate of matches per vacancy is $Aq(\theta), q'(\theta) \leq 0$, where θ is the ratio of vacancies to unemployed or "market tightness", and A governs the search efficiency. From the assumption of constant returns to scale, the Poisson arrival rate of matches per unemployed is $Am(\theta) \equiv A\theta q(\theta)$. The elasticity of the match function $\eta(\theta) \equiv -\frac{q'(\theta)\theta}{q(\theta)} \in [0,1]$, where the bounds are implied by the assumption of constant returns.

Value equations for an unemployed agent U, matched agent W, and the asset prices for a matched job J, vacancy V, in steady states are given by

$$\begin{aligned} rU &= b + Am(\theta)(W - U), \\ r(W+J) &= p - \lambda(W+J-V-U) - \delta(W+J-U), \\ rV &= -c + Aq(\theta)(J-V) - \delta V. \end{aligned} \tag{1}$$

The expected flow to the unemployed rU consists of the unemployment benefit $b \geq 0$, the capital gain $(W-U)$ resulting from a match with a vacancy which occurs at rate $Am(\theta)$. The sum of expected flows to a agent-job match $r(W+J)$, consists of the per period productivity p, the capital loss $(W+J-V-U)$ resulting from a match productivity shock which occurs at rate λ, and the capital loss $(W+J-U)$ resulting from an obsolescence shock which occurs at rate δ. The expected flow to a vacancy rV, consists of the per period search cost $c \geq 0$, the capital gain $(J-V)$ resulting from a match with an unemployed agent which occurs at rate $Aq(\theta)$, and the capital loss resulting from an obsolescence shock.[3)]

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3) In the standard model $V=0$ motivates a vacancy search cost $c>0$. Here we

Let $\beta \in (0,1)$ denote the bargaining share of the unemployed. The match surplus $S = W + J - U - V$. An agent-job match determines the division of the surplus as the outcome of Nash bargaining. The Nash bargaining rule implies

$$(1-\beta)(W-U) = \beta(J-V). \tag{2}$$

In terms of equations, nothing differentiates the current model with the standard search model of unemployment up to this point.

The innovation of the model is the following. Unmatched agents can circumvent search frictions, and create new jobs instead of searching for vacancies. Thus, the value of a new job consists of the sum of the value of a matched agent and a matched job: $W + J$. Each new job requires an exogenous and irreversible one-off investment $I > 0$ to create. The capital stock resulting from this investment embodies the general (non-match specific) component of the job and is freely traded at asset price J. In equilibrium, unmatched agents are indifferent between creating new jobs or searching for existing vacancies. The value of creating a job $W + J$ is then given by

$$U = W + J - I. \tag{3}$$

Since $W > U$, matched agents have no incentives to create new jobs under this condition. Since $V = W + J - U - S = I - S$ and $S > 0$, investors have no incentive to create new vacancies under this condition.[4] I assume that investment is bounded as

allow for the possibility that $c = 0$.

4) Alternatively, we can interpret the unmatched agent creating a new job with an investor who owns the job valued at J, after the investment is sunk. Let I_W, I_J denote the investment of the unmatched agent and investor respectively. The finance of the investment is divided according to

$$W - U - I_W = \beta(J + W - U - I),$$

$$Assumption\ 1\ ;\ \frac{p-b}{r+\delta+\lambda+\beta Am(\theta_0)} \leq I < \frac{p-b}{r+\delta}, \qquad (4)$$

where θ_0 is the equilibrium market tightness associated with setting $V=0$. These conditions imply that $U \geq 0$, $V \geq 0$ (to be shown).

Condition (3) replaces the free entry condition of vacancies $V=0$, in the standard model. That condition implies there is no need to distinguish between match specific shocks λ and obsolescence shocks δ. In the current model, these shocks have different effects because the value of vacancies will be positive $V \geq 0$. The system of equations (1), (2) and (3) determine $\{V, W, J, U, \theta\}$ given $\{r, \lambda, p, A, \beta, \delta, I\}$ and match function $q(\theta)$.

1. Equilibrium

Equations (1), (2) and (3) imply[5)]

$$p-b-(r+\delta)I = (rU-b)\left(1+\frac{\lambda}{\beta Am(\theta)}\right).$$

Thus, the upper bound on investment $\frac{p-b}{r+\delta} > I$, is a necessary and sufficient condition for $rU-b \geq 0$. If this condition is not satisfied there is no entry of jobs.

Equilibrium market tightness is given by[6)]

$$W-U=\beta(J+W-U-V),$$
$$\Rightarrow I_W = \beta I - \beta V,$$
$$\Rightarrow I_J = (1-\beta)I + \beta V.$$

The agent working in the job pays for his share of the investment minus his share of the increase in the investors outside option V, which he cannot appropriate ex post.

5) Using (1), (2) and (3)

$$r(W+J) = p - \delta I - \frac{\lambda(rU-b)}{\beta Am(\theta)} = rU + rI.$$

6) Using (1) and (2)

$$\left(\frac{r+\lambda+\delta+\beta Am(\theta)}{(1-\beta)Aq(\theta)}\right)(c+(r+\delta)V)=p-b-(r+\delta)V. \quad (5)$$

Setting $V=0$ implies the usual formula for the determination of market tightness in the standard model of unemployment. The value of vacancies is given by

$$\begin{aligned}(r+\delta)V &= -c+Aq(\theta)(1-\beta)-Aq(\theta)(1-\beta)V \\ &= -c+(p-b-(r+\delta)I)\frac{(1-\beta)Aq(\theta)}{\beta Am(\theta)+\lambda}.\end{aligned}$$

Thus, $\frac{c}{Aq(\theta_0)(1-\beta)}\leq I$ is a necessary and sufficient condition for $V\geq 0$, where θ_0 is the market tightness evaluated from setting $V=0$ from (5). Noting that $\frac{c}{Aq(\theta_0)(1-\beta)}=\frac{\pi-b}{r+\delta+\lambda+\beta Am(\theta_0)}\leq I$, yields the lower bound for investment. If this condition is not satisfied there is entry and exit of jobs in equilibrium, but there is no matching of vacancies given the search costs.

Substituting in the value of vacancies, equilibrium market tightness is given by

$$\frac{\beta Am(\theta)+\lambda}{r+\delta+(1-\beta)Aq(\theta)}=\frac{p-b-(r+\delta)I}{c+(r+\delta)I}. \quad (6)$$

Given the constraints on investment (4), there exists a unique steady state equilibrium with entry and exit of jobs and matching of vacancies, where market tightness is given by this equation.

$$(r+\delta)V=-cAq(\theta)(J-V)$$

$$rU-b=((r+\delta)V+c)\frac{\beta}{1-\beta}\theta$$

$$\begin{aligned}(r+\lambda+\delta)S &= \frac{(r+\lambda+\delta)((r+\delta)V+c)}{(1-\beta)Aq(\theta)} \\ &= p-b-((r+\delta)V+c)\frac{\beta}{1-\beta}\theta-(r+\delta)V.\end{aligned}$$

Proposition 1 Equilibrium

Given (4), a unique equilibrium with entry and exit of jobs and matching of vacancies exists.

Comparative statics for p, and other variables are summarized as follows.

Proposition 2 Market tightness

Market tightness θ is (i) increasing in p and (ii) decreasing in b, c, I, β, λ

These results for p, b, c, β, λ conform to those reported by Pissarides (2000) in the context of the standard search unemployment model. In contrast, results for A, r are ambiguous as is the result for δ.

From (5), setting $V=0$, the elasticity of tightness θ with respect to productivity p under the standard model is given by

$$\frac{d\theta}{dp}\frac{p}{\theta}=\frac{p}{p-b}\times\frac{r+\lambda+\delta+\beta Am(\theta)}{\eta(r+\lambda+\delta)+\beta Am(\theta)}. \tag{7}$$

Costain and Reiter (2005) and Shimer (2005) report that this elasticity is large empirically. Shimer (2005) and Manovskii and Hagedorn (2008a) observe that only when b is close to p can such large elasticities be generated. Mortensen and Nagypal (2007) and Hall (2007) note that b is unlikely to be so close to p.[7]

From (6), the elasticity of tightness θ with respect to productivity p in the current model is given by

$$\frac{d\theta}{dp}\frac{p}{\theta}=\frac{p}{p-b-(r+\delta)I}\times\left[\frac{\beta Am(\theta)(1-\eta)}{\beta Am(\theta)+\lambda}\frac{(1-\beta)Aq(\theta)\eta}{r+\delta+(1-\beta)Aq(\theta)}\right]^{-1}. \tag{8}$$

7) This debate is discussed further in the calibration section.

In this case, large elasticities can be generated by $b+(r+\delta)I$ close to p.

Substantively, this opens up the possibility that large elasticities can be generated even when b is not close to p.[8] Intuitively, the scarcity of jobs in this framework introduces a wedge between the benefit of unemployment and the benefit of employment, because the value of vacancies is greater than zero. This implies that although the net flow of a match ($p-b-(r+\delta)I$) is small, the gap $p-b$ can be large.

1) Unemployment and vacancies

The total stock of jobs a, measure of job creation e, and stock of vacancies v are given by

$$\begin{aligned} \dot{a} &= e-\delta a, \\ \dot{v} &= \lambda(a-v)-Aq(\theta)v-\delta v. \end{aligned} \tag{9}$$

The labor market clearing condition implies

$$1-\frac{v}{\theta}=a-v. \tag{10}$$

This states that the share of employed agents equals the measure of matched jobs. In steady states, the stock of jobs a, stock of vacancies v, the unemployment rate u, are given by

$$\begin{aligned} a &= \frac{\lambda\theta+Am(\theta)+\delta\theta}{\lambda+Am(\theta)+\delta\theta}, \\ v &= \frac{\lambda\theta}{\lambda+Am(\theta)+\delta\theta}, \\ u &= \frac{v}{\theta}=\frac{\lambda}{\lambda+Am(\theta)+\delta\theta}. \end{aligned}$$

8) Note that as in the standard model this elasticity is independent of the level of search cost c.

The stock of jobs and vacancies are rising in θ. The unemployment rate u is falling in θ. Combining with Proposition 2 implies the following.

Proposition 3 Unemployment and vacancies

The unemployment rate u *is (i) decreasing in* p *and (ii) increasing in* b, c, I, β, λ.

The stock of vacancies v *is (i) increasing in* p *and (ii) decreasing in* b, c, I, β.

A key difference between the current model and standard model is that here, unemployment is a jump variable and vacancies are a state variable. In the standard model, these properties are reversed. Shimer (2005) documents that the persistence and volatility of vacancies is similar to that of unemployment. Thus, modeling only one of these as a state variable requires further assumptions on the other for it to exhibit state variable properties.

Unemployment data may exhibit state variable properties if agents take time to create new jobs. This could happen for instance if those creating new jobs continue to draw unemployment benefits, and there are time or further search lags in creating new jobs. In this case the measure of unemployment would be defined differently.[9]

2) Efficiency

The social planner's problem is given by

$$\max_{v,\theta,K,x} \int_0^\infty e^{-rt}\left[p\left(1-\frac{v}{\theta}\right)+b\frac{v}{\theta}-cv-xI\right]dt$$

$s.t.$(9) and (10).

In the Appendix, I prove the following.

9) In that case, measured unemployment would lie between u and $u+e$.

Proposition 4 Efficiency
The decentralized economy is efficient under the Hosios condition $\beta = \eta$.

Perhaps surprisingly, the entry decision to create new jobs is efficient despite the positive externality that the job creation has on future unemployed agents it may be matched with. From Proposition 2, $\beta > \eta$ $(\beta < \eta)$ implies that market tightness in too low (too high) relative to the social optimum.

2. Asset prices and wages

J, V are the asset prices of matched and vacant jobs respectively, which from (1), (3) and (6) are given by[10)]

$$J = I - \beta S = I - \beta \frac{c + (r+\delta)I}{r + \delta + (1-\beta)Aq(\theta)},$$
$$V = I - S = I - \frac{c + (r+\delta)I}{r + \delta + (1-\beta)Aq(\theta)}. \tag{11}$$

A higher θ is associated with a lower liquidity of jobs (lower $Aq(\theta)$). Thus, both asset prices are falling in θ, and as a result, falling in p from Proposition 2.[11)] The discount rate on a vacant job relative to a matched job is

$$\frac{J - V}{J} = \frac{(1-\beta)S}{I - \beta S},$$

10) Using (1) and (6)

$$\begin{aligned} S &= \frac{rU - b}{\beta Am(\theta)} \\ &= \frac{p - b - (r+\delta)I}{\beta Am(\theta)} \frac{\beta Am(\theta)}{\beta Am(\theta) + \lambda} \\ &= \frac{c + (r+\delta)I}{r + \delta + (1-\beta)Aq(\theta)}. \end{aligned}$$

11) Since $J = V + (1-\beta)S$ imposing $V = 0$, implies that J is rising in θ in the standard model.

which is rising in θ.

The analysis up to this point has been conducted independently of the determination of the wage level. This highlights the fact that any wage arrangement which satisfies the division of match surplus according to (2) is consistent with the model as argued by Pissarides (2007). I proceed by deriving the wage under the typical assumption of continuous wage bargaining and discuss its interpretation in data.

From (1) and (2), the value of a matched agent W, and value of a matched job J can be expressed as

$$\begin{aligned} rW &= w - \lambda\beta S - \delta\beta(W + J - U), \\ rJ &= [p - w] - \lambda(1-\beta)S - \delta(1-\beta)(W + J - U). \end{aligned}$$

Following Pissarides (2000), w has the interpretation of the wage which is continually renegotiated over time between the agent who is working and the owner of the job. $p-w$ has the interpretation of the implied per period profit accruing to the job. This wage is given by[12)]

$$\begin{aligned} w &= \beta(p - rV) + (1-\beta)rU, \\ &= \beta(p + c\theta) + (1-\beta)b + \beta((r+\delta)\theta - r)V. \end{aligned} \tag{12}$$

The elasticity of the wage with respect to p is

$$\frac{dw}{dp}\frac{p}{w} = \frac{p + [c\theta + (r+\delta)\theta V]\dfrac{d\theta}{dp}\dfrac{p}{\theta}}{p + c\theta + \dfrac{1-\beta}{\beta}b + ((r+\delta)\theta - r)V} + \frac{((r+\delta)\theta - r)}{p + c\theta + \dfrac{1-\beta}{\beta}b + ((r+\delta)\theta - r)V}\frac{dV}{dp}p,$$

12) Using (1) and (2) to get

$$rU = b + ((r+\delta)V + c)\frac{\beta}{1-\beta}\theta.$$

$$\frac{dV}{dp}p = \frac{(1-\beta)Aq(\theta)}{\beta Am(\theta)+\lambda}$$

$$\left[p-(p-b-(r+\delta)I)\left(\frac{\beta Am(\theta)+\lambda\eta}{\beta Am(\theta)+\lambda}\right)\frac{d\theta}{dp}\frac{p}{\theta}\right]. \qquad (13)$$

Setting $V=\frac{dV}{dp}p=0$ the wage elasticity coincides with that of the standard model.

The assumption of continuous wage bargaining is independent of the components of the model which predict a relationship between labor productivity and market tightness, unemployment, job creation, and investment. In particular, there may be alternative wage setting arrangements which predict different relationships between wages and contemporaneous labor productivity as argued by Pissarides (2007). Gertler and Trigari (2005) motivate a model of staggered wage setting to simultaneously address the low contemporaneous elasticity of wages with productivity, and the high volatility of wages. In that environment, the wage elasticity (13) calculated under the assumption of continuous wage bargaining is an upper bound on empirically observed wage elasticity, and I interpret it as such in the quantitative analysis which follows.

Ⅲ. Calibration

I set the match function $q(\theta)=\theta^{-\eta}$. The annual interest rate is set at $r=0.05$, andthe productivity within matches is normalized to $p=1$. Following Shimer (2005), the elasticity of the match function is set to $\eta=0.72$. The annual arrival rate of a match specific productivity shock is set at $\lambda=0.4-\delta$.[13] The annual arrival rate of a match per unemployed is set at $Am(\theta)=A\theta^{1-\eta}=5.42$.

13) This ensures job separations occur at rate $\lambda+\delta=0.4$.

Following den Haan, Ramey and Watson (2000) the annual arrival rate of a match per vacancy is set at $Aq(\theta) = A\theta^{-\eta} = 8.52$. Following Hall (2007), the net benefit flow during unemployment (the formal unemployment benefit plus the value of leisure) is set at $b = 0.7$. Thus, using this low value of the net benefit flow during unemployment, the calibration strategy follows Shimer (2005) and Hall (2007) rather than Hagedorn and Manovskii (2008a).

Using equilibrium condition (6), given other parameters, the bargaining share of matched agents β is set to be consistent with a per period vacancy search cost of $c = 0.11$ as reported by Hagedorn and Manovskii (2008a).[14)]

1. Results

Table 1 summarizes the set of benchmark parameters. At these parameters, in the standard model, $\beta = 0.79$ from (mp), and the elasticity of market tightness from (5) is $\frac{d\theta}{dp}\frac{p}{\theta} = 3.4$, which is substantially lower than the empirical estimate of 19.1 reported by Shimer (2005).[15)]

【Table 1】 Benchmark parameters.

$parameter$	r	p	η	λ	$Am(\theta)$	$Aq(\theta)$	b	β
$Value$	0.05	1	0.72	0.4-δ	5.42	8.52	0.7	c=0.11

The new parameters of the extended model are δ, I. Given the formula for the elasticity of market tightness (8), I first report the

14) Hagedorn and Manovskii (2008) report labor search costs of 0.11, and capital search costs of 0.47. I reinterpret the capital cost as part of the cost of job creation I. Results setting $c = 0.58$ instead of $c = 0.11$ are reported in Appendix Table A1. Only the calibrated value of β is somewhat affected.

15) $\beta = 0.40$, $\frac{d\theta}{dp}\frac{p}{\theta} = 3.5$ when using $c = 0.58$.

level of investment in new jobs I. required to generate an elasticity of $\frac{d\theta}{dp}\frac{p}{\theta}=19.1$, for various levels of the obsolescence parameter δ.[16] I experiment over the entire feasible range of δ. Given $\lambda \geq 0$, $\delta \geq 0$ the feasible range of δ is (0, 0.4).[17]

Table 2 reports the results for δ, I and the associated bargaining share β. When most job destruction is sourced from match specific shocks $\lambda \simeq 0.4$, the required level of investment in new jobs is almost 5 times the per period output. When most job destruction is sourced from job obsolescence shocks $\delta \simeq 0.4$, the required level of investment in new jobs is only half the per period output.

【Table 2】 Parameter set generating $\frac{d\theta}{dp}\frac{p}{\theta}=19.1$.

δ	0	0.05	0.1	0.2	0.3	0.4
l	4.84	2.42	1.62	0.97	0.70	0.54
β	0.15	0.16	0.16	0.18	0.19	0.21
u	0.07	0.06	0.05	0.03	0.02	0
$\frac{\delta al}{p(1-u)}$	0	0.13	0.17	0.20	0.21	0.22
$\frac{al}{p(1-u)}$	5.07	2.52	1.67	0.99	0.71	0.54
$\frac{w}{p}$	0.74	0.76	0.77	0.78	0.79	0.80
$\frac{dw}{dp}\frac{p}{w}$	1.05	1.07	1.09	1.16	1.23	1.29

Table 2 reports further long run statistics of interest which were not targeted by the calibration strategy. These are the unemployment rate u, the investment rate $\frac{\delta I}{p(1-u)}$, the capital-output ratio (valuing

16) Mortensen and Nagypal (2007) argue that the low contemporaneous correlation between θ, p warrants a target of 7.6 rather than 19.1. See Hagedorn and Manovskii (2008b) for an argument why the higher elasticity should be targeted.

17) In each case, I verify that Assumption 1 (4) is satisfied.

capital at replacement cost) $\frac{aI}{p(1-u)}$ and labor share of output $\frac{w}{p}$. For δ in the range $(0.05, 0.1)$ the implied unemployment rate, investment rate, capital output ratio are consistent with the data.[18] The implied labor share of output is slightly larger than usual estimates of around $0.67-0.7$, but comparable. Since $\frac{w}{p} \geq b = 0.7$ this outcome is largely due to the estimate of b adopted from Hall (2007). In Appendix Table A2, I report results using $b = 0.6$ which generate wage shares of 0.67 when δ is in the range $(0.05, 0.1)$ without substantially affecting other results.[19]

Table 2 also reports the elasticity of wages $\frac{dw}{dp}\frac{p}{w}$, under the assumption of continuous wage bargaining. Under this assumption, the model predicts a wage elasticity higher than that observed in data. Hagedorn and Manovskii (2008b) report a wage elasticity of 0.45 using CES data and 0.64 using CPS data. As argued in the discussion on wage determination, the elasticity implied by the model under continuous wage bargaining is likely to be an upper bound on the observed elasticity, consistent with what is found here.

Pissarides (2007) suggests the wage elasticity predicted under continuous wage bargaining is comparable to empirical wage elasticities in new matches. He summarizes micro-econometric evidence suggesting this wage elasticity ranges from 1.02-1.47. This is matched by the predictions here. Overall, the extended model of unemployment is consistent with the observed elasticity of market tightness with respect to labor productivity shocks.[20]

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18) This is the case assuming all investment occurs in new jobs, and capital obsolescence coincides with job obsolescence. For $\delta \in (0.05, 0.1)$ these seem reasonable assumptions to make since δ coincides with typical estimates of capital depreciation rates.

19) Moreover, the implied range of self-employment rates $x = a\delta \in (0.05, 0.1)$ accords well with non-agricultural self-employment rates ranging from 6.9% to 7.5% between 1990-2003, reported by the BLS. See Hipple (2004).

20) The elasticity of market tightness, equation (8), implies that if market tightness is sensitive to labor productivity it is likely to be sensitive to unemployment

2. Comparison with model of vacancy creation

Here I consider a simpler model of costly vacancy creation. The specification is somewhat artificial in that I rule out job creation possibilities by unmatched agents to study the mechanics of the quantitative results in a way which facilitates comparison with the standard model. Equilibrium condition (entry) is replaced by

$$V = I. \tag{14}$$

Recall that under the benchmark model, equilibrium condition (entry) implies $V < I$. Thus, here we are assuming that unmatched agents are not able to create jobs (as in the standard model). Under this new condition, the model is the standard unemployment model with positive vacancy creation cost.

【Table 3】 Parameter set generating $\frac{d\theta}{dp}\frac{p}{\theta} = 19.1$.

δ	0	0.05	0.1	0.2	0.3	0.4
l	4.83	2.42	1.61	0.97	0.69	0.54
β	0.14	0.14	0.14	0.14	0.14	0.14
u	0.07	0.07	0.07	0.07	0.07	0.07
$\frac{\delta al}{p(1-u)}$	0	0.13	0.17	0.20	0.22	0.23
$\frac{al}{p(1-u)}$	5.06	2.54	1.67	1.02	0.72	0.57
$\frac{w}{p}$	0.74	0.76	0.76	0.77	0.77	0.77
$\frac{dw}{dp}\frac{p}{w}$	1.01	0.98	0.98	0.97	0.97	0.97

benefits via b as argued by Costain and Reiter (2008) in the context of the standard model. Hagedorn and Manovskii (2008a) argue that the implications of evidence to this dimension of the model is mixed.

Equilibrium equations are derived in the Appendix. Here I report the calibration results using the parameters motivated above. These are reported in Table 3.[21)]

The overall results are very similar to the benchmark model. Thus, I conclude the quantitative performance of the model is driven by introducing positive vacancy costs to the standard unemployment model.

As mentioned above in the discussion of (8), intuitively, the scarcity of jobs in this framework introduces a wedge between the benefit of unemployment and the benefit of employment, because the value of vacancies is greater than zero. This implies that although the net flow of a match $(p-b-(r+\delta)I)$ is small, the gap $p-b$ can be large leading to the results regarding elasticity.

However, a key theoretical motivation in terms for allowing for the occupational choice of unmatched agents, is addressed only by the benchmark model.

Ⅳ. Conclusion

This paper extended the canonical search model of unemployment along two dimensions: (i) allowing agents to create jobs, and (ii) allowing vacancies to be re-matched. Only agents who are unmatched with jobs have incentives to create jobs, and jobs are valued not just for their output in current matches, but also their anticipated output in future matches. Parameter specifications of the extended model conform with empirical dimensions of unemployment dynamics.

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21) Note that $\lambda=0 \Rightarrow \delta=0.4$ corresponds to the case of no trade in vacancies (as in the standard model).

◈ *References* ◈

Costain, J. S. and Reiter, M. (2008), "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models," *Journal of Economic Dynamics and Control*, 32:4, pp. 1120-1155.

den Hann, Wouter, Garey Ramey, and Joel Watson (2000), "Job Destruction and Propagation of Shocks," *American Economic Review*, 90, pp. 482-498.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005), "Over-the-counter Markets," *Econometrica*, 73, pp. 1815-1847.

Gertler, M., and A. Trigari (2006), "Unemployment Fluctuations with Staggered Nash Bargaining," *Journal of Political Economy*, 117(1), pp. 38-86.

Hagedorn, Marcus and Iourii Manovskii (2008a), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98.

Hagedorn, Marcus and Iourii Manovskii (2008b), "The Cyclical Behavior of Equilibrium Unemployment," Vacancies and Wages, mimeo.

Hall, Robert (2007), "Sources and Mechanisms of Cyclical Fluctuations in the Labor Market," mimeo.

Hipple, Steven (2004), Self-Employment in the United States: An Update, BLS, http://www.bls.gov/opub/mlr/2004/07/art2exc.htm

Hornstein, A., P. Krusell, and G. L. Violante (2005), "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanisms," *Federal Reserve Bank of Richmond Economic Quarterly*, 91(3), pp. 19-51.

Hosios, Arthur (1990), "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57:2, pp. 279-98.

Kim, Y. (2008), Liquidity and Selection in Asset Markets with Search Frictions, mimeo USC.

Lagos, R., and G. Rocheteau (2007), "Liquidity in Asset Markets with Search Frictions," *Econometrica*, 77:2, pp. 403-426.

Miao, Jianjun (2006) "A Search Model of Centralized and Decentralized Trade," *Review of Economic Dynamics*, Elsevier for the Society for Economic Dynamics, 9:1, pp. 68-92, January.

Mortensen, Dale and Eva Nagypal (2007), "More on Unemployment and Vacancy Fluctuations," *Review of Economic Dynamics*, 10, pp. 327-347.

Pissarides, C. A. (2000), *Equilibrium Unemployment Theory*, MIT Press, 2nd edition, Cambridge.

Pissarides, C. A. (2009), "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?" *Econometrica*, 77, pp. 1339-1369.

Rust, J., and J. Hall (2003), "Middlemen versus Market Makers: A Theory of Competitive Exchange," *Journal of Political Economy*, 111, pp. 353-403.

Shimer, Robert (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95, pp. 25-49.

Spulber, D. (1996), "Market Making by Price Setting Firms," *Review of Economic Studies*, 63, pp. 559-580.

Weill, P. O. (2007), "Leaning Against the Wind," *Review of Economic Studies*, 74, pp. 1329-1354.

Williams, J. T. (1995) "Pricing Real Assets with Costly Search," *Review of Financial Studies*, 8, pp. 55-90.

Appendix

Proof of efficiency

Let μ, ϕ denote co-state variables associated with the evolution of job stock a, and vacancy stock v respectively, and let τ denote the Lagrange multiplier on the labor market clearing condition. Euler conditions and first-order conditions are given by

$$v:\ -e^{-rt}\left(\frac{p}{\theta}-\frac{b}{\theta}+c\right)+\phi(\lambda+\delta+Aq(\theta))-\dot{\phi}+\tau\left(\frac{1}{\theta}-1\right)=0$$

$$\theta:\ e^{-rt}(p-b)-\phi\eta Am(\theta)-\tau=0$$

$$a:\ \delta\mu-\dot{\mu}-\lambda\phi+\tau=0$$

$$x:\ -e^{-rt}I-\mu=0.$$

Solving for co-state variables and Lagrange multiplier

$$\mu=-e^{-rt}I$$

$$\tau=e^{-rt}(r+\delta)I+\lambda\phi$$

$$\phi=e^{-rt}\frac{p-b-(r+\delta)I}{\eta Am(\theta)+\lambda}.$$

Substituting into the Euler equation for v, and dividing through by e^{-rt}

$$\left(\frac{p}{\theta}-\frac{b}{\theta}+c\right)=\frac{p-b-(r+\delta)I}{\eta Am(\theta)+\lambda}\left(r+\lambda+\delta+Aq(\theta)+\lambda\left(\frac{1}{\theta}-1\right)\right)$$
$$+(r+\delta)I\left(\frac{1}{\theta}-1\right)$$

$$c+(r+\delta)I=\frac{p-b-(r+\delta)I}{\eta Am(\theta)+\lambda}\left(r+\delta+Aq(\theta)+\lambda\frac{1}{\theta}\right)$$
$$-(p-b-(r+\delta)I)\frac{1}{\theta}$$

$$\frac{c+(r+\delta)I}{p-b-(r+\delta)I}=\frac{r+\delta+Aq(\theta)-\eta Aq(\theta)}{\eta Am(\theta)+\lambda}.$$

Comparing with (6), the two conditions for θ are equivalent under the Hosios condition.

Calibration results under alternative specifications

【Table A.1】 Parameter set generating $\frac{d\theta}{dp}\frac{p}{\theta}=19.1$, using $b=0.58$.

δ	0	0.05	0.1	0.2	0.3	0.4
l	4.72	2.37	1.59	0.96	0.69	0.55
β	0.04	0.05	0.06	0.07	0.09	0.10
u	0.07	0.06	0.05	0.03	0.02	0
$\frac{\delta al}{p(1-u)}$	0	0.12	0.16	0.20	0.21	0.22
$\frac{al}{p(1-u)}$	4.94	2.47	1.64	0.98	0.70	0.55
$\frac{w}{p}$	0.73	0.74	0.74	0.75	0.77	0.78
$\frac{dw}{dp}\frac{p}{w}$	0.66	0.75	0.84	1.01	1.17	1.32

【Table A.2】 Parameter set generating $\frac{d\theta}{dp}\frac{p}{\theta}=19.1$, using $c=0.6$.

δ	0	0.05	0.1	0.2	0.3	0.4
l	6.81	3.41	2.28	1.37	0.98	0.77
β	0.11	0.12	0.13	0.14	0.15	0.17
u	0.07	0.06	0.05	0.03	0.02	0
$\frac{\delta al}{p(1-u)}$	0	0.18	0.24	0.28	0.30	0.31
$\frac{al}{p(1-u)}$	7.14	3.55	2.36	1.39	0.99	0.77
$\frac{w}{p}$	0.64	0.66	0.67	0.69	0.70	0.78
$\frac{dw}{dp}\frac{p}{w}$	1.12	1.15	1.19	1.28	1.37	1.82

Equilibrium of vacancy creation model

Using (5) and (14), market tightness and its elasticity is given by

$$\frac{r+\lambda+\delta+\beta Am(\theta)}{(1-\beta)Aq(\theta)}=\frac{p-b-(r+\delta)I}{c+(r+\delta)I},$$
$$\frac{dp}{d\theta}\frac{\theta}{p}=\frac{p}{p-b-(r+\delta)I}\times\frac{r+\lambda+\delta+\beta Am(\theta)}{\eta(r+\lambda+\delta)+\beta Am(\theta)}.$$

The law of motion for the asset stock and the labor market clearing condition are unchanged. The law of motion for the stock of vacancies is

$$\dot{v}=x+\lambda(a-v)-Aq(\theta)v-\delta v\Rightarrow v=a\frac{\lambda+\delta}{\lambda+\delta+Aq(\theta)}.$$

Equilibrium stocks are then given by

$$a=\frac{(\lambda+\delta)\theta+Am(\theta)}{\lambda+\delta+Am(\theta)},$$
$$v=\frac{(\lambda+\delta)\theta}{\lambda+\delta+Am(\theta)},$$
$$u=\frac{v}{\theta}=\frac{\lambda+\delta}{\lambda+\delta+Am(\theta)}.$$

From (13), setting $V=I$, $\frac{dV}{dp}p=0$, the wage elasticity under continuous wage bargaining is given by

$$\frac{dw}{dp}\frac{p}{w}=\frac{p+[c\theta+(r+\delta)\theta I]\frac{d\theta}{dp}\frac{p}{\theta}}{p+c\theta+\frac{1-\beta}{\beta}b+((r+\delta)\theta-r)I}.$$

These equations are used to calculate statistics reported in Table 3.

실업과 일자리의 자산 시장

김 용 진*

논문초록

실업의 탐색 모형을 통해 우리는 경제를 기업과 노동자라는 경제내 두 집단 간의 결합 문제(matching problem)로 분석할 수 있다.이 시점에서 우리는 '노동자들은 왜 스스로 일자리를 만들어 탐색 마찰을 피해갈 수 없는가' 라는 질문과 마주하게 된다. 한편 기업들이 완전 탄력적으로 빈 일자리를 공급할 수 있는 가운데 일자리 공여가 생성되는 과정에 관한 의문점을 제기할 수 있다. 필자는 이 문제들을 사전적으로 동일한 경제주체들이 일자리를 만들고 거래할 수 있는 환경에서 분석하고 모형을 정량적으로 평가할 것이다.

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