Forecasting Hourly Electricity Loads of South Korea: Innovations State Space Modeling Approach

Moonyoung Baek*

Abstracts In this paper we discuss improving forecast accuracy by considering multiple seasonalities in high-frequency (hourly) time series data. Since the existing seasonal forecasting approaches are capable of dealing with a single pattern only, we employ an extended version of the traditional approach proposed by Taylor (2003). Furthermore, we setup a statistical model for which Taylor's double seasoal Holt-Winters expeonential smoothing method is optiaml in the framework of state space approaching, so that the innovations state space models with either single or double seasonalities can be estiamted by a numerical optimization procedure.

> We empirically apply the innovations state space models with a single seasonality (benchmarks) and another model with double seasonalities to forecasting hourly electricity loads of South Korea. As a result, we find that the innovations state space model corresponding to Taylor's double seasonal exponential smoothing method outperforms the other benchmark models with a single seasonality.

KRF Classification : B030104

Keywords : time series forecasting, innovations state space models, exponential smoothing, multiple seasonalities

 ^{*} BK21 Researcher, School of Economics, Yonsei University, e-mail: m.y.baek@ yonsei.ac.kr

Ⅰ**. Introduction**

Along with developments of computer-aided technologies, time series data with high frequencies become available and are prevalent in a relatively shorter interval of time (e.g., half-hourly, hourly, or daily) than typical time series with lower frequencies (monthly, quarterly, or yearly). For example, we can observe hourly time series of electricity loads or hourly series of demand of water or natural gas. We also observe a time series on counted numbers of calling in a service center in every hour, a series of hourly arrivals in an emergency room of a hospital, or a series of hourly traffic in a highway. A distinguishing feature of those high-frequency time series is that they are capable of showing more than one seasonal (periodic) pattern.

When high-frequency series are observed during a day or a week in a calendar, we might be able to find within-day (daily) and within-week (weekly) seasonal patterns at the same time. One of the reasons for the presence of multiple (double) seasonal patterns is that the high-frequency time series can be regarded as results of rational socio-economic behaviors that are periodically repeated in the real world over time-horizons defined customarily (e.g., 24-hour day, 168-hour week, and etc.).

Since traditional time series forecasting approaches (models or methods) typically consider a single seasonal pattern only, it is expected that the presence of multiple seasonalities in high-freuency series would make the existing seasonal forecasting approaches less accurate. This implies that improving forecast accuracy needs to take such multiple seasonal patterns into consideration simultaneously.

In this paper, we deal with the issue of a double seasonality in the framework of innovations state space models using exponential smoothing and examine forecast performance or forecast accuracy,

which is yielded by popular forecast error measures in time series forecasting literature.

The remaining part of this paper is organized as follows. In the next section, we review relevant literature to the developments of extending the traditional Holt-Winters exponential smoothing method and innovations state space models, for which the exponential smoothing method is optimal. We then in Section 3 represent two innovations state space model specifications for the double seasonal Holt-Winters exponential smoothing method. In Section 4, using the hourly time series of elelctricity loads of South Korea, we estimate the innovations state space models and evaluate point forecasts made by those models. We provide concluding remarks in the last section.

Ⅱ**. Literature Review**

We briefly review the existing studies on univariate forecasting approaches and their extensions to consider multiple seasonalities. A simple traditional forecasting approach for single seasonal time series is well known as Holt (1957) and Winters' (1960) exponential smoothing method.1) Since the standard Holt-Winters exponential smoothing method (additive or mutliplicative seasonal version) considers a single seasonal pattern only2), it is in fact incapable of accommodating simultaneously more than one seasonal pattern within its structure, although this limitation has not been seriously

 ¹⁾ Refer to Gardner (1985) and Gardner (2006) to see the historical developments of forecasting approach based on exponential smoothing methods.

 ²⁾ Basically the Holt-Winters exponential smoothing methods are able to deal with one seasonal pattern as well as other two components (level and growth). These methods have been widely used to produce reliable point forecasts with low-frequency (monthly or quarterly) time series.

recognized until time series forecasters confront a double (daily and weekly, typically) seasonality in several hourly time series data.

If two distinct seasonal patterns can be simultaneously incorporated into the scheme of exponential smoothing forecasting approach, a significant improvement in forecast accuracy should be expected. Taylor (2003) first studied this by focusing on two types of seasonal patterns in half-hourly electricity loads in England and Wales. For this he extended the standard Holt-Winters exponential smoothing method by adding an updating equation for the second seasonal pattern and modifying the updating equations for the level and the preexisting seasonal component in a recurrence form. His extended version can successfully capture both daily and weekly seasonal patterns observed in the electricity loads series. On the other hand, Taylor, *et al*. (2006) developed another extension by proposing double multiplicative seasonal Holt-Winters exponential smoothing approach and applied this to obtain point forecasts of electricity demands in Brazil and in England and Wales. In another paper, McSharry and Taylor (2006) employed an additive version of double seasonal Holt-Winters method for producing point forecasts of electricity loads in France.

Taylor's and others' studies show that their double seasonal exponential smoothing approach outperforms not only the single seasonal Holt-Winters method, but also complicated model-based approaches such as double multiplicative seasonal ARIMA model (Taylor, 2003), artificial neural network model, principal-componentanalysis based regression model (Taylor, *et al*., 2006), and periodic ARMA model (McSharry and Taylor, 2006).

Neverthless, the studies on Taylor's double seasonal Holt-Winters exponential smoothing methods have a theoretical limitation: the double seasonal exponential smoothing methods are just an

computational algorithm, not a statistical model, and they simply give point forecasts only.3) This gives researchers an additional motivation to find an appropriate statistical model in which the exponential smoothing method with two seasonal patterns underlies.4) We show in the next section appropriate model setups for Taylor's double exponential smoothing method with a multiplicative seasonality.

Ⅲ**. Theory**

1. The innovations state space model for the double multiplicative seasonal Holt-Winters exponential smoothing method

1.1 Additive error model

We setup an additive error version of an innovations state space model for the double multiplicative seasonal Holt-Winters exponential smoothing method. This innovations state space model consists of the following an observation equation and four state equations: s of the following an observation

ons:
 $y_t = (l_{t-1} + b_{t-1})s_{1,t-m_1}s_{2,t-m_2} + \epsilon_t$
 $l_t = (l_{t-1} + b_{t-1}) + \alpha_1 \frac{\epsilon_t}{s_{1,t-m_1}s_{2,t-m_2}}$

$$
y_t = (l_{t-1} + b_{t-1})s_{1,t-m_1}s_{2,t-m_2} + \epsilon_t
$$
 (1a)

$$
l_t = (l_{t-1} + b_{t-1}) + \alpha_1 \frac{\epsilon_t}{s_{1,t-m_1}s_{2,t-m_2}}
$$
 (1b)

 ³⁾ Thus, the forecasting strategy based on exponential smoothing methods is referred to as a method-based forecasting approach.

 ⁴⁾ See Ord, *et al*. (1997), Koehler, *et al*. (2001), Hyndman, *et al*. (2002, 2005), and Hyndman, *et al*. (2008) for discussions on the model-based forecasting approaches based on exponential smoothing methods with a single seasonality.

306 Moonyoung Baek

Nonyoung Back

\n
$$
b_{t} = b_{t-1} + \alpha_{2} \frac{\epsilon_{t}}{s_{1,t-m_{1}} s_{2,t-m_{2}}}
$$
\n
$$
s_{1,t} = s_{1,t-m_{1}} + \alpha_{3} \frac{\epsilon_{t}}{(l_{t-1} + b_{t-1}) s_{2,t-m_{2}}}
$$
\n(1d)

\n
$$
s_{2,t} = s_{2,t-m_{2}} + \alpha_{4} \frac{\epsilon_{t}}{(l_{t-1} + b_{t-1}) s_{1,t-m_{1}}}
$$
\n(1e)

$$
s_{1,t} = s_{1,t-m_1} + \alpha_3 \frac{\epsilon_t}{(l_{t-1} + b_{t-1})s_{2,t-m_2}}
$$
 (1d)

$$
s_{2,t} = s_{2,t-m_2} + \alpha_4 \frac{\epsilon_t}{(l_{t-1} + b_{t-1})s_{1,t-m_1}}
$$
 (1e)

where l_t , b_t , and s_t are level, growth and seasonal component in a time series observed at time t, respectively; $\epsilon_t \sim \text{WN}(0, \sigma_{\epsilon}^2)$ (white noise) and $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ are the model parameters in the above state space model. Substituting the following ϵ_t obtained from Eq.(1a)

$$
\epsilon_t = y_t - E[y_t | I_{t-1}] = y_t - y_{t|t-1}
$$

$$
= y_t - (l_{t-1} + b_{t-1})s_{1,t-m_1}s_{2,t-m_2}
$$

into each state equation (Eqs. (1b) \sim (1e)), we obtain the following exponential smoothing form: y_t $(t_{t-1} + b_{t-1})$.

ach state equation (E

ential smoothing form
 $l_t = \alpha_1 \frac{y_t}{s_{1,t-m_1}s_{2,t-m_2}}$

$$
l_t = \alpha_1 \frac{y_t}{s_{1,t-m_1} s_{2,t-m_2}} + (1 - \alpha_1)(l_{t-1} + b_{t-1})
$$
 (2a)

$$
b_t = \frac{\alpha_2}{\alpha_1} (l_t - l_{t-1}) + (1 - \frac{\alpha_2}{\alpha_1}) b_{t-1}
$$
 (2b)

$$
l_{t} = \alpha_{1} \frac{y_{t}}{s_{1,t-m_{1}}s_{2,t-m_{2}}} + (1 - \alpha_{1})(l_{t-1} + b_{t-1})
$$
(2a)
\n
$$
b_{t} = \frac{\alpha_{2}}{\alpha_{1}}(l_{t} - l_{t-1}) + (1 - \frac{\alpha_{2}}{\alpha_{1}})b_{t-1}
$$
(2b)
\n
$$
s_{1,t} = \left(\frac{\alpha_{3}}{1 - \alpha_{1}} \cdot \frac{l_{t}}{l_{t-1} + b_{t-1}}\right) \frac{y_{t}}{l_{t} \cdot s_{2,t-m_{2}}}
$$

\n
$$
+ (1 - \frac{\alpha_{3}}{1 - \alpha_{1}} \cdot \frac{l_{t}}{l_{t-1} + b_{t-1}})s_{1,t-m_{1}}
$$
(2c)
\n
$$
s_{2,t} = \left(\frac{\alpha_{4}}{1 - \alpha_{1}} \cdot \frac{l_{t}}{l_{t-1} + b_{t-1}}\right) \frac{y_{t}}{l_{t} \cdot s_{1,t-m_{1}}}
$$

$$
s_{2,t} = \left(\frac{\alpha_4}{1 - \alpha_1} \bullet \frac{t_t}{t_{t-1} + b_{t-1}}\right) \frac{y_t}{t_t \bullet s_{1,t-m_1}}
$$

Forecasting Hourly Electricity Loads of South Korea 307

Forecasting Hourly Electricity Loads of South Korea
+
$$
(1 - \frac{\alpha_4}{1 - \alpha_1} \cdot \frac{l_t}{l_{t-1} + b_{t-1}})s_{2,t-m_2}
$$
 (2d)

Here, I_{t-1} is the information set available up to time $t-1$.

1.2 Multiplicative error model

In this subsection, we provide the multiplicative error version of the innovations state space model for the double multiplicative seasonal exponential smoothing method as follows:

$$
y_t = (l_{t-1} + b_{t-1})s_{1,t-m_1}s_{2,t-m_2}(1+\epsilon_t)
$$
 (3a)

$$
l_t = (l_{t-1} + b_{t-1})(1 + \alpha_1 \epsilon_t)
$$
\n(3b)

$$
b_t = b_{t-1} + \alpha_2 (l_{t-1} + b_{t-1}) \epsilon_t
$$
\n(3c)

$$
s_{1,t} = s_{1,t-m_1}(1+\alpha_3 \epsilon_t)
$$
 (3d)

$$
s_{2,t} = s_{2,t-m_2}(1 + \alpha_4 \epsilon_t) \tag{3e}
$$

Substituting ϵ_t obtained from the observation equation (Eq.(3a))

$$
\epsilon_t = \frac{y_t - (l_{t-1} + b_{t-1})s_{1,t-m_1}s_{2,t-m_2}}{(l_{t-1} + b_{t-1})s_{1,t-m_1}s_{2,t-m_2}}
$$

into each state equation (Eq.(3b) \sim Eq.(3e)), we can obtain the following exponential smoothing form: ach state equation

ach state equation

ing exponential smoot
 $l_t = \alpha_1 \frac{y_t}{s_{1,t-m_1}s_{2,t-m_2}}$

$$
l_t = \alpha_1 \frac{y_t}{s_{1,t-m_1} s_{2,t-m_2}} + (1 - \alpha_1)(l_{t-1} + b_{t-1})
$$
 (4a)

$$
b_t = \frac{\alpha_2}{\alpha_1} (l_t - l_{t-1}) + (1 - \frac{\alpha_2}{\alpha_1}) b_{t-1}
$$
 (4b)

$$
l_t = \alpha_1 \frac{y_t}{s_{1,t-m_1} s_{2,t-m_2}} + (1 - \alpha_1)(l_{t-1} + b_{t-1})
$$

$$
b_t = \frac{\alpha_2}{\alpha_1} (l_t - l_{t-1}) + (1 - \frac{\alpha_2}{\alpha_1}) b_{t-1}
$$

$$
s_{1,t} = (\frac{\alpha_3}{1 - \alpha_1} \cdot \frac{l_t}{l_{t-1} + b_{t-1}}) \frac{y_t}{l_t \cdot s_{2,t-m_2}}
$$

308 Monyoung Back
\n
$$
+ (1 - \frac{\alpha_3}{1 - \alpha_1} \cdot \frac{l_t}{l_{t-1} + b_{t-1}}) s_{1,t-m_1}
$$
\n
$$
s_{2,t} = (\frac{\alpha_4}{1 - \alpha_1} \cdot \frac{l_t}{l_{t-1} + b_{t-1}}) \frac{y_t}{l_t \cdot s_{1,t-m_1}}
$$
\n
$$
+ (1 - \frac{\alpha_4}{1 - \alpha_1} \cdot \frac{l_t}{l_{t-1} + b_{t-1}}) s_{2,t-m_2}
$$
\n(4d)

We should note that both additive and multiplicative error versions of the multiplicative seasonal innovations state space models have identical exponential smoothing form (i.e., Eq.(2) or Eq.(4)) from Eq.(1) (additive error version) and Eq.(3) (multiplicative error version).

Finally from this we have the common exponential smoothing form as follows: from Eq.(1) (
plicative error version
lly from this we has follows:
 $l_t = \alpha \frac{y_t}{s_{1,t-m_1}s_{2,t-m_2}}$
 $b_t = \beta (l_t - l_{t-1}) + (1$

$$
l_t = \alpha \frac{y_t}{s_{1,t-m_1}s_{2,t-m_2}} + (1-\alpha)(l_{t-1} + b_{t-1})
$$
 (5a)

$$
b_t = \beta (l_t - l_{t-1}) + (1 - \beta) b_{t-1}
$$
\n(5b)

plicative error version).

\nIly from this we have the common exponential smooth solutions:

\n
$$
l_{t} = \alpha \frac{y_{t}}{s_{1,t-m_{1}}s_{2,t-m_{2}}} + (1-\alpha)(l_{t-1} + b_{t-1}) \qquad (5a)
$$
\n
$$
b_{t} = \beta(l_{t}-l_{t-1}) + (1-\beta)b_{t-1} \qquad (5b)
$$
\n
$$
s_{1,t} = \gamma_{1} \frac{y_{t}}{(l_{t-1} + b_{t-1})s_{2,t-m_{2}}} + (1-\gamma_{1} \cdot \frac{l_{t}}{l_{t-1} + b_{t-1}})s_{1,t-m_{1}} \qquad (5c)
$$
\n
$$
s_{2,t} = \gamma_{2} \frac{y_{t}}{(l_{t-1} + b_{t-1})s_{1,t-m_{1}}} + (1-\gamma_{2} \cdot \frac{l_{t}}{l_{t-1} + b_{t-1}})s_{2,t-m_{2}} \qquad (5d)
$$

$$
+(1-\gamma_1 \bullet \frac{\tau_t}{l_{t-1}+b_{t-1}})s_{1,t-m_1}
$$
(5c)

$$
s_{2,t} = \gamma_2 \frac{y_t}{(l_{t-1}+b_{t-1})s_{1,t-m_1}}
$$

$$
+(1-\gamma_2 \bullet \frac{l_t}{l_{t-1}+b_{t-1}})s_{2,t-m_2}
$$
(5d)

where we let

$$
\alpha = \alpha_1
$$
, $\beta = \frac{\alpha_2}{\alpha_1}$, $\gamma_1 = \frac{\alpha_3}{1 - \alpha_1}$, and $\gamma_2 = \frac{\alpha_4}{1 - \alpha_1}$,

and the restrictions on the smoothing parameters are:

$$
0 \le \alpha \le 1
$$
, $0 \le \beta \le 1$, $0 \le \gamma_1 \le 1$, and $0 \le \gamma_2 \le 1$.

respectively. We first (directly) estimate the smoothing parameters $(\alpha, \beta, \gamma_1, \text{ and } \gamma_2)$ in (5) by least squares method for minimizing the mean squred one-step-ahead errors⁵⁾, and we indirectly⁶⁾ calculate the estimates of the model parameters $(\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ α_4) by reverting the estimated smoothing parameters into

$$
\alpha_1 = \alpha
$$
, $\alpha_2 = \beta \alpha$, $\alpha_3 = \gamma_1 (1 - \alpha)$, and $\alpha_4 = \gamma_2 (1 - \alpha)$.

Ⅳ**. Empirical Applications**

1. Description of data

We use a high-frequency time series of hourly electricity loads in South Korea between 2004 and 2005, which were provided by Korea Power Exchange (KPX). We choose a full sample period starting from January 25 in 2004 to February 5 in 2005. Table 1 provides the summary information on the samples for parameter estimation (denoted as within-sample) and forecast evaluations (denoted as post-sample).7)

 ⁵⁾ In order to trigger a numerical optimization procedure, we need initial values of the three components and the starting values of the smoothing parameters. (We follow Taylor (2003)'s methodology to obtain the initial values of the components.)

 ⁶⁾ This allows for a simple form of restrictions. We use *optim*(∙) function of R-system to implement the required numerical optimization.

 ⁷⁾ At the current circumstance, we avoid including two major holiday seasons such as 'Seolnal' in winter and 'Chuseok' in fall to cause a calendar effect. The calendar effect with the two holidays in Korea is very strong, which is observed with unusually huge decreases in hourly electricity loads

| Sample | Start from | | End to | Week Day Obs. | |
|--------|------------|--|--------|---------------|--|
| | | Full 00h, 01-25-2004(Sun) 23h, 02-05-2005(Sat) 52 364 8736 | | | |
| | | Within-00h, 01-25-2004(Sun) 23h, 09-25-2004(Sat) 35 273 6552 | | | |
| | | Post- 00h, 10-03-2004(Sun) 23h, 02-05-2005(Sat) 18 91 2184 | | | |

【Table 1】Sample information of the series of hourly electricity loads

Note: 00h indicates 12 a.m. (midnight) and 23h indicates 11 p.m. of a day

In particular, we note from Figure 1 that the hourly series shows strong daily (seasonal length, $m_1 = 24$) and weekly ($m_2 =$ 168) patterns.

【Figure 1】Time-series plot of the observations in the within sample (the top panel) and its partial plot (the low panel).

during the holidays. The existence of the calendar effect could seriously affect forecast performance without appropriately controlling the effect. We want to bypass modeling the calendar effect at this time and to keep the theoretically equivalent relation between the method-based approach and the model-based approach using exponential smoothing.

2. Estimation of innovations state space models using the within-sample

We estimate three innovations state space models – two benchmarks with a single seasonalty having different seasonal lengths and another model with a double seasonality – using the within-sample. We estimate the model specifications with additive error, additive trend, and multiplicative seasonal pattern. This specification is denoted as AAM.8) Table 2 summerizes the estimation results and two popular error measures such as root mean squared error (RMSE) and mean absolute percentage error (MAPE).

【Table 2】Estimation of ISSM using the within-sample

| | | | | Model level (α_1) growth (α_2) daily (α_3) weekly (α_4) RMSE MAPE | | |
|------------------------------|-----------------------|------|------|---|-------------|--|
| | HW_{24}^{AAM} 0.83 | 0.00 | 0.17 | | 544.73 1.11 | |
| | HW_{168}^{AAM} 0.42 | 0.00 | | 0.58 | 587.07 1.23 | |
| $DSHW_{(24.168)}^{AAM}$ 0.62 | | 0.00 | 0.38 | 0.06 414.47 0.86 | | |

Note 1: HW_{24}^{AAM} and HW_{168}^{AAM} indicate the innovations state model specifications with either 24-hour daily or 168-hour weekly seasonal patterns; $DSHW^{AAM}_{(24,168)}$ indicates the innovatoins state space model specificaiton with both daily and weekly seasonal patterns. Note 1: HW_{24}^{2} and HW_{168}^{2}
with either 24-hous
 $DSHW_{(24,168)}^{AM}$ indicate
with both daily and
Note 2: The two error measur
 $RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}e_t^2}$

Note 2: The two error measures above are defined as:

$$
RMSE = \sqrt{\frac{1}{T} \sum_{t=1}^{T} e_t^2}
$$
 and

$$
MAPE = \frac{1}{T} \sum_{t=1}^{T} \left| \frac{100 e_t}{y_t} \right|
$$

where $e_t = y_t - y_{t|t-1}$, $t = 1, 2, \dots, T$, which is the one-step-ahead forecast error at time t .

 ⁸⁾ There is another specification, denoted as MAM, which indicates the multiplicative error version, Eq.(3). As shown in Section 3, both model specifications have identical estimates of the smoothing parameters because their exponential smoothing forms (Eq.(2) and Eq.(4)) are the same and they are expressed as a single (common) exponential smoothing form, Eq.(5).

We find that the model specification with 24-hour seasonal pattern (HW_{24}^{AAM}) has smaller MAPE (1.11) than the model with longer seasonal pattern (seasonal length = 168 hours). Even though HW_{168}^{AAM} can be regarded as a more generalized version of HW_{24}^{AAM} with longer 168-hour seasonal pattern, HW_{168}^{AAM} AАM surprisingly produces larger error measures. This implies that updating information every 24 hour could be better in reducing errors than updating every 168 hour. It is also noteworthy to pay attention on the change in the estimated model parameter for level (i.e., α_1) between HW_{24}^{AAM} and HW_{168}^{AAM} . The smaller estimate of from HW_{168}^{AAM} implies that level smoothing spreads over the 168-hour seasonal pattern by introducing a longer seasonality.

Model specification $\mathit{DSHW}_{(24,168)}^{AAM}$ yields the smallest RMSE (414.47) and MAPE (0.86) from the within-sample. This indicates that the performance of the model with a double seasonality improves the existing two benchmark specifications: HW_{24}^{AAM} and HW_{168}^{AAM} . As mentioned earlier, if we unavoidably had to choose one of seasonal patterns in a high-frequency time series, then we would better select the shorter 24-hour pattern. When we omit one of the two seasonal patterns from $DSHW_{(24,168)}^{AAM}$, the increase of MAPE in HW_{168}^{AAM} (i.e, by disregarding the daily seasonal pattern) is apparently larger than the increase of MAPE in HW^{AAM}_{24} without the longer weekly pattern. Frequent (every 24 hour) updating information seems to be more important than wide updating.

3. Evaluation on forecasting accuracy from the post-sample

We produce point forecasts for the post-sample by each innovations

state space model. Table 3 shows the post-sample evaluations made by the three innovations state space model specifications.

| μ able 3) Evaluation of forecasting accuracy from the post-sample | | | | | | |
|---|-------------|-------------|-------------|--|--|--|
| Model | RMSE | MAPE | MASE | | | |
| HW_{24}^{AAM} | 535.39 | 1.0502 | 0.4100 | | | |
| HW_{168}^{AAM} | 605.23 | 1.2297 | 0.4883 | | | |
| $DSHW^{AAM}_{(24,168)}$ | 376.46 | 0.7210 | 0.2852 | | | |
| Note: MASE indicates 'mean absolute scaled error', which is another error measure suggested by Hyndman and Koehler (2006): this is defined as $\textit{MASE}\!=\!\left.\frac{1}{T}\!\Sigma_{t=1}^{T}\right \!\frac{e_t}{\left.\frac{1}{T-1}\Sigma_{t=2}^{T} y_t-y_{t-1} \right }$ | | | | | | |
| where $e_t = y_t - y_{t+t-1}$, $t = 1, 2, \cdots, T$. | | | | | | |
| | | | | | | |

【Table 3】Evaluation of forecasting accuracy from the post-sample

$$
MASE = \frac{1}{T} \sum_{t=1}^{T} \left| \frac{e_t}{\frac{1}{T-1} \sum_{t=2}^{T} |y_t - y_{t-1}|} \right|
$$

where $e_t = y_t - y_{t|t-1}$, $t = 1, 2, \dots, T$.

According to Table 3, the innovations state space model specified by the double seasonal pattern $(DSHW_{(24,168)}^{AAM})$ outperforms both benchmarks $(HW_{24}^{AAM}$ and $HW_{168}^{AAM})$ as it yields the smallest RMSE (376.46), MAPE (0.7210), and MASE (0.2852). Therefore, we argue that the forecasting approach with a double seasonality gives effectlvely better forecasts than the traditional forecasting approaches with only one seasonal pattern.⁹⁾ We also note that the same rank of forecasting accuracy is observed as in the within-sample. Figure 2 shows the actual electricity loads and their forecasts produced by the innovations state space model with double seasonalities $(DSHW_{(24,168)}^{AAM})$ in the post-sample (out-of-sample).

 ⁹⁾ Even though we do not implement a formal test on significance of the forecast accuracy improvement between the double seasonal model and the two benchmark models, we could try a nonparametric test using a bootstrap techniqe to examine the significance. (We might confront a heavy burden of computational work to do this.)

314 Moonyoung Baek

Ⅴ**. Concluding Remarks**

In this paper, we have discussed the state space modeling approaches based on the seasonal exponential smoothing methods. It is known in Ord, *et all* (1997) that the traditional single seasonal Holt-Winters exponential smoothing method underlies the innovations state space model with a single seasonality. We explicitly show that the double seasonal Holt-Winters exponential smooithing method proposed by Taylor (2003) can be fully integrated into an innovations state space model with a double seasonal pattern. In emprical section, we estimate two benchmark innovations state space models and a double seasonal innovations state space model using hourly electricity loads and empirically evaluate their forecast performance. We find that the innovations state space model with a double seasonal pattern outperforms the other benchmarks with a single seasonality. This means that we

could improve forecast accuracy by accomodating two different seasonal patterns simultaneously.

Received: December 9, 2010. Revised: December 14, 2010. Accepted: December 15, 2010.

- Gardner, Everette S. (1985), "Exponential Smoothing: The State of the Art," Journal of Forecasting, $4(1)$, pp. 1-28.
- Gardner, Everette S. (2006), "Exponential Smoothing: The State of the Art - Part II," International Journal of Forecasting, $22(4)$, pp.637-666.
- Holt, C. C. (1957), "Forecasting Seasonals and Trends by Exponentially Weighted Moving Averages," in ONR Memorandum, Volume 52. Carnegie Institution of Technology.
- Hyndman, R. J. and A. B. Koehler (2006), "Another Look at Measures of Forecast Accuracy," International Journal of Forecasting, 22(4), pp.679-688.
- Hyndman, Rob J., A. B. Koehler, and J. Keith Ord (2005), "Prediction Intervals for Exponential Smoothing Using two New Classes of State Space Models," *Journal of Forecasting*, 24, pp. 17-37.
- Hyndman, Rob J., A. B. Koehler, Ralph. D. Snyder, and Simone Grose (2002), "Astate Space Framework for Automatic Forecasting Using Exponential Smoothing Methods," International Journal of Forecasting, 18, pp.439-454.
- Hyndman, Rob J., Anne B. Koehler, Keith Ord, and Ralph D. Snyder, Forecasting with Exponential Smoothing: The State Space Approach, Springer, August 2008
- McSharry, Patrick E. and James W. Taylor (2006), "Evaluation of Short-Term Forecasting Methods for Electricity Demand in France," RTE-VT workshop, Paris, pages 1-6.
- Ord, J. K., A. B. Koehler, and R. D. Snyder (1997), "Estimation and Prediction for Aclass of Dynamic Nonlinear Statistical Models," Journal of the American Statistical Association, 92(440), pp.1621-1629.
- Taylor, J. W (2003), "Short-Term Electricity Demand Forecasting Using Double Seasonal Exponential Smoothing," Journal of the Operational Research Society, 54, pp.799-805.
- Taylor, James. W., Lilian M. de Menezes, and Patrick E. McSharry (2006), "A Comparison of Univariate Methods for Forecasting Electricity Demand up to a Day Ahead," International Journal of Forecasting, 22, pp.1-16.
- Winters, P. R (1960), "Forecasting Sales by Exponentially Weighted Moving Averages," Management Science, 6, pp.324-342.

한국의 시간당 전력수요 예측: Innovations State Space 모형접근

백 문 영*

논문초록

본 논문에서 우리는 매우 짧은 간격(시간당)으로 관찰되는 시계열 자료의 다중계절성을 고려하여 시계열 예측정확성을 개선하려는 것에 대해서 논의 한다. 기존의 시계열 예측방법들은 단지 하나의 계절성 패턴만을 다룰 수 있 으므로, 우리는 Taylor(2003)에 의해 제안된 전통적 시계열 예측방법의 확 장형을 선정하였다. 더 나아가 우리는 상태 공간 모형화의 틀에서 Taylor의 이중 계절성 지수평활법이 적정하게 되는 통계적 모형을 설정해 보고, 단일 계절성 혹은 이중 계절성을 반영하는 innovations state space 모형을 수 치적 적정화 과정을 통해 추정하여 본다.

한국의 전력수요(부하) 시계열 자료를 이용하여, 우리는 단일 계절성 이 노베이션 상태 공간모형(기준모형, benchmarks)과 이중 계절성 모형을 시계열 예측에 적용하여 본다. 그 결과, 우리는 Taylor의 이중 계절성 지수 평활방법과 대응하는 innovations state space 모형이 단일 계절성만을 반영하는 두 개의 기준모형보다 우월하다는 것을 발견하였다.

주제분류: B030104 핵심 주제어: 시계열 예측, 이노베이션 상태 공간모형, 지수평활, 다중 계절성

 ^{*} 연세대학교 상경대학 경제학과 BK21 연구원, e-mail: m.y.baek@yonsei.ac.kr