한국경제학보 제18권 제1호
The Korean Journal of Economics Vol. 18, No. 1(Spring 2011)

# Input-Output Multiplier Analysis through the Decomposition by Factors of the Leontief Inverse: A Regional Case Study on the Korean Economy

Ho Un Gim* · Koonchan Kim**

Abstracts

The Leontief inverse $C^f$, also termed as the output requirements matrix for final demand, gave a rise to the formulation of the ordinary input-output multipliers. In this paper, we discuss on the existence and concept of four useful input and output requirements matrices: the Leontief inverse, the output requirements matrix for output, $C^g$, obtained from the decomposition by factors of the Leontief inverse, and two input requirements matrices $\Gamma^f$ and $\Gamma^g$ for final demand and output respectively. This paper then focuses on the formulation and analysis of various input-output multipliers based on these four requirements matrices; the multipliers based on $\Gamma^f$ and $C^f$ measure the total requirements of changes in final demand and the multipliers based on $\Gamma^g$ and $C^g$ compute the total requirements of changes in gross output. A unifying framework that describes completely how these four requirements matrices are related to inputs, output, and final demand is presented, and a comprehensive empirical analysis is performed in the Korean economy to study the difference of the effects of these newly defined multipliers.

KRF Classification : B031002
Keywords : Input-Output Multiplier, Decomposition by Factors, Leontief Inverse, Requirements Matrices, Output Requirements

* Corresponding Author, Professor, Department of Economics and Finance, Keimyung University, e-mail: houn@kmu.ac.kr
** Professor, Department of Mathematics, Keimyung University, e-mail: kmkim@kmu.ac.kr

# Ⅰ. Introduction

The ordinary (traditional) input-output multipliers are probably the most important tools in measuring the total impact upon output, employment, and income when there is a change in the exogenous final demand $f$. These well known multipliers are based primarily on the Leontief inverse $C^f = (I - A)^{-1}$ which can only be associated with the exogenous final demand, where $I$ is the identity matrix and $A$ the input coefficient matrix.

The development of new and different types of multipliers is necessary and its demand is increasing as can be seen from the applications considered, for example, in Rickman and Schwer (1995), Lenzen (2001), Dietzenbacher (2002), and Liew (2005). In particular, as an attempt for the development of new multipliers that accept output as an initial change, Oosterhaven and Stelder (2002) introduced the concept of net multipliers[1)] to avoid double-counting impacts when $v_c'(I - A)^{-1}$ is multiplied with the endogenous total output $x$, instead of the exogenous final demand $f$, where $v_c'$ represents a row with value-added coefficients.

In this paper we first discuss the existence of altogether four different output and input requirements matrices, each of which

1) For further notes and comments on "net multipliers" see de Mesnard (2002, 2007), Oosterhaven (2004, 2007), and Dietzenbacher (2005). It turned out that the net multiplier is only a homogenous formula with no causal link, but it can be viewed as a new key sector indicator. The literatures mentioned above, however, did not point out a fundamental problem. That is, the Leontief inverse $C^f$ should not be multiplied by the total output $x$, apart from the problems of double counting and overestimation. The reason is that the subscript $j$ in the element $c_{ij}^f$ of $C^f$ represents the final demand of sector $j$ and the subscript $j$ in the element $x_j$ of $x$ represents the total output of sector $j$, and so the multiplication of $C^f$ by $x$ does not make sense since there is no "consecutive connection" (refer to footnote 8 for its concept) between elements in $C^f$ and $x$.

has a distinct characteristic: two input requirements matrices $\Gamma^f$ and $\Gamma^g$ for final demand and output respectively, the concept of output requirements matrix $C^g$ for output obtained from the decomposition factors of the Leontief inverse, and the Leontief inverse $C^f$ which represents the output requirements matrix for final demand. We outline the characteristics of the four requirements matrices and present a unifying framework which describes completely how they are related to inputs, output, and final demand.

Then, in addition to the ordinary multipliers driven from the Leontief inverse, we define other various types of multipliers on the basis of three other output and input requirements matrices. We show in particular that the multipliers developed based on the output (or input) requirements matrix $C^g$ (or $\Gamma^g$) for output can be advantageously used for computing the impact when the initial change is output. We perform an empirical analysis on the Korean economy to examine the difference of the effects of the decomposition factors and these newly defined multipliers by comparing the Spearman's rank correlation coefficients. The importance of the requirements matrices is further demonstrated in the justification of the Hawkins-Simon conditions and in the backward and forward linkage analysis.

This paper begins with a brief overview of the development of four input and output requirements matrices. Throughout this paper, the notation $C^f = (c_{ij}^f)$ is used for the Leontief inverse ($L$ or $C$ is used in some other literatures) to be consistent with other notations in this paper. The superscripts '$f$' and '$g$' given in the requirements matrices stand for final demand and gross output, respectively.

# Ⅱ. Preliminaries

The Leontief inverse $\boldsymbol{C}^f = (\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{A}^2 + \cdots$ is termed as the output requirements matrix for final demand, and its $ij$th element $c_{ij}^f$ represents the direct and indirect output requirements of commodity $i$ to support a unit of final demand of commodity $j$. It is also well-known that the notion of direct and indirect input requirements of commodity $i$ to 'support' a unit of final demand of commodity $j$, denoted by $\gamma_{ij}^f$, and $c_{ij}^f$ are related by:[2]

$$\gamma_{ii}^f = c_{ii}^f - 1, \text{ and } \gamma_{ij}^f = c_{ij}^f \text{ for } i \neq j \qquad (1)$$

In matrix notation, equation (1) can be written as $\boldsymbol{\Gamma}^f = \boldsymbol{C}^f - \boldsymbol{I} = \boldsymbol{A} + \boldsymbol{A}^2 + \cdots$,[3] where $\boldsymbol{\Gamma}^f = (\gamma_{ij}^f)$.

Jeong (1982, 1984) pointed out that the two notions, the notion of $\gamma_{ij}^f$ and the notion of direct and indirect input requirements of commodity $i$ to 'produce' a unit of gross output of commodity $j$, denoted by $\gamma_{ij}^g$, are different in the open static input-output model. Gim and Kim (1998), then, attempted to find the general relation between them. The expression below is the complemented general relation between $\gamma_{ij}^f$ and $\gamma_{ij}^g$ (Gim and Kim, 2009):

$$\gamma_{ii}^f = \gamma_{ii}^g c_{ii}^f, \;\; \gamma_{ij}^f = \gamma_{ij}^g c_{jj}^f, i \neq j \qquad (2)$$

where

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2) This relation is defined in Jeong (1984, p. 475) and Gim and Kim (1998, 2005), etc.

3) The inverse $\boldsymbol{C}^f$ represents the direct and indirect output requirements matrix for final demand and $\boldsymbol{I}$ the total output itself to be delivered to final demand. Therefore, the direct and indirect input requirements matrix for final demand $\boldsymbol{\Gamma}^f$ has the same meaning as that of $\boldsymbol{C}^f - \boldsymbol{I}$.

$$\gamma^g_{ii} = 1 - \frac{1}{c^f_{ii}} = a_{ii} + \sum_{j=1, j \neq i}^{n} a_{ij} \frac{c^f_{ji}}{c^f_{ii}}, \tag{3}$$

$$\gamma^g_{ij} = \frac{c^f_{ij}}{c^f_{jj}} = a_{ij} + \sum_{k=1, k \neq j}^{n} a_{ik} \frac{c^f_{kj}}{c^f_{jj}}, i \neq j \tag{4}$$

for $i, j = 1, 2, \cdots, n$. As we call the Leontief inverse $\boldsymbol{C}^f$ the output requirements matrix for final demand, the requirements matrices $\boldsymbol{\Gamma}^f = (\gamma^f_{ij})$ and $\boldsymbol{\Gamma}^g = (\gamma^g_{ij})$ are termed the input requirements matrix for final demand and the input requirements matrix for output, respectively. Notice that if we let $\boldsymbol{K}$ be the diagonal matrix whose $i$th diagonal element is the $i$th diagonal element of $\boldsymbol{C}^f$ (i.e., $c^f_{ii}$), then it follows from equation (2) that $\boldsymbol{\Gamma}^g = \boldsymbol{\Gamma}^f \boldsymbol{K}^{-1}$.

As we illustrated above, the two requirements matrices $\boldsymbol{\Gamma}^f = (\gamma^f_{ij})$ and $\boldsymbol{\Gamma}^g = (\gamma^g_{ij})$ can be easily obtained once $\boldsymbol{C}^f = (c^f_{ij})$ is computed. Recently Gim and Kim (2005, 2008) further showed that $\gamma^g_{ij}$ can be decomposed into the direct and the technical indirect effects, $\gamma^f_{ij}$ into the direct, the technical indirect, and the interrelated indirect effects, and $c^f_{ij}$ into four different parts as the final demand, the direct, the technical indirect, and the interrelated indirect effects. The derivation of the decomposition is briefly reviewed below.

To begin with, as the Leontief inverse $\boldsymbol{C}^f$ can be written in a power series of $\boldsymbol{A}$ (see Waugh, 1950), each element $c^f_{ij}$ with $n$ sectors can be expressed as:

$$\begin{aligned} c^f_{ij} = \delta_{ij} + a_{ij} + \Big( & \sum_{r_1=1}^{n} a_{ir_1} a_{r_1 j} + \sum_{r_2=1}^{n} \sum_{r_1=1}^{n} a_{ir_1} a_{r_1 r_2} a_{r_2 j} \\ & + \cdots + \sum_{r_k=1}^{n} \sum_{r_{k-1}=1}^{n} \cdots \sum_{r_1=1}^{n} a_{ir_1} a_{r_1 r_2} \cdots a_{r_{k-1} r_k} a_{r_k j} + \cdots \Big) \end{aligned} \tag{5}$$

where

$$\delta_{ij} = \begin{cases} 1, & \text{if } i = j \\ 0, & otherwise \end{cases}$$

for $i, j = 1, 2, \cdots, n$. The third term in the right-hand side of equation (5) is well-known to be the total cumulative indirect effect. Moreover, in terms of only the elements of $\boldsymbol{A}$, the element $\gamma_{ij}^{g}$ given in equations (3) and (4) with $n$ sectors can be formulated and represented by:[4)]

$$\gamma_{ij}^{g} = a_{ij} + \Big( \sum_{\substack{r_1 = 1 \\ r_1 \neq j}}^{n} a_{ir_1} a_{r_1 j} + \sum_{\substack{r_2 = 1 \\ r_2 \neq j}}^{n} \sum_{\substack{r_1 = 1 \\ r_1 \neq j}}^{n} a_{ir_1} a_{r_1 r_2} a_{r_2 j} + \cdots + \sum_{\substack{r_k = 1 \\ r_k \neq j}}^{n} \sum_{\substack{r_{k-1} = 1 \\ r_{k-1} \neq j}}^{n} \cdots \sum_{\substack{r_1 = 1 \\ r_1 \neq j}}^{n} a_{ir_1} a_{r_1 r_2} \cdots a_{r_{k-1} r_k} a_{r_k j} + \cdots \Big) \tag{6}$$

for $i, j = 1, 2, \cdots, n$. The above expression is generated by considering only for the interindustry interdependence which reflects the complete technical relationship of production. Equation (6), thus, can be written as $\gamma_{ij}^{g} = a_{ij} + t_{ij}$, where $t_{ij}$ is the second term in the right-hand side of equation (6). The factor $t_{ij}$ is referred to as the technical indirect effect[5)] since it is derived from only the purely technical relation between inputs and output. Finally, let $r_{ij}$ be the remainder after excluding the technical indirect effect from the total cumulative indirect effect. That is, $r_{ij}$ can be obtained by subtracting $t_{ij}$ from the third term in the right-hand side of equation (5), which yields:

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4) See Appendix A (A1) for a brief illustration of deriving the expression of $\gamma_{ij}^{g}$.

5) See Appendix A (A2) for the concept of technical indirect effect and footnote 8 of Gim and Kim (2009).

$$r_{ij} = a_{ij}a_{jj} + \Big(\sum_{r_1=1}^{n} a_{ir_1}a_{r_1j}a_{jj} + \sum_{\substack{r_2=1 \\ r_2 \neq j}}^{n} a_{ij}a_{jr_2}a_{r_2j}\Big)$$

$$+ \Big(\sum_{r_2=1}^{n}\sum_{r_1=1}^{n} a_{ir_1}a_{r_1r_2}a_{r_2j}a_{jj} + \sum_{\substack{r_3=1 \\ r_3 \neq j}}^{n}\sum_{r_1=1}^{n} a_{ir_1}a_{r_1j}a_{jr_3}a_{r_3j}$$

$$+ \sum_{\substack{r_3=1 \\ r_3 \neq j}}^{n}\sum_{\substack{r_2=1 \\ r_2 \neq j}}^{n} a_{ij}a_{jr_2}a_{r_2r_3}a_{r_3j}\Big) + \cdots$$

for $i,j = 1,2,\cdots,n$. The factor $r_{ij}$ is referred to as the interrelated indirect effect.[6] Consequently, each element of the Leontief inverse $c_{ij}^f$ can be written as $c_{ij}^f = \delta_{ij} + a_{ij} + t_{ij} + r_{ij}$.

It follows that the above decomposition results can be summarized in matrix notation as $\boldsymbol{\Gamma}^g = \boldsymbol{A} + \boldsymbol{T}$, $\boldsymbol{\Gamma}^f = \boldsymbol{A} + \boldsymbol{T} + \boldsymbol{R}$ and $\boldsymbol{C}^f = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{T} + \boldsymbol{R}$, where $\boldsymbol{I}$, $\boldsymbol{A} = (a_{ij})$, $\boldsymbol{T} = (t_{ij})$, and $\boldsymbol{R} = (r_{ij})$ represent the initial effect, the direct effect, the technical indirect effect, and the interrelated indirect effect, respectively. From the decomposition by factors, it is clear that the two notions $\gamma_{ij}^f$ and $\gamma_{ij}^g$ differ by $r_{ij}$ (i.e., $\boldsymbol{\Gamma}^f - \boldsymbol{\Gamma}^g = \boldsymbol{R}$).

Moreover, due to the decomposition of the Leontief inverse by the four factors, a total of eleven combinations is possible as shown below, among which three of them are already used as $\boldsymbol{C}^f$, $\boldsymbol{\Gamma}^f$, and $\boldsymbol{\Gamma}^g$:

$$(\boldsymbol{I}, \boldsymbol{A}), (\boldsymbol{I}, \boldsymbol{T}), (\boldsymbol{I}, \boldsymbol{R}), (\boldsymbol{A}, \boldsymbol{T}), (\boldsymbol{A}, \boldsymbol{R}), (\boldsymbol{T}, \boldsymbol{R});$$
$$(\boldsymbol{I}, \boldsymbol{A}, \boldsymbol{T}), (\boldsymbol{I}, \boldsymbol{T}, \boldsymbol{R}), (\boldsymbol{A}, \boldsymbol{T}, \boldsymbol{R}), (\boldsymbol{I}, \boldsymbol{A}, \boldsymbol{R});$$
$$(\boldsymbol{I}, \boldsymbol{A}, \boldsymbol{T}, \boldsymbol{R})$$

By taking the combination $(\boldsymbol{I}, \boldsymbol{A}, \boldsymbol{T})$, Gim and Kim (2009)

6) See Appendix A (A2) for the concept of interrelated indirect effect and footnote 9 of Gim and Kim (2009).

further introduced the concept of *the (direct and indirect) output requirements matrix for output*,7) which is defined as:

$$\boldsymbol{C}^g = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{T}$$

More specifically, the $ij$th element $c^g_{ij}$, given by $\delta_{ij} + a_{ij} + t_{ij}$, of $\boldsymbol{C}^g$ is defined as *the direct and indirect output requirements of commodity* $i$ *to produce a unit of gross output of commodity* $j$. Based on the new concept, they built a new open static 'output-output' (O-O) model in comparison to the open static input-output (I-O) model.

## Ⅲ. The Overall Framework

The four requirements matrices we reviewed in the previous section are the two output requirements matrices $\boldsymbol{C}^f$ and $\boldsymbol{C}^g$ for final demand and output and the two input requirements matrices $\boldsymbol{\Gamma}^f$ and $\boldsymbol{\Gamma}^g$ for final demand and output, respectively:

$$\boldsymbol{C}^f = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{T} + \boldsymbol{R};$$
$$\boldsymbol{C}^g = \boldsymbol{I} + \boldsymbol{A} + \boldsymbol{T};$$
$$\boldsymbol{\Gamma}^f = \boldsymbol{A} + \boldsymbol{T} + \boldsymbol{R};$$
$$\boldsymbol{\Gamma}^g = \boldsymbol{A} + \boldsymbol{T}.$$

In this section we present an overall framework for how the

7) This concept differs from the mixed type of input-output model called the output-to-output multiplier in Miller and Blair (2009). Also for the total-flow approach, see Szyrmer (1992) and Szyrmer and Walker (1983). The new concept describes the system which relates output for final demand $\boldsymbol{x}$ to output for output $\boldsymbol{o}$ as shown in Figures 1 and 2.

output and input requirements matrices $\boldsymbol{C}^f$, $\boldsymbol{C}^g$, $\boldsymbol{\Gamma}^f$, and $\boldsymbol{\Gamma}^g$ between inputs, output, and final demand are connected. If we denote $\boldsymbol{f}$ as the final demand vector, $\boldsymbol{x}$ as the total output vector for a given final demand vector $\boldsymbol{f}$, $\boldsymbol{p}$ as the total input vector for a given final demand vector $\boldsymbol{f}$, $\boldsymbol{q}$ as the total input vector for a given total output vector $\boldsymbol{x}$, and $\boldsymbol{o}$ as the total output vector for a given total output vector $\boldsymbol{x}$, then we have, for example:

$$\boldsymbol{x} = \boldsymbol{C}^f \boldsymbol{f},\ \boldsymbol{p} = \boldsymbol{\Gamma}^f \boldsymbol{f},\ \boldsymbol{q} = \boldsymbol{\Gamma}^g \boldsymbol{x},\ \text{and}\ \boldsymbol{o} = \boldsymbol{C}^g \boldsymbol{x}$$

In words, when the final demand $\boldsymbol{f}$ is given, then the total output requirements $\boldsymbol{x}$ to be produced is computed by multiplying the output requirements matrix for final demand $\boldsymbol{C}^f$ on $\boldsymbol{f}$. Similarly, when the total output $\boldsymbol{x}$ is given, the total input requirements $\boldsymbol{q}$ to be needed is calculated by multiplying the input requirements matrix for output $\boldsymbol{\Gamma}^g$ on $\boldsymbol{x}$. Furthermore, when the output requirements matrix for output $\boldsymbol{C}^g$ is multiplied on $\boldsymbol{x}$, the total output requirements $\boldsymbol{o}$ to be produced for the given total output $\boldsymbol{x}$ can be estimated (see Figure 1).

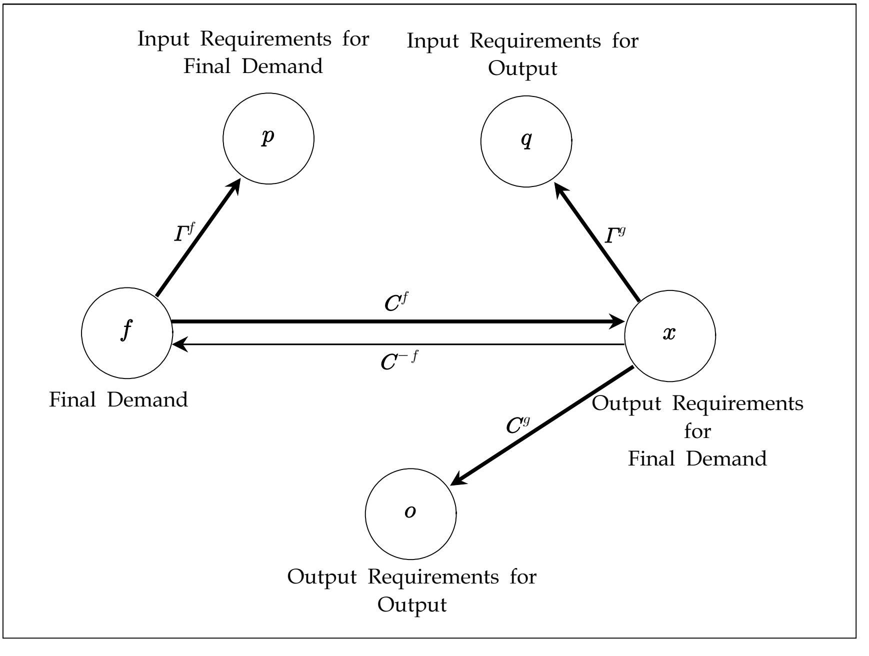
Notice that the so-called "consecutive connection"[8] exists between elements in $\boldsymbol{C}^f$ and $\boldsymbol{f}$. That is, an element $c_{ij}^f$ of the Leontief inverse represents the direct and indirect output requirements of commodity $i$ to support a unit of final demand of commodity $j$ and an element $f_j$ of $\boldsymbol{f}$ represents the final demand of commodity $j$. When the two elements $c_{ij}^f$ and $f_j$ are multiplied to form $c_{ij}^f f_j$, the product $c_{ij}^f f_j$ makes sense in the interpretation

8) The concept of consecutive connection in the input-output model means that the term which should be post-multiplied (or pre-multiplied) by $\boldsymbol{C}^f$ is restricted to only the final demand $\boldsymbol{f}$ (or the gross output $\boldsymbol{x}$) to have a proper economic meaning of interindustry interdependence among sectors and variables.

of interindustry interdependence because the subscript $j$ in $c_{ij}^{f}$ and $f_j$ represents the final demand of commodity $j$.

Likewise, there exists a consecutive connection between elements in $\boldsymbol{C}^g$ and $\boldsymbol{x}$. The product $c_{ij}^{g}x_j$ makes sense in the interpretation of intersectoral interreliance because the subscript $j$ in $c_{ij}^{g}$ and $x_j$ means the same gross output of commodity $j$. Apparently the consecutive connection between elements in $\boldsymbol{\Gamma}^f$ and $\boldsymbol{f}$ and that in $\boldsymbol{\Gamma}^g$ and $\boldsymbol{x}$ also hold.[9] With the notation $\boldsymbol{C}^{-f}$ for the inverse of $\boldsymbol{C}^f$, which is given by $(\boldsymbol{I}-\boldsymbol{A})$, a framework can be constructed as shown in Figure 1.

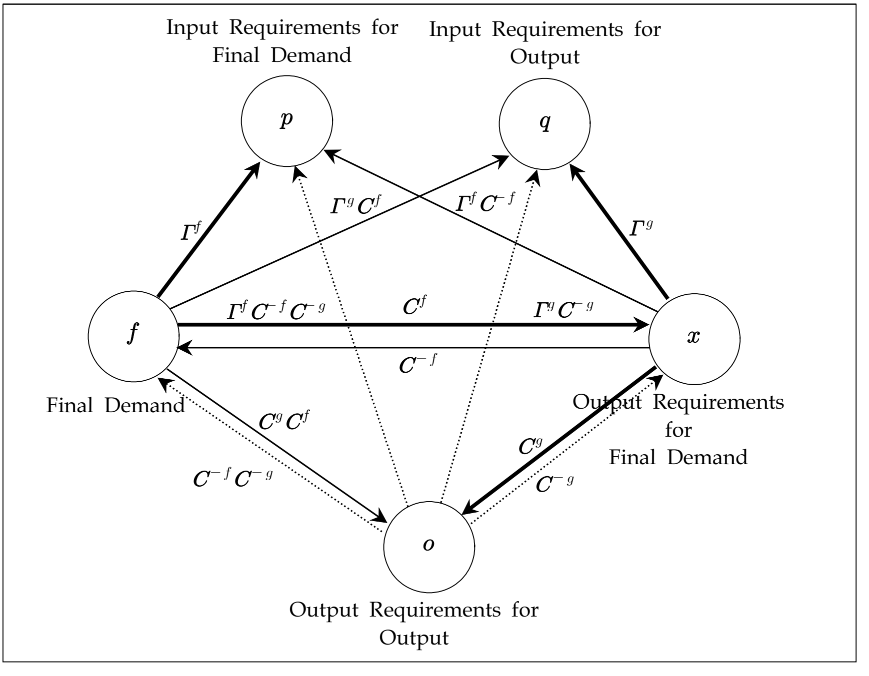
【Figure 1】 Framework of the Requirements Matrices




9) We should be cautious in the case of premultiplying $\boldsymbol{A}$ to $\boldsymbol{C}^f$ or $\boldsymbol{C}^g$. The input coefficient matrix $\boldsymbol{A}$ is explicitly connected or relevant to the solution of the input-output model $\boldsymbol{x}=\boldsymbol{C}^f\boldsymbol{f}$. Therefore, $\boldsymbol{AC}^f$ makes sense, but $\boldsymbol{AC}^g$ does not make sense since the two systems $\boldsymbol{x}=\boldsymbol{C}^f\boldsymbol{f}$ and $\boldsymbol{o}=\boldsymbol{C}^g\boldsymbol{x}$ are different.

If the inverse of $C^g$, denoted by $C^{-g}$, exists[10] (most of the time it exists), then a complete unifying framework for the output and input requirements matrices between inputs, output, and final demand can be established as shown in Figure 2 (cf. Gim, 2008, chapter v).

【Figure 2】 Complete Framework of the Requirements Matrices




The more elaborated diagram above enables one to compute the required amount (effect variable) when a certain initial amount

10) $C^g$ can be derived from the balance equation $o = Bo + x$, where $B$ can be defined as "output" coefficient matrix and $o$ the gross output vector for output. The $ij$th element of $B$, $b_{ij}$, can be given as the ratio of output to output, that is, $b_{ij} = w_{ij}/o_j$, where $w_{ij}$ represents the flow of output from sector $i$ to sector $j$ and $o_j$ the $j$th element of $o$. Solving $o$ from $o = Bo + x$ gives $o = (I - B)^{-1}x = C^g x$ with $C^g = (I - B)^{-1}$. If we assume $0 \le b_{ij} < 1$ for all $i$ and $j$, then $C^g = (I - B)^{-1}$ and $C^{-g} = (I - B) = (I + A + T)^{-1}$ can also exist.

(cause variable) is provided. For example, $q = \Gamma^g C^f f$, $x = C^{-g} o$, $o = C^g C^f f$, etc. Note that $\Gamma^g C^f$ represents the product of two matrices $\Gamma^g$ and $C^f$. The four fundamental requirements matrices $C^f$, $C^g$, $\Gamma^f$, and $\Gamma^g$ are represented by the arrows with the thick-solid line as in Figure 1. $C^{-f}$ and the composition of two matrices among $C^f$, $C^g$, $\Gamma^f$, $\Gamma^g$, and $C^{-f}$ are shown by the arrows with the thin-solid line. $C^{-g}$ and the composition of more than two matrices involving $C^{-g}$ are represented by the arrows with the dotted line. However, for all the empirical studies we have conducted, the inverse existed. The arrows for the inverses of $\Gamma^f$ and $\Gamma^g$ are excluded from the diagram since some empirical studies showed that the inverses do not exist. More specifically, most of the entries in the twenty-fifth row sector (public administration and defense, see Appendix B) of $\Gamma^f$ and $\Gamma^g$ are all turned out to be zero in the wake of the inherent property possessed by the sector.

## Ⅳ. Formulation of Multipliers Based on the Requirements Matrices

In this section we rigorously define and formulate other various multipliers[11] based on the output requirements matrix $C^g$ for output and on the input requirements matrices, $\Gamma^f$ and $\Gamma^g$, for final demand and output respectively in addition to presenting the ordinary multipliers on the basis of the Leontief inverse $C^f$.

Due to the formulation below, a significant advantage for us is

11) We consider here only the open (comprises endogenous and exogenous sectors) and type I models with the household sector exogenous.

that there are now two different types of multipliers: multipliers that deal with change in final demand and multipliers that handle change in gross output. More specifically, multipliers based on $\boldsymbol{C}^f$ and $\Gamma^f$ measure output (or input), employment, and income effects when the initial change occurs in final demand $\boldsymbol{f}$. Whereas, multipliers derived on the basis of $\boldsymbol{C}^g$ and $\Gamma^g$ enable us to measure output (or input), employment, and income effects when the initial change arises in gross output $\boldsymbol{x}$.

### 4.1. Output and Input Multipliers

The output multiplier, which is based on $\boldsymbol{C}^f$, for sector $j$ is defined as the total output requirements in all sectors of the economy that are needed to support a unit of final demand of sector $j$. If we let $\boldsymbol{c}_j^f$ be the $j$th column vector of $\boldsymbol{C}^f$, then the output multiplier for sector $j$ is given by:

$$\mu_j^O = \boldsymbol{i}' \cdot \boldsymbol{c}_j^f$$

where $\boldsymbol{i}'$ represents a row vector with all ones. The output multipliers for all sectors on the basis of $\boldsymbol{C}^f$ can be represented compactly in matrix form as:

$$\mu^O = \boldsymbol{i}'(\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{i}'\boldsymbol{C}^f$$

We define another output multiplier based on $\boldsymbol{C}^g$. For sector $j$, it is defined as the total output requirements in all sectors of the economy that are needed to produce a unit of gross output of sector $j$. With $\boldsymbol{c}_j^g$ the $j$th column vector of $\boldsymbol{C}^g$, the output multiplier for sector $j$ is given by:

$$\dot{\mu}_j^O = \boldsymbol{i}' \cdot \boldsymbol{c}_j^g$$

The output multipliers for all sectors on the ground of $\boldsymbol{C}^g$ can be represented as:

$$\dot{\mu}^O = \boldsymbol{i}'\boldsymbol{C}^g$$

The input multiplier,[12] which is based on $\Gamma^f$, for sector $j$ can be defined as the total input requirements in all sectors of the economy that are necessary to support a unit of final demand of sector $j$. With $\gamma_j^f$ the $j$th column vector of $\Gamma^f$, the input multiplier for sector $j$ is given by:

$$\hat{\mu}_j^I = \boldsymbol{i}' \cdot \gamma_j^f$$

The input multipliers for all sectors on the basis of $\Gamma^f$ can be represented as:

$$\hat{\mu}^I = \boldsymbol{i}'\boldsymbol{\Gamma}^f$$

We define another input multiplier based on $\Gamma^g$. For sector $j$, it is defined as the total input requirements in all sectors of the economy that are necessary to produce a unit of gross output of sector $j$. With $\gamma_j^g$ the $j$th column vector of $\Gamma^g$, the input multiplier for sector $j$ is given by:

$$\tilde{\mu}_j^I = \boldsymbol{i}' \cdot \gamma_j^g$$

12) The term "input multiplier" used in this context is entirely different from the same term "input (or supply) multiplier" drawn from supply-side input-output model (see Miller and Blair, 2009).

The input multipliers for all sectors on the ground of $\Gamma^g$ can be represented as:

$$\tilde{\mu}^I = i'\Gamma^g$$

### 4.2. Employment Multipliers

The employment multiplier, which is based on $C^f$, for sector $j$ is defined as the ratio of total employment effects (total employment changes) in all sectors of the economy resulting from one unit change for sector $j$ final demand, to the direct employment effect for sector $j$ output, $l_j$. If we let $c_j^f$ be the $j$th column vector of $C^f$, then the employment multiplier for sector $j$ is given by:

$$\mu_j^L = l_c' \cdot c_j^f / l_j$$

The employment multipliers for all sectors on the basis of $C^f$ can be represented compactly as:

$$\mu^L = l_c' C^f < l_c >^{-1}$$

Above, $l_c'$ represents the employment coefficient vector, $< l_c >^{-1}$ the diagonal inverse of the sectoral labor-output (input) ratios, and $l_j$ the $j$th element of $l_c$.

The employment multiplier, which is based on $C^g$, for sector $j$ can be defined as the ratio of total employment effects in all sectors of the economy resulting from one unit change for sector $j$ output, to the direct employment effect for sector $j$ output, and it is given by:

$$\dot{\mu}_j^L = \boldsymbol{l}_c' \cdot \boldsymbol{c}_j^g / l_j$$

The employment multipliers for all sectors on the ground of $\boldsymbol{C}^g$ can be represented as:

$$\dot{\mu}^L = \boldsymbol{l}_c' \boldsymbol{C}^g < \boldsymbol{l}_c >^{-1}$$

The employment multiplier, which is based on $\boldsymbol{\Gamma}^f$, for sector $j$ can be defined as the ratio of input employment effects in all sectors of the economy resulting from one unit change for sector $j$ final demand, to the direct employment effect for sector $j$ input $l_j$, and it is given by:

$$\widehat{\mu_j}^L = \boldsymbol{l}_c' \cdot \gamma_j^f / l_j$$

The employment multipliers for all sectors on the basis of $\boldsymbol{\Gamma}^f$ can be represented as:

$$\hat{\mu}^L = \boldsymbol{l}_c' \boldsymbol{\Gamma}^f < \boldsymbol{l}_c >^{-1}$$

The employment multiplier, which is based on $\boldsymbol{\Gamma}^g$, for sector $j$ can be defined as the ratio of input employment effects in all sectors of the economy resulting from one unit change for sector $j$ output, to the direct employment effect for sector $j$ input, and it is given by:

$$\tilde{\mu}_j^L = \boldsymbol{l}_c' \cdot \gamma_j^g / l_j$$

The employment multipliers for all sectors on the ground of $\boldsymbol{\Gamma}^g$

can be represented as:

$$\tilde{\mu}^{L} = l_c{}' \Gamma^{g} < l_c >^{-1}$$

For all employment multipliers based on $C^f$, $C^g$, $\Gamma^f$ and $\Gamma^g$, the same expression $< l_c >^{-1}$ can be used since it is well known that the sectoral labor-input ratios are the same as the sectoral labor-output ratios. That is, the (direct) employment coefficients are logically identical whether the denominators in $\mu^L$ and $\dot{\mu}^L$, and in $\hat{\mu}^L$ and $\tilde{\mu}^L$ are given as total output (the former two multipliers) or total input (the latter two multipliers).

### 4.3. Income Multipliers

The income multiplier, which is based on $C^f$, for sector $j$ is defined as the ratio of total income effects (total income changes) in all sectors of the economy resulting from one unit change for sector $j$ final demand, to the direct income effect for sector $j$ output, $y_j$. If we let $c_j^f$ be the $j$th column vector of $C^f$, then the income multiplier for sector $j$ is given by:

$$\mu_j^Y = y_c{}' \cdot c_j^f / y_j$$

and the income multipliers for all sectors on the basis of $C^f$ can be represented compactly in matrix form as:

$$\mu^{Y} = y_c{}' C^f < y_c >^{-1}$$

Above, $y_c{}'$ represents the income coefficient vector, $< y_c >^{-1}$ the diagonal inverse of the sectoral income-output (input) ratios, and

$y_j$ the $j$th element of $\boldsymbol{y}_c$.

The income multiplier, which is based on $\boldsymbol{C}^g$, for sector $j$ can be defined as the ratio of total income effects in all sectors of the economy resulting from one unit change for sector $j$ output, to the direct income effect for sector $j$ output, and it is given by:

$$\dot{\mu}_j^Y = \boldsymbol{y}_c{}' \cdot \boldsymbol{c}_j^g / y_j$$

The income multipliers for all sectors on the ground of $\boldsymbol{C}^g$ can be represented as:

$$\dot{\mu}^Y = \boldsymbol{y}_c{}' \boldsymbol{C}^g < \boldsymbol{y}_c >^{-1}$$

The income multiplier, which is based on $\boldsymbol{\Gamma}^f$, for sector $j$ can be defined as the ratio of input income effects in all sectors of the economy resulting from one unit change for sector $j$ final demand, to the direct employment effect for sector $j$ input $y_j$, and it is given by:

$$\hat{\mu}_j^Y = \boldsymbol{y}_c{}' \cdot \gamma_j^f / y_j$$

The income multipliers for all sectors on the basis of $\boldsymbol{\Gamma}^f$ can be represented as:

$$\hat{\mu}^Y = \boldsymbol{y}_c{}' \boldsymbol{\Gamma}^f < \boldsymbol{y}_c >^{-1}$$

The income multiplier, which is based on $\boldsymbol{\Gamma}^g$, for sector $j$ can be defined as the ratio of input income effects in all sectors of the economy resulting from one unit change for sector $j$ output, to the direct employment effect for sector $j$ input, and it is given by:

$$\tilde{\mu}_j^Y = \boldsymbol{y}_c{}' \cdot \gamma_j^g / y_j$$

The income multipliers for all sectors on the ground of $\Gamma^g$ can be represented as:

$$\tilde{\mu}^Y = \boldsymbol{y}_c{}' \Gamma^g < \boldsymbol{y}_c >^{-1}$$

For all the income multipliers based on $\boldsymbol{C}^f$, $\boldsymbol{C}^g$, $\Gamma^f$ and $\Gamma^g$, the same expression $< \boldsymbol{y}_c >^{-1}$ can be used as in the case with the employment multipliers since it is also well known that the sectoral income-input ratios are the same as the sectoral income-output ratios.

Beneficially, for instance, the multipliers on the ground of the output requirements matrix $\boldsymbol{C}^g$ (or $\Gamma^g$) can be adequately and effectively used when the initial change is output, without incurring double-counting impacts and overestimation problem raised and discussed by Oosterhaven and Stelder (2002), and Oosterhaven (2007) in their development of net multipliers concept. Table 1 gives the summary on the multipliers based on the four different output and input requirements matrices.

## Ⅴ. Multiplier Analysis for the Korean Economy

This section presents the results of the analysis on multiplier effects and rank correlation coefficients based on the requirements matrices $\boldsymbol{C}^f$, $\boldsymbol{C}^g$, $\Gamma^f$, and $\Gamma^g$. The 28 basic sectors given in the 2000 Input-Output Tables of Korea (Bank of Korea, 2003) are used (see Appendix B) in this empirical analysis. The transactions are at producers' prices in one million Korean Won. The type of inverse

【Table 1】Summary on Various Types of Multipliers

| **Requirements Matrix** | $C^f$ | $C^g$ | $\Gamma^f$ | $\Gamma^g$ |
|---|---|---|---|---|
| Impact Effect | Direct and Indirect Output Requirements for Final Demand | Direct and Indirect Output Requirements for Output | Direct and Indirect Input Requirements for Final Demand | Direct and Indirect Input Requirements for Output |
| Decomposition Factors | $I, A, T, R$ | $I, A, T$ | $A, T, R$ | $A, T$ |
| Output Multipliers | $\mu^O = i' C^f$<br>$\mu_j^O = i' \cdot c_j^f$ | $\dot{\mu}^O = i' C^g$<br>$\dot{\mu}_j^O = i' \cdot c_j^g$ | - | - |
| Input Multipliers | - | - | $\hat{\mu}^I = i' \Gamma^f$<br>$\hat{\mu}_j^I = i' \cdot \gamma_j^f$ | $\tilde{\mu}^I = i' \Gamma^g$<br>$\tilde{\mu}_j^I = i' \cdot \gamma_j^g$ |
| Employment Multipliers | $\mu^L = l_c' C^f < l_c >^{-1}$<br>$\mu_j^L = l_c' \cdot c_j^f / l_j$ | $\dot{\mu}^L = l_c' C^g < l_c >^{-1}$<br>$\dot{\mu}_j^L = l_c' \cdot c_j^g / l_j$ | $\hat{\mu}^L = l_c' \Gamma^f < l_c >^{-1}$<br>$\hat{\mu}_j^L = l_c' \cdot \gamma_j^f / l_j$ | $\tilde{\mu}^L = l_c' \Gamma^g < l_c >^{-1}$<br>$\tilde{\mu}_j^L = l_c' \cdot \gamma_j^g / l_j$ |
| Income Multipliers | $\mu^Y = y_c' C^f < y_c >^{-1}$<br>$\mu_j^Y = y_c' \cdot c_j^f / y_j$ | $\dot{\mu}^Y = y_c' C^g < y_c >^{-1}$<br>$\dot{\mu}_j^Y = y_c' \cdot c_j^g / y_j$ | $\hat{\mu}^Y = y_c' \Gamma^f < y_c >^{-1}$<br>$\hat{\mu}_j^Y = y_c' \cdot \gamma_j^f / y_j$ | $\tilde{\mu}^Y = y_c' \Gamma^g < y_c >^{-1}$<br>$\tilde{\mu}_j^Y = y_c' \cdot \gamma_j^g / y_j$ |

【Table 2】 Output and Input Multipliers

| Sector | Direct Effect($A$) | Technical Indirect Effect($T$) | Interrelated Indirect Effect($R$) | Output and Input Multipliers | | | |
|---|---|---|---|---|---|---|---|
| | | | | Output Multipliers | | Input Multipliers | |
| | | | | $C^f$ | $C^g$ | $\Gamma^f$ | $\Gamma^g$ |
| 1 | .3758(22)* | .4297(19) | .0909(17) | 1.8964(22) | 1.8055(22) | .8964(22) | .8055(22) |
| 2 | .3659(24) | .3923(22) | .0495(22) | 1.8077(25) | 1.7582(24) | .8077(25) | .7582(24) |
| 3 | .7299(7) | .5398(13) | .2842(9) | 2.5539(13) | 2.2697(10) | 1.5539(13) | 1.2697(10) |
| 4 | .7029(9) | .5450(12) | .6513(6) | 2.8992(10) | 2.2479(12) | 1.8992(10) | 1.2479(12) |
| 5 | .7343(6) | .4080(20) | .8139(4) | 2.9562(7) | 2.1423(14) | 1.9562(7) | 1.1423(14) |
| 6 | .6878(11) | .9022(4) | .1963(13) | 2.7863(12) | 2.5900(5) | 1.7863(12) | 1.5900(5) |
| 7 | .6721(12) | .4825(16) | .1083(16) | 2.2629(18) | 2.1546(13) | 1.2629(18) | 1.1546(13) |
| 8 | .7541(4) | .3691(23) | .8782(3) | 3.0014(5) | 2.1232(17) | 2.0014(5) | 1.1232(17) |
| 9 | .6604(13) | .5882(11) | .2718(10) | 2.5204(14) | 2.2486(11) | 1.5204(14) | 1.2486(11) |
| 10 | .7891(2) | .2538(27) | 1.3361(1) | 3.3790(1) | 2.0429(19) | 2.3790(1) | 1.0429(19) |
| 11 | .6554(15) | 1.0671(2) | .2264(12) | 2.9489(9) | 2.7225(2) | 1.9489(9) | 1.7225(2) |
| 12 | .6929(10) | .7810(8) | .4895(7) | 2.9634(6) | 2.4739(7) | 1.9634(6) | 1.4739(7) |
| 13 | .7268(8) | .4006(21) | .9333(2) | 3.0607(4) | 2.1274(16) | 2.0607(4) | 1.1274(16) |
| 14 | .7433(5) | .8956(5) | .3160(8) | 2.9549(8) | 2.6389(4) | 1.9549(8) | 1.6389(4) |
| 15 | .7604(3) | .7930(7) | .6866(5) | 3.2400(3) | 2.5534(6) | 2.2400(3) | 1.5534(6) |
| 16 | .6599(14) | 1.0552(3) | .0740(20) | 2.7891(11) | 2.7151(3) | 1.7891(11) | 1.7151(3) |
| 17 | .5440(19) | .4385(18) | .1479(15) | 2.1304(20) | 1.9825(20) | 1.1304(20) | .9825(20) |
| 18 | .5604(18) | .8517(6) | .0143(27) | 2.4264(15) | 2.4121(8) | 1.4264(15) | 1.4121(8) |
| 19 | .3699(23) | .3369(24) | .0331(24) | 1.7399(26) | 1.7068(26) | .7399(26) | .7068(26) |
| 20 | .5951(16) | .7375(9) | .0221(25) | 2.3547(16) | 2.3326(9) | 1.3547(16) | 1.3326(9) |
| 21 | .5828(17) | .4835(15) | .2627(11) | 2.3290(17) | 2.0663(18) | 1.3290(17) | 1.0663(18) |
| 22 | .4153(21) | .3360(25) | .1592(14) | 1.9105(21) | 1.7513(25) | .9105(21) | .7513(25) |
| 23 | .3122(27) | .1986(28) | .0879(18) | 1.5987(28) | 1.5108(28) | .5987(28) | .5108(28) |
| 24 | .2874(28) | .2632(26) | .0558(21) | 1.6064(27) | 1.5506(27) | .6064(27) | .5506(27) |
| 25 | .3181(26) | .5006(14) | .0000(28) | 1.8187(24) | 1.8187(21) | .8157(24) | .8187(21) |
| 26 | .3293(25) | .4742(17) | .0207(26) | 1.8242(23) | 1.8035(23) | .8242(23) | .8035(23) |
| 27 | .4988(20) | .6382(10) | .0410(23) | 2.1780(19) | 2.1370(15) | 1.1780(19) | 1.1370(15) |
| 28 | .9438(1) | 1.2528(1) | .0839(19) | 3.2805(2) | 3.1966(1) | 2.2805(2) | 2.1966(1) |
| Average | .5881 | .5863 | .2977 | 2.4721 | 2.1744 | 1.4721 | 1.1744 |

* The number in the parenthesis is the rank.

matrix is $\boldsymbol{C}^f = (\boldsymbol{I} - \boldsymbol{A})^{-1}$ and the three requirements matrices $\boldsymbol{C}^g$, $\boldsymbol{\Gamma}^f$, and $\boldsymbol{\Gamma}^g$ are computed from the decomposition factors $\boldsymbol{A}$, $\boldsymbol{T}$, and $\boldsymbol{R}$ of $\boldsymbol{C}^f$ and by using the general relation given in equation (2). Appendix C shows the employment and income coefficients for the 28 sectors.

Table 2 presents the values of output and input multipliers defined in the previous section. The two output multipliers based on $\boldsymbol{C}^f$ and $\boldsymbol{C}^g$ and the two input multipliers on the basis of $\boldsymbol{\Gamma}^f$ and $\boldsymbol{\Gamma}^g$ are shown in the last four columns. For example, the output multiplier for sector 1 on the ground of $\boldsymbol{C}^f$, $\mu_1^O$, is 1.8964 units, which means the total output requirements in all sectors of the economy that are needed to support a unit of final demand of sector 1. It consists of four different parts: the final demand (1), the direct effect (0.3758), the technical indirect effect (0.4297), and the interrelated indirect effect (0.0909). The input multiplier for sector 1 based on $\boldsymbol{\Gamma}^g$, $\tilde{\mu}_1^I$, is 0.8055 units, which implies the total input requirements in all sectors of the economy that are necessary to produce a unit of output of sector 1. It consists of two parts: the direct effect (0.3758) and the technical indirect effect (0.4297).

【Table 3】 SRCC for the Output and Input Multipliers

| Combinations | SRCC |
|---|---|
| $\boldsymbol{A}$ & $\boldsymbol{C}^f$, $\boldsymbol{A}$ & $\boldsymbol{\Gamma}^f$ | .9464 |
| $\boldsymbol{T}$ & $\boldsymbol{C}^g$, $\boldsymbol{T}$ & $\boldsymbol{\Gamma}^g$ | .9102 |
| $\boldsymbol{R}$ & $\boldsymbol{C}^f$, $\boldsymbol{R}$ & $\boldsymbol{\Gamma}^f$ | .7252 |
| $\boldsymbol{C}^f$ & $\boldsymbol{\Gamma}^f$, $\boldsymbol{C}^g$ & $\boldsymbol{\Gamma}^g$ | 1 |
| $\boldsymbol{C}^f$ & $\boldsymbol{C}^g$, $\boldsymbol{\Gamma}^f$ & $\boldsymbol{\Gamma}^g$, $\boldsymbol{C}^f$ & $\boldsymbol{\Gamma}^g$, $\boldsymbol{C}^g$ & $\boldsymbol{\Gamma}^f$ | .7159 |

The Spearman's rank correlation coefficients (SRCC, Spearman, 1904), $\gamma_s$, are also calculated to observe the relevant structural economic characteristics of each sector between the component matrices and the output and input multipliers and between the output and input multipliers based on the requirements matrices. The results are shown in Table 3.

The SRCC between $\boldsymbol{A}$ and $\boldsymbol{C}^f$ (and also $\boldsymbol{A}$ and $\boldsymbol{\Gamma}^f$) is 0.9464 and that between $\boldsymbol{T}$ and $\boldsymbol{C}^g$ (and also $\boldsymbol{T}$ and $\boldsymbol{\Gamma}^g$) is 0.9102. The higher values (close to 1) imply that the entire priority ranks between combinations are almost the same throughout the whole sectors. The SRCC between the output multipliers based on $\boldsymbol{C}^g$ and the input multipliers on the basis of $\boldsymbol{\Gamma}^g$, for example, is equal to 1 because their difference is the identity matrix $\boldsymbol{I}$. The SRCC for all the combinations in the last row of Table 3 turned out to be 0.7159. It can be observed that the difference of each combination is either $\boldsymbol{R}$ or $\boldsymbol{I}+\boldsymbol{R}$, which gives an implication that the effect of $\boldsymbol{R}$ should be treated meaningfully in this output and input multipliers case.

Table 4 presents the values of employment multipliers for four different matrices. The employment coefficient given in the third column of Appendix C and the direct employment effect shown in the second column of Table 4 are the same notion and hence they are the same. The computed employment multipliers based on the output requirements matrices $\boldsymbol{C}^f$ and $\boldsymbol{C}^g$ are obtained by dividing the total employment effects by the direct employment effect in each sector. The calculated employment multipliers on the ground of the input requirements matrices $\boldsymbol{\Gamma}^f$ and $\boldsymbol{\Gamma}^g$ are obtained similarly by dividing the input employment effects by the direct employment effect in each sector.

The employment multiplier for sector 1 based on $\boldsymbol{C}^f$, $\mu_1^L$, is 1.2130, which means the ratio of total employment effects (0.0706) in all sectors of the economy resulting from one unit change for sector 1 final demand, to the direct employment effect for sector 1

【Table 4】 Employment Multipliers for Four Different Matrices

| Sector | Direct Employment Effect | Employment Multipliers | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Output Requirements Matrices | | | | Input Requirements Matrices | | | |
| | | $C^f$ | | $C^g$ | | $\Gamma^f$ | | $\Gamma^g$ | |
| | | Total[a] | Multipliers | Total[a] | Multipliers | Input[b] | Multipliers | Input[b] | Multipliers |
| 1 | .0582(1)* | .0706(1) | 1.2130(26) | .0693(1) | 1.1914(26) | .0124(13) | .2130(26) | .0111(9) | .1914(26) |
| 2 | .0072(16) | .0140(22) | 1.9476(17) | .0136(20) | 1.8895(18) | .0068(25) | .9476(17) | .0064(24) | .8895(18) |
| 3 | .0048(20) | .0437(4) | 9.1053(2) | .0366(5) | 7.6225(2) | .0389(1) | 8.1053(2) | .0318(1) | 6.6225(2) |
| 4 | .0109(12) | .0306(7) | 2.8061(13) | .0238(11) | 2.1868(14) | .0197(4) | 1.8061(13) | .0129(6) | 1.1868(14) |
| 5 | .0063(18) | .0263(11) | 4.1669(8) | .0180(16) | 2.8494(9) | .0200(3) | 3.1669(8) | .0117(7) | 1.8494(9) |
| 6 | .0132(9) | .0309(6) | 2.3443(14) | .0290(6) | 2.1968(13) | .0177(5) | 1.3443(14) | .0158(3) | 1.1968(13) |
| 7 | .0003(27) | .0100(27) | 33.4590(1) | .0092(26) | 30.6720(1) | .0097(19) | 32.4590(1) | .0089(16) | 29.6720(1) |
| 8 | .0037(24) | .0182(20) | 4.9164(4) | .0118(22) | 3.1972(6) | .0145(10) | 3.9164(4) | .0081(17) | 2.1972(6) |
| 9 | .0063(17) | .0184(19) | 2.9184(11) | .0162(17) | 2.5754(10) | .0121(14) | 1.9184(11) | .0099(14) | 1.5754(10) |
| 10 | .0019(26) | .0140(23) | 7.3599(3) | .0072(27) | 3.7897(4) | .0121(15) | 6.3599(3) | .0053(26) | 2.7897(4) |
| 11 | .0107(13) | .0232(15) | 2.1660(16) | .0217(14) | 2.0306(16) | .0125(12) | 1.1660(16) | .0110(10) | 1.0306(16) |
| 12 | .0072(15) | .0217(16) | 3.0080(10) | .0181(15) | 2.5078(12) | .0145(11) | 2.0080(10) | .0109(13) | 1.5078(12) |
| 13 | .0039(22) | .0185(18) | 4.7513(6) | .0119(21) | 3.0526(7) | .0146(9) | 3.7513(6) | .0080(18) | 2.0526(7) |
| 14 | .0091(14) | .0259(12) | 2.8450(12) | .0232(13) | 2.5470(11) | .0168(6) | 1.8450(12) | .0141(5) | 1.5470(11) |
| 15 | .0042(21) | .0200(17) | 4.7595(5) | .0151(19) | 3.6066(5) | .0158(8) | 3.7595(5) | .0109(11) | 2.6066(5) |
| 16 | .0133(8) | .0291(8) | 2.1909(15) | .0285(8) | 2.1417(15) | .0158(7) | 1.1909(15) | .0152(4) | 1.1417(15) |
| 17 | .0023(25) | .0105(26) | 4.5484(7) | .0094(25) | 4.0853(3) | .0082(21) | 3.5484(7) | .0071(22) | 3.0853(3) |
| 18 | .0126(10) | .0240(13) | 1.9057(19) | .0239(10) | 1.8967(17) | .0114(16) | .9057(19) | .0113(8) | .8967(17) |
| 19 | .0413(2) | .0490(3) | 1.1866(27) | .0487(3) | 1.1782(27) | .0077(23) | .1866(27) | .0074(21) | .1782(27) |
| 20 | .0333(3) | .0556(2) | 1.6700(21) | .0552(2) | 1.6591(20) | .0223(2) | .6700(21) | .0219(2) | .6591(20) |
| 21 | .0153(7) | .0267(10) | 1.7422(20) | .0244(9) | 1.5957(21) | .0114(17) | .7422(20) | .0091(15) | .5957(21) |
| 22 | .0037(23) | .0123(24) | 3.3223(9) | .0108(24) | 2.9166(8) | .0086(20) | 2.3223(9) | .0071(23) | 1.9166(8) |
| 23 | .0110(11) | .0171(21) | 1.5538(22) | .0162(18) | 1.4723(23) | .0061(26) | .5538(22) | .0052(27) | .4723(23) |
| 24 | .0062(19) | .0121(25) | 1.9456(18) | .0115(23) | 1.8585(19) | .0059(27) | .9456(18) | .0053(25) | .8585(19) |
| 25 | .0155(6) | .0233(14) | 1.5016(23) | .0233(12) | 1.5016(22) | .0078(22) | .5016(23) | .0078(19) | .5016(22) |
| 26 | .0213(5) | .0290(9) | 1.3608(24) | .0288(7) | 1.3517(24) | .0077(24) | .3608(24) | .0075(20) | .3517(24) |
| 27 | .0317(4) | .0430(5) | 1.3562(25) | .0426(4) | 1.3437(25) | .0113(18) | .3562(25) | .0109(12) | .3437(25) |
| 28 | – | – | – | – | – | – | – | – | – |
| Average | .0120 | .0266 | 4.1500 | .0240 | 3.5154 | .0134 | 3.1500 | .0108 | 2.5154 |

* The number in the parenthesis is the rank.
[a] Total employment effects.
[b] Input employment effects.

output (0.0582). Also, the employment multiplier for sector 1 on the basis of $\varGamma^g$, $\tilde{\mu}_1^L$, is 0.1914,[13] which implies the ratio of input employment effects (0.0111) in all sectors of the economy resulting from one unit change for sector 1 output, to the direct employment effect for sector 1 input (0.0582).

The Spearman's rank correlation coefficients are calculated between the direct and the total employment effects and between the employment multipliers based on the requirements matrices. The values of SRCC are relatively high and close to 1 as shown in Table 5. The SRCC for each combination in the last row of Table 5 is 0.9872. The difference of each combination is again either $\boldsymbol{R}$ or $\boldsymbol{I}+\boldsymbol{R}$. In this case, we conclude that the effect of $\boldsymbol{R}$, for the most sectors, is the smallest among the four decomposition factors and that the entire priority ranks of the effects of employment multipliers are almost the same throughout all over the sectors.

【Table 5】 SRCC for the Employment Multipliers

| Combinations | SRCC |
|---|---|
| Direct & Total($\boldsymbol{C}^g$) | .8840 |
| Total($\boldsymbol{C}^f$) & Total($\boldsymbol{C}^g$) | .9634 |
| Input($\boldsymbol{\varGamma}^f$) & Input($\boldsymbol{\varGamma}^g$) | .8547 |
| $\boldsymbol{C}^f$ & $\boldsymbol{\varGamma}^f$, $\boldsymbol{C}^g$ & $\boldsymbol{\varGamma}^g$ | 1 |
| $\boldsymbol{C}^f$ & $\boldsymbol{C}^g$, $\boldsymbol{\varGamma}^f$ & $\boldsymbol{\varGamma}^g$, $\boldsymbol{C}^f$ & $\boldsymbol{\varGamma}^g$, $\boldsymbol{C}^g$ & $\boldsymbol{\varGamma}^f$ | .9872 |

Table 6 gives the values of income multipliers for four different matrices. The income coefficient given in the fifth column of Appendix C and the direct income effect shown in the second column of Table 6 are the same notion and hence they are the

13) This value is computed by SAS/IML program. Therefore, a real calculated value (.1907=.0111/.0582) is a little different from the value computed by the program.

【Table 6】 Income multipliers for four different matrices

| Sector | Direct Income Effect | Income Multipliers | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Output Requirements Matrices | | | | Input Requirements Matrices | | | |
| | | $C^f$ | | $C^g$ | | $\Gamma^f$ | | $\Gamma^g$ | |
| | | Total[a] | Multipliers | Total[a] | Multipliers | Input[b] | Multipliers | Input[b] | Multipliers |
| 1 | .5677(4)* | .8542(2) | 1.5047(22) | .8251(4) | 1.4534(24) | .2865(22) | .5047(22) | .2574(22) | .4534(24) |
| 2 | .6202(1) | .8938(1) | 1.4411(25) | .8770(1) | 1.4140(25) | .2736(24) | .4411(25) | .2568(23) | .4140(25) |
| 3 | .1456(26) | .7375(24) | 5.0651(3) | .6292(19) | 4.3214(3) | .5919(3) | 4.0651(3) | .4836(5) | 3.3214(3) |
| 4 | .2489(18) | .7957(7) | 3.1970(11) | .6082(21) | 2.4436(16) | .5468(10) | 2.1970(11) | .3593(15) | 1.4436(16) |
| 5 | .2126(21) | .7842(12) | 3.6888(8) | .5464(25) | 2.5701(12) | .5716(7) | 2.6888(8) | .3338(17) | 1.5701(12) |
| 6 | .2545(16) | .7880(9) | 3.0961(12) | .7293(12) | 2.8657(6) | .5335(12) | 2.0961(12) | .4748(6) | 1.8657(6) |
| 7 | .0692(27) | .6523(28) | 9.4256(2) | .6022(22) | 8.7028(2) | .5831(4) | 8.4256(2) | .5330(2) | 7.7028(2) |
| 8 | .1826(23) | .7417(22) | 4.0621(6) | .4964(27) | 2.7183(8) | .5591(9) | 3.0621(6) | .3138(18) | 1.7183(8) |
| 9 | .2562(15) | .7677(18) | 2.9964(13) | .6762(16) | 2.6395(11) | .5115(13) | 1.9964(13) | .4200(10) | 1.6395(11) |
| 10 | .1529(25) | .7498(20) | 4.9038(4) | .4146(28) | 2.7117(9) | .5969(2) | 3.9038(4) | .2617(21) | 1.7117(9) |
| 11 | .2872(13) | .7859(10) | 2.7364(15) | .7280(13) | 2.5347(13) | .4987(15) | 1.7364(15) | .4408(8) | 1.5347(13) |
| 12 | .2390(19) | .7725(16) | 3.2323(10) | .6395(18) | 2.6758(10) | .5335(11) | 2.2323(10) | .4005(11) | 1.6758(10) |
| 13 | .2133(20) | .7778(15) | 3.6466(9) | .5221(26) | 2.4479(15) | .5645(8) | 2.6466(9) | .3088(19) | 1.4479(15) |
| 14 | .1997(22) | .7805(14) | 3.9086(7) | .6866(15) | 3.4383(5) | .5808(5) | 2.9086(7) | .4869(4) | 2.4383(5) |
| 15 | .1626(24) | .7391(23) | 4.5453(5) | .5624(24) | 3.4587(4) | .5765(6) | 3.5453(5) | .3998(12) | 2.4587(4) |
| 16 | .2738(14) | .7829(13) | 2.8593(14) | .7618(9) | 2.7823(7) | .5091(14) | 1.8593(14) | .4880(3) | 1.7823(7) |
| 17 | .2525(17) | .6816(26) | 2.6993(16) | .6254(20) | 2.4769(14) | .4291(18) | 1.6993(16) | .3729(13) | 1.4769(14) |
| 18 | .3684(9) | .8011(6) | 2.1745(19) | .7967(5) | 2.1627(18) | .4327(17) | 1.1745(19) | .4283(9) | 1.1627(18) |
| 19 | .5641(5) | .8430(4) | 1.4944(24) | .8305(3) | 1.4722(23) | .2789(23) | .4944(24) | .2664(20) | .4722(23) |
| 20 | .3341(11) | .7848(11) | 2.3490(17) | .7774(7) | 2.3270(17) | .4507(16) | 1.3490(17) | .4433(7) | 1.3270(17) |
| 21 | .3211(12) | .7439(21) | 2.3167(18) | .6604(17) | 2.0565(19) | .4228(19) | 1.3167(18) | .3393(16) | 1.0565(19) |
| 22 | .3507(10) | .6527(27) | 1.8610(21) | .5999(23) | 1.7105(21) | .3020(21) | .8610(21) | .2492(24) | .7105(21) |
| 23 | .5791(3) | .8238(5) | 1.4226(26) | .7879(6) | 1.3605(28) | .2447(26) | .4226(26) | .2088(27) | .3605(28) |
| 24 | .5395(6) | .7610(19) | 1.4106(28) | .7406(11) | 1.3727(27) | .2215(28) | .4106(28) | .2011(28) | .3727(27) |
| 25 | .4852(7) | .7271(25) | 1.4986(23) | .7271(14) | 1.4986(22) | .2418(27) | .4986(23) | .2419(26) | .4986(22) |
| 26 | .6002(2) | .8518(3) | 1.4191(27) | .8455(2) | 1.4087(26) | .2516(25) | .4191(27) | .2453(25) | .4087(26) |
| 27 | .3935(8) | .7720(17) | 1.9619(20) | .7588(10) | 1.9284(20) | .3785(20) | .9619(20) | .3653(14) | .9284(20) |
| 28 | .0562(28) | .7911(8) | 14.0760(1) | .7640(8) | 13.5950(1) | .7349(1) | 13.0760(1) | .7078(1) | 12.5950(1) |
| Average | .3311 | .7728 | 3.3926 | .6864 | 2.9481 | .4538 | 2.3926 | .3675 | 1.9481 |

* The number in the parenthesis is the rank.

[a] Total employment effects.

[b] Input employment effects.

same. The computed income multipliers based on the output requirements matrices $\boldsymbol{C}^f$ and $\boldsymbol{C}^g$ are obtained by dividing the total income effects by the direct income effect in each sector. The calculated income multipliers on the basis of the input requirements matrices $\boldsymbol{\Gamma}^f$ and $\boldsymbol{\Gamma}^g$ are obtained similarly by dividing the input income effects by the direct income effect in each sector.

The income multiplier for sector 1 on the ground of $\boldsymbol{C}^f$, $\mu_1^Y$, is 1.5047, which means the ratio of total income effects (0.8542) in all sectors of the economy resulting from one unit change for sector 1 final demand, to the direct income effect for sector 1 output (0.5677). Also, the income multiplier for sector 1 based on $\boldsymbol{\Gamma}^g$, $\tilde{\mu}_1^Y$, is 0.4534, which implies the ratio of input income effects (0.2574) in all sectors of the economy resulting from one unit change for sector 1 output, to the direct income effects for sector 1 input (0.5677).

The Spearman's rank correlation coefficients are calculated between the direct and the total income effects and between income multipliers on the basis of the requirements matrices. The results are shown in Table 7. Except the last two rows, the Spearman's coefficients in this case turned out a little smaller than the case of employment multipliers. The coefficients between income multipliers for four different requirements matrices are the same (1 or 0.9392), again by the same reason given for the output, input, and employment multipliers in Tables 3 and 5. Thus, we conclude that the overall priority ranks of the effects of income multipliers are nearly the same throughout the whole sectors.

As a matter of course, in every case of the requirements matrices, the value of each multiplier for output, input, employment and income cannot be compared directly with the value of the multiplier with the same name, respectively since their initial, exogenous (or endogenous) changes and the corresponding multiplier effects of the changes for the specific sectors are entirely different.

【Table 7】 SRCC for the Income Multipliers

| Combinations | SRCC |
|---|---|
| Direct & Total($C^g$) | .7324 |
| Total($C^f$) & Total($C^g$) | .7247 |
| Input($\Gamma^f$) & Input($\Gamma^g$) | .6760 |
| $C^f$ & $\Gamma^f$, $C^g$ & $\Gamma^g$ | 1 |
| $C^f$ & $C^g$, $\Gamma^f$ & $\Gamma^g$, $C^f$ & $\Gamma^g$, $C^g$ & $\Gamma^f$ | .9392 |

# Ⅵ. Other Economic Analyses on the Requirements Matrices

## 6.1. The Contribution Ratio of Each Decomposition Factor

In this subsection we analyzed the contribution ratio of the effect of each decomposition factor ($I$, $A$, $T$, $R$) for the output requirements ($C^f$) in the period from 1995 to 2003. The numerical values in Table 8 are the averages for the factors and the data for the year 2000 are obtained from the last line of Table 2. The data (not shown in this paper) for the years 1995 and 2003, respectively, are calculated from the 1995 and 2003 Input-Output Tables of Korea. It is presumed that the contribution ratio of each effect for total output requirements is about 0.41 : 0.23 : 0.23 : 0.13 in all three instances. That is, of the total output requirements, 41% is the unit change, about 23% comes from each of the direct effect and the technical indirect effect, and about 13% from the interrelated indirect effect. As for a specific example, if $\Delta x = 1$ by the initial change in final demand (i.e., $1 = C^f \Delta f$), then 0.41 is the unit change, about 0.23 comes from the direct effect, about 0.23 from the technical indirect effect, and about 0.13 from the interrelated indirect effect.

The outcome of contribution ratio analysis could provide a significant meaning for the discovery or derivation of a more concrete relation equation between each individual effect and the total output requirements.

**【Table 8】 Ratio of Each Effect to Output Requirements**

| Year | Unit Change ($I$) | Direct Effect ($A$) | Technical Indirect Effect ($T$) | Interrelated Indirect Effect ($R$) | Total Output Requirements ($C^f$) |
|---|---|---|---|---|---|
| 1995 | 1.0000 | .5534 | .5524 | .3363 | 2.4421 |
| | (41%*) | (23%) | (23%) | (14%) | (100%) |
| 2000 | 1.0000 | .5881 | .5863 | .2977 | 2.4721 |
| | (40%) | (24%) | (24%) | (12%) | (100%) |
| 2003 | 1.0000 | .5805 | .5652 | .2814 | 2.4271 |
| | (41%) | (24%) | (23%) | (12%) | (100%) |

* The number in the parenthesis is the ratio (e.g., 41=(1.0000/2.4421)×100).

We performed the Pearson correlation analysis to measure the closeness of the relationship between the decomposition factors as shown in Table 9. The result shows that the Pearson correlation coefficients $\gamma$ between $C^f$ and $A$, for example, is 0.9458 (obtained from the last row and the second column), the largest among three decomposition factors $A$, $T$, and $R$ of $C^f$. This implies that from the structural point of view in production the contribution of decomposition factor $A$ to $C^f$ is much higher than the other two factors $T$ and $R$. Among three joint combinations $A+T$, $T+R$, and $A+R$, the Pearson coefficient $\gamma$ between $C^f$ and $T+R$

comes out to be the largest value (0.9890 from the last row and the sixth column).

【Table 9】 Correlation Matrix between Decomposition Factors

| | $A$ | $T$ | $R$ | $A+T$ | $T+R$ | $A+R$ | $C^f$ |
|---|---|---|---|---|---|---|---|
| $A$ | 1.0000 | | | | | | |
| $T$ | .5282 | 1.0000 | | | | | |
| $R$ | .5828 | −0.2247 | 1.0000 | | | | |
| $A+T$ | .8121 | .9244 | .1073 | 1.0000 | | | |
| $T+R$ | .8873 | .4978 | .7333 | .7406 | 1.0000 | | |
| $A+R$ | .8001 | .0282 | .9528 | .3787 | .8677 | 1.0000 | |
| $C^{f}$* | .9458 | .5201 | .7034 | .7822 | .9890 | .8679 | 1.0000 |

* $C^f = I + A + T + R$.

### 6.2. The Verification of the Hawkins-Simon Conditions

The notion of $\gamma^g_{ii}$, which means the direct and indirect input requirements of that commodity itself to produce a unit of gross output of commodity $i$, can be used to grasp the economic interpretation of the Hawkins-Simon (H-S) conditions (Hawkins and Simon, 1949, p. 248; Jeong, 1982; Fujita, 1991). Its interpretation is well-known to mean "to produce one unit of a commodity, the direct and indirect input requirements of that commodity itself must not exceed one unit." Coincidently, this interpretation is precisely the definition of $\gamma^g_{ii}$. From the relation $\gamma^g_{ij} = a_{ij} + t_{ij}$, we have:

$$\gamma^g_{ii} = a_{ii} + t_{ii}$$

where $a_{ii}$ represents the direct requirement and $t_{ii}$ the technical indirect requirement. Table 10 shows the numerical values of $\gamma^g_{ii}$ for the 28 basic sectors given in the 2000 Input-Output Tables of

Korea.

【Table 10】 Direct and Technical Indirect Requirements

| Sector | $a_{ii}$ | $t_{ii}$ | $\gamma_{ii}^{g}$ | Sector | $a_{ii}$ | $t_{ii}$ | $\gamma_{ii}^{g}$ |
|---|---|---|---|---|---|---|---|
| 1 | .0481 | .0530 | .1011 | 15 | .3030 | .0034 | .3064 |
| 2 | .0000 | .0613 | .0613 | 16 | .0385 | .0028 | .0413 |
| 3 | .1324 | .0506 | .1830 | 17 | .1133 | .0177 | .1310 |
| 4 | .3391 | .0039 | .3430 | 18 | .0003 | .0096 | .0099 |
| 5 | .4097 | .0063 | .4160 | 19 | .0352 | .0094 | .0446 |
| 6 | .1019 | .0080 | .1099 | 20 | .0000 | .0162 | .0162 |
| 7 | .0318 | .0540 | .0858 | 21 | .1896 | .0082 | .1978 |
| 8 | .4233 | .0154 | .4387 | 22 | .1670 | .0077 | .1747 |
| 9 | .1754 | .0035 | .1789 | 23 | .1368 | .0104 | .1472 |
| 10 | .5554 | .0063 | .5617 | 24 | .0658 | .0259 | .0917 |
| 11 | .1072 | .0090 | .1162 | 25 | .0000 | .0000 | .0000 |
| 12 | .2392 | .0102 | .2494 | 26 | .0174 | .0073 | .0247 |
| 13 | .4464 | .0065 | .4529 | 27 | .0293 | .0055 | .0348 |
| 14 | .1560 | .0056 | .1616 | 28 | .0000 | .0369 | .0369 |

As can be readily observed, all the values are less than one unit. Therefore, the concept of $\gamma_{ii}^{g}$ can be valuably used to check whether the input-output tables constructed under the different research objectives hold the correct economic interpretation of the H-S conditions.

### 6.3. Backward and Forward Linkages

The four requirements matrices $\boldsymbol{C}^{f}$, $\boldsymbol{C}^{g}$, $\boldsymbol{\Gamma}^{f}$, and $\boldsymbol{\Gamma}^{g}$ can be used to define a various type of backward and forward linkage effects.[14)]

14) In this subsection, we are introducing the Rasmussen's (1956) type linkages. We have differing definitions of measures in the literature from different perspectives and attempts. A comprehensive survey up to the recent works on these linkages can be found, for example, in Cai and Leung (2004). Based on the forward-oriented Ghosh model (Ghosh, 1958; de Mesnard, 2009), Ghosh row sums (denoted as $\beta_{i\,\bullet}$) are suggested to

Recall that the backward linkage indicates the interconnection of a particular sector (when increased its output) to those sectors from which it purchases inputs, and the forward linkage indicates the interconnection of a particular sector (when increased its output) to those sectors to which it sells its output (cf. Miller and Blair, 1985, p. 323). The backward linkage effects are explained by the impact coefficient $\boldsymbol{IC}$ and the forward linkage effects are indicated by the sensitivity coefficient $\boldsymbol{SC}$.

For the output requirements matrix for final demand $\boldsymbol{C}^f$, the impact coefficient for sector $j$ is defined as the ratio of the amount of purchases of sector $j$ from all the endogenous sectors (the $j$th column sum of $\boldsymbol{C}^f$) to the average of the amounts of purchases of all the sectors $j=1,2,\cdots,n$ from all the endogenous sectors (the sum of all the elements in $\boldsymbol{C}^f$ divided by $n$), when a unit of final demand in sector $j$ has been occurred. It can be written as $\boldsymbol{IC}_j = \boldsymbol{i}' \cdot \boldsymbol{c}_j^f / [(\boldsymbol{i}' \boldsymbol{C}^f \boldsymbol{i})/n)$, where $\boldsymbol{i}'$ is the $1 \times n$ sum vector and $\boldsymbol{c}_j^f$ is the $j$th column vector of $\boldsymbol{C}^f$. Thus, the impact coefficients for all the sectors with respect to $\boldsymbol{C}^f$ can be represented in matrix notation as:

$$\boldsymbol{IC} = \boldsymbol{i}' \boldsymbol{C}^f (\boldsymbol{K}^{\boldsymbol{C}^f})^{-1}$$

where $\boldsymbol{K}^{\boldsymbol{C}^f}$ is the diagonal matrix whose diagonal elements are all identically given as the sum of all the elements in $\boldsymbol{C}^f$ divided by $n$.

The sensitivity coefficient of sector $i$ for the output requirements

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replace Leontief row sums (denoted as $\alpha_{i\,\bullet}$) as an alternative forward linkage measure. The 'price-model' reinterpretation of the Ghosh model by Diezenbacher (1997) gives a more plausible interpretation of $\beta_{i\,\bullet}$ as sector $i$'s 'value impact' on the rest of economy through a price transmission mechanism (cf. Cai and Leung, 2004, p. 68). Oosterhaven (1996) also mentioned that the standard quantity interpretation of the Ghosh model is economically highly implausible.

matrix for final demand $C^f$ is defined as the ratio of the amount of sells of sector $i$'s output to all the endogenous sectors (the $i$th row sum of $C^f$) to the average of the amounts of sells of all the sectors $j=1,2,\cdots,n$ from all the endogenous sectors (the sum of all the elements in $C^f$ divided by $n$), when a unit of final demand in all the endogenous sectors has been occurred. It can be expressed as $SC_i = c_i^f \cdot i / [(i' C^f i)/n)$, where $c_i^f$ is the $i$th row vector of $C^f$. Thus, the sensitivity coefficients for all the sectors based on $C^f$ can be represented in matrix notation by:

$$SC = i'(C^f)'(K^{C^f})^{-1}$$

The impact and sensitivity coefficients on the basis of other requirements matrices $C^g$, $\Gamma^f$, and $\Gamma^g$ can be analogously defined and they are summarized in Table 11. In it, $K^{C^g}$ (also $K^{\Gamma^f}$ and $K^{\Gamma^g}$) is the diagonal matrix whose diagonal elements are all identically given as the sum of all the elements in $C^g$ (also $\Gamma^f$ and $\Gamma^g$) divided by $n$.

【Table 11】 Impact and Sensitivity Coefficients

| Requirements Matrices | Impact Coefficients for Backward Linkage | Sensitivity Coefficients for Forward Linkage |
|---|---|---|
| $C^f$ (Output Requirements for Final Demand) | $IC = i' C^f (K^{C^f})^{-1}$ | $SC = i'(C^f)'(K^{C^f})^{-1}$ |
| $C^g$ (Output Requirements for Output) | $IC = i' C^g (K^{C^g})^{-1}$ | $SC = i'(C^g)'(K^{C^g})^{-1}$ |
| $\Gamma^f$ (Input Requirements for Final Demand) | $IC = i' \Gamma^f (K^{\Gamma^f})^{-1}$ | $SC = i'(\Gamma^f)'(K^{\Gamma^f})^{-1}$ |
| $\Gamma^g$ (Input Requirements for Output) | $IC = i' \Gamma^g (K^{\Gamma^g})^{-1}$ | $SC = i'(\Gamma^g)'(K^{\Gamma^g})^{-1}$ |

【Table 12】 Impact and Sensitivity Coefficients for Four Different Matrices

| Sector | Impact Coefficient $IC_j$ | | | | Sensitivity Coefficient $SC_i$ | | | |
|---|---|---|---|---|---|---|---|---|
| | $C^f$ | $C^g$ | $\Gamma^f$ | $\Gamma^g$ | $C^f$ | $C^g$ | $\Gamma^f$ | $\Gamma^g$ |
| 1 | .7671(22)* | .8303(22) | .6089(22) | .6859(22) | 1.0070(10) | 1.0424(10) | 1.0118(10) | 1.0784(10) |
| 2 | .7312(25) | .8086(24) | .5487(25) | .6456(24) | 1.5742(4) | 1.5429(4) | 1.9643(4) | 2.0052(4) |
| 3 | 1.0331(13) | 1.0438(10) | 1.0556(13) | 1.0811(10) | 1.0085(9) | 1.0716(9) | 1.0143(9) | 1.1325(9) |
| 4 | 1.1728(10) | 1.0338(12) | 1.2901(10) | 1.0626(12) | .7797(18) | .7759(19) | .6301(18) | .5851(19) |
| 5 | 1.1958(7) | .9852(14) | 1.3288(7) | .9727(14) | 1.2941(7) | 1.2501(6) | 1.4939(7) | 1.4631(6) |
| 6 | 1.1271(12) | 1.1911(5) | 1.2134(12) | 1.3539(5) | .5771(23) | .6284(23) | .2898(23) | .3119(23) |
| 7 | .9154(18) | .9909(13) | .8579(18) | .9831(13) | 1.3504(6) | 1.3168(5) | 1.5885(6) | 1.5865(5) |
| 8 | 1.2141(5) | .9765(17) | 1.3596(5) | .9564(17) | 2.1823(1) | 2.0078(1) | 2.9854(1) | 2.8660(1) |
| 9 | 1.0195(14) | 1.0341(11) | 1.0328(14) | 1.0632(11) | .6970(21) | .7311(21) | .4911(21) | .5022(21) |
| 10 | 1.3669(1) | .9395(19) | 1.6161(1) | .8880(19) | 2.1658(2) | 1.8778(2) | 2.9577(2) | .6252(2) |
| 11 | 1.1929(9) | 1.2521(2) | 1.3239(9) | 1.4667(2) | .7011(20) | .7404(20) | .4980(20) | .5193(20) |
| 12 | 1.1987(6) | 1.1377(7) | 1.3337(6) | 1.2550(7) | .8564(16) | .8644(16) | .7588(16) | .7490(16) |
| 13 | 1.2381(4) | .9784(16) | 1.3998(4) | .9600(16) | 1.3537(5) | 1.2490(7) | 1.5939(5) | 1.4610(7) |
| 14 | 1.1953(8) | 1.2136(4) | 1.3280(8) | 1.3955(4) | .5517(24) | .5952(25) | .2471(24) | .2505(25) |
| 15 | 1.3106(3) | 1.1743(6) | 1.5216(3) | 1.3227(6) | .7679(19) | .7833(18) | .6103(19) | .5987(18) |
| 16 | 1.1282(11) | 1.2487(3) | 1.2153(11) | 1.4604(3) | .4814(27) | .5369(27) | .1291(27) | .1426(27) |
| 17 | .8618(20) | .9117(20) | .7679(20) | .8366(2) | .9762(12) | .9758(14) | .9600(12) | .9551(14) |
| 18 | .9815(15) | 1.1093(8) | .9690(15) | 1.2024(8) | .5459(25) | .5964(24) | .2375(25) | .2527(24) |
| 19 | .7038(26) | .7850(26) | .5026(26) | .6018(26) | .9738(13) | .9857(13) | .9560(13) | .9735(13) |
| 20 | .9525(16) | 1.0728(9) | .9202(16) | 1.1347(9) | .9477(14) | 1.0086(12) | .9122(14) | 1.0159(12) |
| 21 | .9421(17) | .9503(18) | .9028(17) | .9080(18) | .8634(15) | .8846(15) | .7706(15) | .7863(15) |
| 22 | .7728(21) | .8054(25) | .6185(21) | .6397(25) | .7898(17) | .8280(17) | .6470(17) | .6816(17) |
| 23 | .6467(28) | .6948(28) | .4067(28) | .4349(28) | 1.2341(8) | 1.2317(8) | 1.3931(8) | 1.4289(8) |
| 24 | .6498(27) | .7131(27) | .4119(27) | .4688(27) | 1.7530(3) | 1.7441(3) | 2.2645(3) | 2.3777(3) |
| 25 | .7357(24) | .8364(21) | .5541(24) | .6971(21) | .4045(28) | .4599(28) | .0000(28) | .0000(28) |
| 26 | .7379(23) | .8294(23) | .5599(23) | .6842(23) | .6574(22) | .6861(22) | .4246(22) | .4189(22) |
| 27 | .8810(19) | .9828(15) | .8002(19) | .9682(15) | .5184(26) | .5711(26) | .1912(26) | .2058(26) |
| 28 | 1.3270(2) | 1.4701(1) | 1.5491(2) | 1.8704(1) | .9873(11) | 1.0141(11) | .9766(11) | 1.0261(11) |
| Average | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

* The number in the parenthesis is the rank.

The computed values of impact and sensitivity coefficients based on the four requirements matrices are shown in Table 12. The analysis is also performed on the same type of inverse matrix $C^f = (I - A)^{-1}$ given in the 2000 Input-Output Tables of Korea. The result shows that the sector with the highest impact coefficient associated with $C^f$ is the sector 10 of primary metal products whose value is 1.3669. The sector with the highest sensitivity coefficient with respect to $\Gamma^g$ is the sector 8 of chemical products whose value is 2.8660. If the impact coefficient (or sensitivity coefficient) is higher than the average value of 1, then we interpret that its sector has relatively high backward linkage effect (or forward linkage effect). Whereas, if the impact coefficient (or sensitivity coefficient) is lower than the average value of 1, then we construe that its sector has relatively low backward linkage effect (or forward linkage effect).

【Table 13】 Spearman's Rank Correlation Coefficients for Impact and Sensitivity Coefficients

| Impact Coefficients | | | | Sensitivity Coefficients | | | |
|---|---|---|---|---|---|---|---|
| Comb. | SRCC | Comb. | SRCC | Comb. | SRCC | Comb. | SRCC |
| $C^f$ & $\Gamma^f$ | 1 | $C^f$ & $C^g$ | .7159 | $C^f$ & $\Gamma^f$ | 1 | $C^f$ & $C^g$ | .9951 |
| $C^g$ & $\Gamma^g$ | 1 | $\Gamma^f$ & $\Gamma^g$ | .7159 | $C^g$ & $\Gamma^g$ | 1 | $\Gamma^f$ & $\Gamma^g$ | .9951 |
| | | $C^f$ & $\Gamma^g$ | .7159 | | | $C^f$ & $\Gamma^g$ | .9951 |
| | | $C^g$ & $\Gamma^f$ | .7159 | | | $C^g$ & $\Gamma^f$ | .9951 |

The Spearman's rank correlation coefficients (SRCC), $\gamma_s$, are calculated for the impact and sensitivity coefficients on a number of different combinations (Comb.) as shown in Table 13. The rank for each sector can be found from Table 12. The SRCC for each combination in the forth column of impact coefficients is 0.7159 and that of the sensitivity coefficients is 0.9951. Since the difference of each combination in them is either $\boldsymbol{R}$ or $\boldsymbol{I}+\boldsymbol{R}$, it implies that the interrelated effect $\boldsymbol{R}$ for those combinations in impact coefficients is higher than those in sensitivity coefficients.

## Ⅶ. Conclusions

In this paper, we discussed the existence of altogether four different types of output and input requirements matrices, each of which has a distinct characteristic: two input requirements matrices $\boldsymbol{\Gamma}^f$ and $\boldsymbol{\Gamma}^g$ for final demand and output respectively, one output requirements matrix $\boldsymbol{C}^g$ for output obtained from the decomposition factors of the Leontief inverse, and $\boldsymbol{C}^f$ the output requirements matrix for final demand. In particular, we emphasized that the concept of output requirements matrix $\boldsymbol{C}^g$ for output has a characteristic that relates output for final demand $\boldsymbol{x}$ to output for output $\boldsymbol{o}$ as shown in Figure 2, as if the Leontief inverse $\boldsymbol{C}^f$ relates final demand $\boldsymbol{f}$ to output for final demand $\boldsymbol{x}$. We drew diagrams which describe overall and unifying frameworks for the connections of $\boldsymbol{C}^f$, $\boldsymbol{C}^g$, $\boldsymbol{\Gamma}^f$, and $\boldsymbol{\Gamma}^g$ for final demand, inputs, and output.

In addition to the ordinary multipliers that are based on the Leontief inverse, we then defined and summarized other various types of output, input, employment, and income multipliers based on three other output and input requirements matrices. An advantage of utilizing the multipliers on the basis of $\boldsymbol{C}^g$ (and $\boldsymbol{\Gamma}^g$) can be seen by considering the concept of net multipliers introduced by Oosterhaven and Stelder (2002). Their net multipliers had been developed as a remedy to solve the double-counting impacts and overestimation problem when the Leontief inverse is multiplied by the total output $x$. However, apart from all these double-counting and overestimation issues, there is a fundamental problem that one should not overlook. That is, there is no consecutive connection between the Leontief inverse $(I-A)^{-1}$ and $x$, and hence $(I-A)^{-1}$ can not be multiplied by $x$. Thus, if the measurement of total output (or input) effects are required when the initial change is output, the multipliers on the ground of the requirements matrices $C^g$ (or $\Gamma^g$) are more adequate and can be more effectively used.

A preliminary but extensive empirical study was done on the output and input requirements matrices in the Korean economy to examine the differences of the effects of decomposition factors $A$, $T$, and $R$ and various input-output multipliers newly defined. The results of the analysis on multiplier effects showed that the effect of each multiplier for output, input, employment and income based on the requirement matrices $C^f$, $C^g$, $\Gamma^f$, and $\Gamma^g$ was (also) different from one another. For the cases of employment and income multipliers, there were close structural similarities and regularities in the whole priority ranks of sectors characterized and counted by the effects of various multipliers since the Spearman's rank correlation coefficients were close to 1's (cf. 0.9872 from Table 5 and 0.9392 from Table 7). When comparing the averages of multiplier effects, it was shown in general that $C^f > C^g > \Gamma^f > \Gamma^g$ because of the decomposition results of the Leontief inverse by the four factors.

We analyzed the contribution ratio of the effect of each decomposition factor ($I$, $A$, $T$, and $R$) for the output requirements for final demand ($C^f$) in three instances. It is shown that the contribution ratio of each effect for $C^f$ was about 0.41 : 0.23 : 0.23 : 0.13. Moreover, the Pearson correlation coefficient $\gamma$ between $C^f$

and $A$ turned out to be 0.9458, the largest among three decomposition factors ($A$, $T$, and $R$). From the structural point of view in production, this implies that the contribution of decomposition factor $A$ to $C^f$ is much higher than the other two factors $T$ and $R$.

The importance of the input requirements matrix for output $\Gamma^g$ was further demonstrated by showing that the concept of $\Gamma^g$ and the economic interpretation of the Hawkins-Simon conditions exactly coincide with each other. It implies that $\Gamma^g$ can be used to verify whether the input-output tables constructed under the different research objectives hold the correct interpretation of the Hawkins-Simon conditions. In addition, the four requirements matrices $C^f$, $C^g$, $\Gamma^f$, and $\Gamma^g$ were applied to define a various types of backward and forward linkage effects.

In conclusion, our main concern and interest can be summarized as choosing the most suitable output and input requirements matrices, which exactly coincide with the specific purposes of research, among four different requirements matrices according to the variables affecting (or affected by) the initial change. We hope that the findings and results obtained in this paper can also be utilized in many different areas such as in energy and environmental input-output models, intensity analysis of resources, comparison analysis of impact between final demand and gross output, etc.

*Received: February 8, 2011. Revised: April 18, 2011. Accepted: May 12, 2011.*

## ◈ *References* ◈

Bank of Korea (2003), *2000 Input-Output Tables*.

Cai, J. and P. Leung (2004), "Linkage Measures: A Revisit and a Suggested Alternative," *Economic Systems Research*, Vol. 16, pp. 65-85.

Dietzenbacher, E. (1997), "In Vindication of the Ghosh Model: A Reinterpretation as a Price Model," *Journal of Regional Science*, Vol. 37, pp. 629-651.

____________ (2002), "Interregional Multipliers: Looking Backward, Looking Forward," *Regional Studies*, Vol. 36, pp. 125-136.

____________ (2005), "More on Multipliers," *Journal of Regional Science*, Vol. 45, No. 2, pp. 421-426.

Fujita, Y. (1991), "A further Note on a Correct Economic Interpretation of the Hawkins-Simon Conditions," *Journal of Macroeconomics*, Vol. 13, pp. 381-384.

Ghosh, A. (1958), "Input-Output Approach to an Allocation System," *Econometrica*, Vol. 25, pp. 58-64.

Gim, H. U. (2008), "Why Do We Develop a New 'Output-Output Model'? With Attention to Basic Concepts, Model Building, and Applications," *Business Management Review*, Vol. 42, No. 1, pp. 1-18.

________ (2008), "How Do We Utilize a New 'Output-Output Model' and 'Output-Output Table'? Based on an Alternative Analysis to the Input-Output Model and the Input-Output Table," *Kyong Je Hak Yon Gu*, Vol. 57, No. 2, pp. 115-157.

________ (2009), "Why Does the Overestimation of Economic Impact Occur in the Input-Output Model?," *Kyong Je Hak Yon Gu*, Vol. 56, No. 1, pp. 31-56.

Gim, H. U. and K. Kim (1998), "The General Relation between Two Different Notions of Direct and Indirect Input Requirements," *Journal of Macroeconomics*, Vol. 20, No. 1, pp. 199-208.

__________________ (2005), "The Decomposition by Factors in Direct and Indirect Requirements: with Applications to Estimating the Pollution Generation," *The Korean Economic Review*, Vol. 21, No. 2, pp. 309-325.

________________ (2008), "Note on the Decomposition by Factors in Direct and Indirect Requirements," *The Korean Economic Review*, Vol. 24, No. 1, pp. 259-282.

________________ (2009), "A Study on the Building of a New 'Output-Output Model' and Its Usefulness: Based on a Comparative Analysis of the Input-Output Model," *The Annals of Regional Science*, Vol. 43, No. 3, pp. 807-829.

Hawkins, D. and H. A. Simon (1949), "Note: Some Conditions of Macroeconomic Stability," *Econometrica*, Vol. 17, pp. 245-248.

Jeong, K. J. (1982), "Direct and Indirect Requirements: A Correct Economic Interpretation of the Hawkins-Simon Conditions," *Journal of Macroeconomics*, Vol. 4, pp. 349-356.

________ (1984), "The Relation between Two Different Notions of Direct and Indirect Input Requirements," *Journal of Macroeconomics*, Vol. 6, pp. 473-476.

Lenzen, M. (2001), "A Generalized Input-Output Multiplier Calculus for Australia," *Economic Systems Research*, Vol. 13, pp. 65-92.

Leontief, W. (1936), "Quantitative Input-Output Relations in the Economic System of the United States," *The Review of Economics and Statistics*, Vol. 18, pp. 105-125.

Liew, C. J. (2005), "Dynamic Variable Input-Output (VIO) Model and Price-Sensitive Dynamic Multipliers," *The Annals of Regional Science*, Vol. 39, pp. 607-627.

de Mesnard, L. (2002), "Note about the Concept of Net Multipliers," *Journal of Regional Science*, Vol. 42, No. 3, pp. 545-548.

__________ (2007), "A Critical Comment on Oosterhaven-Stelder Net Multipliers," *The Annals of Regional Science*, Vol. 41, pp. 249-271.

__________ (2009), "Is the Ghosh Model Interesting?," *Journal of Regional Science*, Vol. 49, pp. 361-372.

Miller, R. E. and P. D. Blair (2009), *Input-Output Analysis: Foundations and Extensions, Cambridge*, Cambridge University Press.

Oosterhaven, J. (1996), "Leontief versus Ghoshian Price and Quantity

Models," *Southern Economic Journal*, Vol. 62, pp. 750-759.

______________ (2004), "On the Definition of Key Sectors and the Stability of Net Versus Gross Multipliers," Research Report 04C01, Research School SOM, University of Groningen, Available at: http://som.rug.nl.

______________ (2007), "The Net Multipliers is a New Key Sector Indicator: Reply to de Mesnard's Comment," *The Annals of Regional Science*, Vol. 41, pp. 273-283.

Oosterhaven, J. and D. Stelder (2002), "Net Multipliers Avoid Exaggerating Impacts: With a Bi-regional Illustration for the Dutch Transportation Sector," *Journal of Regional Science*, Vol. 42, No. 3, pp. 533-543.

Parikh, A. (1975), "Various Definitions of Direct and Indirect Requirements in Input-Output Analysis," *The Review of Economics and Statistics*, Vol. 57, pp. 375-377.

Rasmussen, P. N. (1956), *Studies in Intersectoral Relations*, North-Holland, Amsterdam.

Rickman, D. S. and R. K. Schwer (1995), "A Comparison of the Multipliers of IMPLAN, REMI, and RIMS II: Benchmarking Ready-Made Models for Comparison," *The Annals of Regional Science*, Vol. 29, pp. 363-374.

Spearman, C. (1904), "The Proof and Measurement of Association between Two Things," *American Journal of Psychology*, Vol. 15, pp. 72-101.

Szyrmer, J. M. (1992), "Input-Output Coefficients and Multipliers from a Total Flow Perspective," *Environment and Planning A*, Vol. 24, pp. 921-937.

Szyrmer, J. M. and R. Walker (1983), "Interregional Total Flow: A Concept and Application to a U.S. Input-Output Model," *The Review of Regional Studies*, Vol. 13, pp. 12-21.

Waugh, F. V. (1950), "Inversion of the Leontief Matrix by Power Series," *Econometrica*, Vol. 18, pp. 142-154.

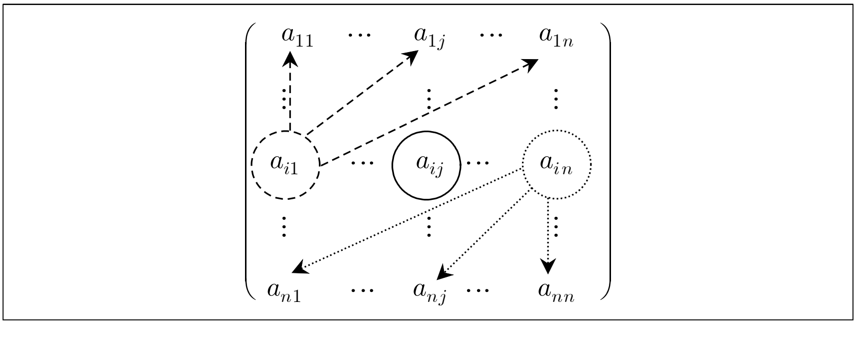
# APPENDIX

## A. The General Relation and the Concepts of Technical and Interrelated Indirect Effects

### A1. A procedure for the Formulation of the General Expression of $\gamma_{ij}^{g}$

In formulating $\gamma_{ij}^{g}$ with $n$ sectors, all elements in the $i$th row of $\boldsymbol{A}$ are considered for the direct and indirect effects. To begin with, $a_{ij}$ is the direct effect and the other elements $a_{i1}, a_{i2}, \cdots, a_{i(j-1)}$, $a_{i(j+1)}, \cdots, a_{in}$ in the $i$th row induce the technical indirect effects. Each one of these $a_{ir_1}, r_1 = 1, 2, \cdots, j-1, j+1, \cdots, n$, is technically connected with the elements in the corresponding row $r_1$ which is the second subscript of $a_{ir_1}$. Hence, it should be postmultiplied by the elements in row $r_1$ of $\boldsymbol{A}$ (see Figure A1 below).

【Figure A1】Connections of Elements $a_{ir_1}$ with the Elements in the Corresponding Rows $r_1$ of $\boldsymbol{A}$




After being multiplied, the resultant terms have the form $a_{ir_1}a_{r_1r_2}, r_2 = 1, 2, \cdots, n$, and all the terms $a_{ir_1}a_{r_1j}$ with $r_1 \neq j$ become the first-round indirect effect. For the terms $a_{ir_1}a_{r_1r_2}$ with $r_2 \neq j$, further postmultiplication can be followed since each element $a_{r_1 r_2}$ in $a_{ir_1}a_{r_1 r_2}$ can be technically connected with the elements in row $r_2$ of $\boldsymbol{A}$. Then, after the second postmultiplication, the resultant terms have the form $a_{ir_1}a_{r_1 r_2}a_{r_2 r_3}$, $r_3 = 1, 2, \cdots, n$, and all the terms $a_{ir_1}a_{r_1 r_2}a_{r_2 j}$ with $r_1 \neq j$ and $r_2 \neq j$ become the second-round indirect effect. The terms $a_{ir_1}a_{r_1 r_2}a_{r_2 j}$ with $r_1 = j$ or $r_2 = j$ are not included in $\gamma_{ij}^{g}$ (the terms not included in $\gamma_{ij}^{g}$ will be called the interrelated indirect effect (see Section A2 below)).

For the terms with $r_3 \neq j$, further postmultiplication can be followed because each element $a_{r_2 r_3}$ in $a_{ir_1}a_{r_1 r_2}a_{r_2 r_3}$ is technically connected with the elements in row $r_3$ of $\boldsymbol{A}$. The above process can be continued for the third-round indirect effect, the forth-round indirect effect, and on and on. The sum of the direct effect $a_{ij}$, the first-round indirect effect, the second-round indirect effect, and so on, yields $\gamma_{ij}^{g}$ given in equation (6).

## A2. The Concepts of the Technical Indirect Effect $t_{ij}$ and the Interrelated Indirect Effect $r_{ij}$

We consider $\gamma_{11}^{g}$ with $n = 3$ and only up to the second-round indirect effects to illustrate the concepts of technical indirect effect $t_{ij}$ and interrelated indirect effect $r_{ij}$.

From equation (5) in Section 2, we have:

$$
\begin{aligned}
c_{11}^{f} = 1 + a_{11} + (&a_{12}a_{21} + a_{12}a_{22}a_{21} + a_{12}a_{23}a_{31} + a_{13}a_{31} \\
&+ a_{13}a_{32}a_{21} + a_{13}a_{33}a_{31}) + (a_{11}a_{11} + a_{11}a_{11}a_{11} + a_{11}a_{12}a_{21} \\
&+ a_{11}a_{13}a_{31} + a_{12}a_{21}a_{11} + a_{13}a_{31}a_{11})
\end{aligned}
$$

and the general expression (6) gives:

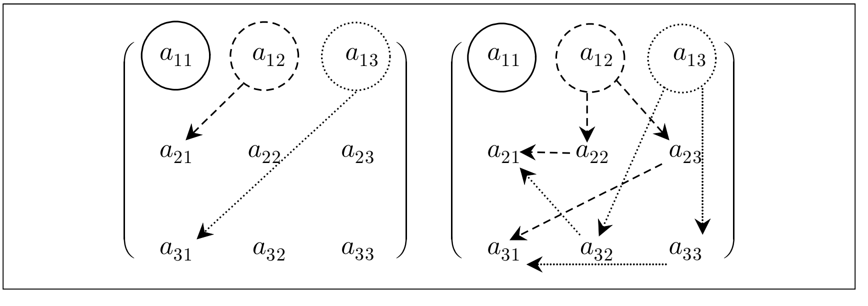
$$\gamma_{11}^{g} = a_{11} + (a_{12}a_{21} + a_{12}a_{22}a_{21} + a_{12}a_{23}a_{31} + a_{13}a_{31} + a_{13}a_{32}a_{21} + a_{13}a_{33}a_{31})$$

The term $a_{11}$ (marked by solid circle) is the direct effect and

$$t_{11} = a_{12}a_{21} + a_{12}a_{22}a_{21} + a_{12}a_{23}a_{31} + a_{13}a_{31} + a_{13}a_{32}a_{21} + a_{13}a_{33}a_{31}$$

Then $t_{11}$ consists of the first-round indirect effect terms $a_{12}a_{21}$ and $a_{13}a_{31}$ and of the second-round indirect effect terms $a_{12}a_{22}a_{21}$, $a_{12}a_{23}a_{31}$, $a_{13}a_{32}a_{21}$, and $a_{13}a_{33}a_{31}$. These are all induced by the 1st row non-diagonal elements $a_{12}$ and $a_{13}$. Figure A2 below shows the connections of up to the second-round indirect effects induced by the non-diagonal elements in the first row of $\boldsymbol{A}$ (marked by dotted circles) for $\gamma_{11}^{g}$.

【Figure A2】Connections of the First-Round (Left) and the Second-Round (Right) Indirect Effects for $\gamma_{11}^{g}$




Each term in $t_{11}$ indicates "pure" (or complete) indirect-effect relation between inputs and output. Thus, naming after the "technical" coefficient matrix $\boldsymbol{A}$ and the purely "technical" relation between inputs and output in production system, $t_{11}$ ($t_{ij}$ in general) is called the "technical indirect effect."

For the concept of interrelated indirect effect, there are all together six terms that are not included in $\gamma_{11}^{g}$: $a_{11}a_{11}$, $a_{11}a_{11}a_{11}$, $a_{11}a_{12}a_{21}$, $a_{11}a_{13}a_{31}$, $a_{12}a_{21}a_{11}$, and $a_{13}a_{31}a_{11}$. The sum of these terms is defined as:

$$
\begin{aligned}
r_{11} = {} & a_{11}a_{11} + a_{11}a_{11}a_{11} + a_{11}a_{12}a_{21} + a_{11}a_{13}a_{31} \\
& + a_{12}a_{21}a_{11} + a_{13}a_{31}a_{11}
\end{aligned}
$$

Notice that each term in $r_{11}$ contains (is induced by) the direct effect $a_{11}$ or a term in the technical indirect effect $t_{11}$ (e.g., $a_{11}a_{11}$ contains the direct effect $a_{11}$ and $a_{12}a_{21}a_{11}$ contains $a_{12}a_{21}$ in $t_{11}$). Since every term in $r_{11}$ is "interrelated" with a term in $\gamma_{11}^{g}$, $r_{11}$ ($r_{ij}$ in general) is called the "interrelated" indirect effect.

## B. Sector Classifications (28 Sectors)

【Table B1】 2000 Input-Output Tables of Korea

| 15 Commodity Sectors | 28 Commodity Sectors |
|---|---|
| 1. Agriculture, Forestry and Fishery | 1. Agriculture, Forestry and Fishery |
| 2. Mining | 2. Mining |
| 3. Food and Beverages | 3. Food and Beverages |
| 4. Textiles and Leather | 4. Textiles and Leather |
| 5. Lumber, Paper and Publishing | 5. Lumber and Wood Products<br>6. Paper, Printing and Publishing |
| 6. Petroleum and Chemicals | 7. Petroleum and Coal Products<br>8. Chemical Products |
| 7. Non-metallic Mineral Products | 9. Non-metallic Mineral Products |
| 8. Metal Products and Machinery | 10. Primary Metal Products<br>11. Metal Products<br>12. General Industrial Machinery |
| 9. Electronic and Electric Equipment | 13. Electronic and Electric Equipment<br>14. Measuring, Medical and Optical Instruments |
| 10. Transportation Equipment and Other Manufactured Products | 15. Transportation Equipment<br>16. Other Manufactured Products |
| 11. Electric, Gas and Water Services | 17. Electric, Gas and Water Services |
| 12. Construction | 18. Construction |
| 13. Commerce, Restaurants and Hotels | 19. Commerce<br>20. Restaurants and Hotels |
| 14. Transportation and Communications | 21. Transportation and Warehousing<br>22. Communications and Broadcasting |
| 15. Finance and Public Service | 23. Finance and Insurance<br>24. Real Estate and Rental<br>25. Public Administration and Defense<br>26. Education and Medical Service<br>27. Social and Other Services<br>28. Others |

## C. Employment and Income Coefficients

| Sector | Number of Employed[a] | Employment Coefficient[b] | Income[c] | Income Coefficient | Total Output (Input) |
|---|---|---|---|---|---|
| 1 | 2,228,849 | .0582 | 21,735,406 | .5677 | 38,286,604 |
| 2 | 19,010 | .0072 | 1,642,504 | .6202 | 2,648,206 |
| 3 | 283,191 | .0048 | 8,604,612 | .1456 | 59,086,107 |
| 4 | 510,769 | .0109 | 11,666,654 | .2489 | 46,871,861 |
| 5 | 106,049 | .0063 | 3,584,881 | .2126 | 16,863,033 |
| 6 | 130,638 | .0132 | 2,518,661 | .2545 | 9,897,778 |
| 7 | 18,196 | .0003 | 3,676,097 | .0692 | 53,147,849 |
| 8 | 323,732 | .0037 | 16,183,010 | .1826 | 88,626,862 |
| 9 | 108,921 | .0063 | 4,399,757 | .2562 | 17,173,290 |
| 10 | 112,408 | .0019 | 8,821,533 | .1529 | 57,688,957 |
| 11 | 224,679 | .0107 | 6,034,155 | .2872 | 21,007,052 |
| 12 | 310,562 | .0072 | 10,308,756 | .2390 | 43,132,012 |
| 13 | 559,465 | .0039 | 30,373,129 | .2133 | 142,426,688 |
| 14 | 62,218 | .0091 | 1,358,920 | .1997 | 6,804,980 |
| 15 | 310,816 | .0042 | 12,128,487 | .1626 | 74,613,704 |
| 16 | 133,456 | .0133 | 2,739,177 | .2738 | 10,004,276 |
| 17 | 71,944 | .0023 | 7,949,336 | .2525 | 31,488,310 |
| 18 | 1,248,774 | .0126 | 36,572,766 | .3684 | 99,268,646 |
| 19 | 2,887,769 | .0413 | 39,399,824 | .5641 | 69,844,226 |
| 20 | 1,369,922 | .0333 | 13,744,140 | .3341 | 41,143,520 |
| 21 | 782,502 | .0153 | 16,428,003 | .3211 | 51,160,891 |
| 22 | 126,761 | .0037 | 11,885,589 | .3507 | 33,890,617 |
| 23 | 698,958 | .0110 | 36,738,420 | .5791 | 63,435,436 |
| 24 | 850,221 | .0062 | 74,148,424 | .5395 | 137,433,450 |
| 25 | 674,749 | .0155 | 21,157,304 | .4852 | 43,601,282 |
| 26 | 1,552,498 | .0213 | 43,699,013 | .6002 | 72,807,642 |
| 27 | 969,499 | .0317 | 12,037,049 | .3935 | 30,592,192 |
| 28 | – | – | 1,685,600 | .0562 | 29,982,300 |
| Total | 16,676,556 | | 461,221,207 | | 1,392,927,771 |
| Average | | .0120 | | .3311 | |

[a] In persons.
[b] Person/one million Korean Won.
[c] Income (in one million Korean Won) = compensation of employees + operating surplus.

# 레온티에프 역행렬의 요인별 분해를 통한 투입·산출 승수분석: 한국 경제에 대한 사례 연구를 중심으로

김 호 언* · 김 군 찬**

## 논문초록

지금까지 레온티에프 역행렬 $C^f$(최종수요에 대한 생산유발계수행렬)을 통하여 투입·산출승수를 유도하였다. 본 연구에서는 레온티에프 역행렬의 완전한 요인별 분해를 통하여 2개의 투입유발계수행렬($\Gamma^f$와 $\Gamma^g$)과 산출물에 대한 생산유발계수행렬($C^g$)을 각각 유도하였다. 이제 $C^f$와 3개의 새로운 유발계수행렬($C^g$, $\Gamma^f$, $\Gamma^g$)을 통하여 다양한 투입·산출승수를 정의하고자 한다. $C^f$와 $\Gamma^f$ 행렬을 통해서는 외생적 최종수요의 변화에 대해서, $C^g$ 와 $\Gamma^g$ 행렬을 통해서는 외생적(혹은 내생적) 산출물의 변화에 대해서 투입 및 생산유발액을 각각 구할 수 있다. 아울러 4개의 유발계수행렬이 최종수요, 생산유발액, 투입유발액 등과의 상호 의존관계를 구체적인 도형으로 도화하였다. 부가적으로 4개의 유발계수행렬에 대한 실제적 유용성을 제고하기 위하여 한국 경제에 대한 다양한 경험적 연구를 수행하였다.

주제분류 : B031002
핵심 주제어 : 투입·산출승수, 요인별 분해, 레온티에프 역행렬, 유발계수행렬, 산출유발액

* 교신저자, 계명대학교 경제금융학과 교수, e-mail: houn@kmu.ac.kr
** 계명대학교 수학과 교수, e-mail: kmkim@kmu.ac.kr