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Impact of Bidder Information in Common Value First-Price Auction*

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Abstracts

This paper examines how the seller's revenue and bidders' payoffs in the common value setup are affected as bidders get increasingly better informed. For doing so, we consider three information structures in a common value first-price auction with two bidders where one bidder gets increasingly better informed of his rival's signal while the latter's information remains the same. It is shown that both bidder's payoff and seller's revenue can fall as the former gets better informed of his rival's signal. Also, the seller's revenue tends to be the lowest in the intermediate case where the bidder may or may not be informed of the rival's signal depending on what signal he holds for himself. So the seller's revenue is changing non-monotonically with bidder's information.

KRF Classification : B030200
Keywords : Common value, First-price auction, Informed bidder, Non-monotonicity

Ⅰ. Introduction

The information held by bidders in auction plays a crucial role

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in determining the performance of auction mechanisms. In this regard, Milgrom and Weber (1982) have established the famous linkage principle that in the common and affiliated values setup, publicly revealing the seller's information to bidders always enhances the seller's revenue, thus reducing bidders' payoffs. This also implies that the more information gets revealed to bidders, the higher revenue obtains, that is the seller's revenue is monotonically increasing with the amount of information known to bidders. However, some recent studies have shown that the linkage principle can fail under some circumstances: bidders demand more than one unit (Perry and Reny, 1999); bidders are constrained by their budgets (Fang and Parreiras, 2003); bidders cannot observe the identity of bidders who have dropped in an English auction (Feinberg and Tennenholtz, 2005). Another case of the detrimental effect of enhanced information is reported by Kim (2008) where only one of two bidders in the first-price common value auction gets better informed with the effect of reducing the seller's revenue.[1)]

In this paper, we ask how the seller's revenue and bidders' payoffs in the common values setup are affected as one bidder gets increasingly better informed of his rival's signal. This comparative statics is potentially important since bidders, who are partially informed of the underlying value of auctioned object, might want to learn their rivals' information and thereby estimate the value of the object more precisely.[2)] To be specific, we consider the following three information structures in the common value first-price auction where there are a couple of bidders and binary signals: (i) the standard information structure, denoted I^N,

1) Note that the learning of extra information in Kim (2008) is different from that in the linkage principle in that the learning occurs asymmetrically.

2) The information acquisition decision is not modeled in this paper, though.

where each bidder is privately informed of only one signal; (ii) the information structure, I^F where one bidder, say bidder 2, is informed of the signal of bidder 1 as well, who remains to be informed of his own signal only; (iii) the information structure, I^P, where bidder 2 is partically informed of bidder 1's signal in the sense that the former obseves the latter's *if his own signal is high*, while bidder 1 is informed of his own signal. Note that bidders have nontrivial information in all three information structures. More importantly, the bidder 2's information on bidder 1's signal increases going from I^N to I^P and also from I^P to I^F. The information structure I^P has not been studied in the literature yet and does have a novel feature that whether one bidder is informed of the other's signal is dependent on the information he already holds. This information structure can easily arise when bidders first learn some partial information about the object and, based on that information, decide whether to learn further information (e.g. rival's signal as in here). We characterize the equilibrium bidding strategy in each information structure and calculate the equilibrium revenue for the seller.

The first main result relates to the ranking of the seller's revenue in the above three cases: the seller's revenue tends to fall going from I^N to I^P and then rises going from I^P to I^F, implying that the seller's revenue tends to be the lowest in the intermediate information structure I^P. So the seller's revenue is not monotonically changing-neither always increasing nor always decreasing-with bidder's information on rival's signal. Our result shows that the impact of bidder information on the auction revenue can be more complex than might be expected from the linkage principle. Another important result is that the payoff of bidder 2 is lower in I^F than in I^P, which means that his payoff falls as he gets better informed. This is because bidder 2 getting better informed in I^F induces bidder 1 to bid more aggressively.[3] This result says that bidders in auction may get hurt by learning more information if it changes others' response accordingly. One lesson we can draw from the above results is that one should pay more attention to the details of information structure in order to properly evaluate the impact of bidder information on the performance of auction mechanisms.

Fang and Morris (2004) and Kim and Che (2004) investigate in the private values setup how the standard auctions are affected by bidders' information about rivals' values. They show that the revenue equivalence between first-price and second-price auctions breaks down. Moreover, Kim and Che (2004) establish a non-monotonicity result similar to this paper. Unlike these papers, Tian and Xiao (2009) endogenize the information structure by assuming that two bidders can pay a fixed cost to acquire some information about rival's value. Our paper differs from this literature since we deal with the common values setup. We believe that bidders' incentive to get informed about rivals' signal is more pronounced in the common values environment where bidders are not fully informed of the value of auctioned object, than in the private values setup where they are. This paper builds upon the analyses of Cambell and Levin (2000) and Kim (2008) that characterize the equilibrium strategy in the information structure I^F and use it to compare the seller's revenues in I^N and I^F. A technical contribution of this paper lies in characterizing the equilibrium bidding strategy for the information structure I^P, which turns out to be quite non-trivial. Section 2 of this paper introduces the model. Section 3 provides the equilibrium characterization and

3) Of course, if the enhanced information of one bidder goes unnoticed by his rival, then the former cannot get worse-off by getting better informed.

revenue/payoff comparison. Section 4 concludes.

Ⅱ. Model

Consider one seller who adopts the first-price auction to sell an indivisible item to two bidders. The item, which is of no value to the seller, yields both bidders the same (common) value v, which is equally likely to be 0 or 1. Each bidder $i=1,2$ observes a binary signal $s_i \in \{0,1\}$. Assume that two signals s_1 and s_2 are independently drawn conditional on the realized value of v as follows: For each $i=1,2$,

$$\alpha := \mathrm{Prob}[s_i = 0 | v = 0] = \mathrm{Prob}[s_i = 1 | v = 1]$$

with $\alpha > \frac{1}{2}$ for affiliation. Note that the higher α becomes, two signals get more positively correlated. Let $p(s_1, s_2)$ denote the probability of the realized signal profile being (s_1, s_2), and $p(s_{-i} | s_i)$ the probability of having s_{-i} realized conditional on i having drawn s_i. Note that

$$p(s_{-i} = y \,|\, s_i = x) \\ = \begin{cases} \alpha^2 + (1-\alpha)^2 & \text{if } x = y \\ 2\alpha(1-\alpha) < p(s_{-i} = x \,|\, s_i = x) & \text{if } x \neq y \end{cases} \tag{1}$$

Also, the expected value of the object conditional on a realization (s_1, s_2), written $\nu(s_1, s_2)$, can be calculated as

$$v(0,0) = \frac{(1-\alpha)^2}{\alpha^2 + (1-\alpha)^2}, v(1,1) = \frac{\alpha^2}{\alpha^2 + (1-\alpha)^2},$$

and

$$v(1,0) = v(0,1) = \frac{1}{2}.$$

Also, the expected value of the object conditional upon s_i being realized, written $v(s_i)$, is

$$v(0) = 1 - \alpha \text{ and } v(1) = \alpha.$$

We consider three information structures, denoted I^N, I^P, and I^F: In all three structures, bidder 1 observes s_1 only. As for bidder 2's information, he observes s_2 only in I^N (not informed) and both s_1 and s_2 in I^F (fully informed). In the information structure I^P, bidder 2 observes s_1 as well as s_2 if $s_2 = 1$ while he only observes s_2 if $s_2 = 0$ (partially informed). So whether bidder 2 is informed of the rival's signal is type-dependent in I^P. This information structure can arise if bidder 2 first draws one signal (i.e. s_2) and then decides whether to acquire his rival's signal s_1 depending on what signal s_2 he has drawn.[4] In this paper, however, we do not explore or endogenize the information acquisition process, which is certainly an important issue and deserves being an independent topic for future research. Note that bidder 2's information gradually increases from I^N to I^P and to I^F. Each information structure is assumed to be commonly known to bidders. The information structure I^N corresponds to the standard setup of Milgrom and Weber (1982), and going from I^N to I^P or to I^F, only bidder 2's information improves. In contrast,

4) Another possible scenario is where bidder 2 observes s_1 only when he draws $s_2 = 0$. While we cannot rule this case out a priori, the analysis of this case has been elusive so we restrict our attention to the information structure I^P.

the linkage principle of Milgrom and Weber (1982) assumes that the seller's public announcement causes all bidders' information to equally improve.

Ⅲ. Analysis: Equilibrium and Revenue

We derive the equilibrium bidding strategy for each information structure, and calculate the bidders' payoffs and the seller's revenue in equilibrium and compare them across information structures. Since this is a common values setup, the seller's (expected) revenue equals the expected value of the item minus the bidders' payoff. Note that the expected value of the item is equal to 1/2 given the binary value assumption.

For the equilibrium bidding strategies in I^N and I^F, we replicate the derivation in Campbell and Levin (2000). Then, we will analyze the equilibrium bidding strategies in I^P. Most of time, bidders employ mixed strategies in equilibrium, whose cumulative distribution for bidder i in information structure k is denoted as H_i^k. Similar notations will apply to other equilibrium quantities.

1. Information Structure I^N and I^F

• Equilibrium bidding strategy in information structure I^N: The equilibrium bidding strategy is symmetric. If $s_i = 0$, then bidder i bids $v(0,0)$ with probability one while if $s_i = 1$, he randomizes in the interval $[v(0,0), p(0\mid 1)v(0,0) + p(1\mid 1)v(1,1)]$ following the distribution given by

$$H^N(x) := \frac{p(0|1)(x - v(0,0))}{p(1|1)(v(1,1) - x)}.$$

As a result, two bidders obtain a symmetric ex ante payoff

$$\pi^N := p(1,0)(v(1,0) - v(0,0)) \tag{2}$$

and the seller's revenue is $R^N := 0.5 - 2\pi^N$.

• Equilibrium bidding strategy in information structure I^F: If $s_1 = 0$, bidder 1 randomizes in the interval $[v(0,0), v(0)]$ with distribution given by

$$H_1^F(x|0) := \frac{v(0,1) - v(0)}{v(0,1) - x}.$$

If $(s_1, s_2) = (0,0)$, bidder 2 bids $v(0,0)$ with probability one while if $(s_1, s_2) = (0,1)$, he randomizes in the interval $[v(0,0), v(0)]$ following the distribution given by

$$H_2^F(x|0,1) := \frac{p(0|0)(x - v(0,0))}{p(1|0)(v(0,1) - x)}.$$

If $s_1 = 1$, bidder 1 randomizes in the interval $[v(1,0), v(1)]$ with distribution given by

$$H_1^F(x|1) := \frac{v(1,1) - v(1)}{v(1,1) - x}.$$

If $(s_1, s_2) = (1,0)$, bidder 2 bids $v(1,0)$ with probability one while if $(s_1, s_2) = (1,1)$, he randomizes in the interval $[v(1,0), v(1)]$

following the distribution given by

$$H_2^F(x|1,1) := \frac{p(0|1)(x - v(1,0))}{p(1|1)(v(1,1) - x)}.$$

As a result, the equilibrium payoffs of bidder 1 and 2 are

$$\pi_1^F := 0 \tag{3}$$
$$\pi_2^F := p(0,1)(v(0,1) - v(0)) + p(1,1)(v(1,1) - v(1)), \tag{4}$$

while the seller's revenue is $R^F := 0.5 - \pi_2^F$.

One can compare bidders' payoffs and seller's revenue as follows:[5)]

Proposition 1 *Bidder 2's payoff is higher in I^F than in I^N (i.e. $\pi_2^F > \pi^N$) while the total payoff of two bidders is lower in I^F. So the seller's revenue is higher in I^F than in I^N (i.e. $R^F > R^N$).*

Proof of Propostion 1 First, it is straightforward to check the followings:

$$v(1,1) - v(1) = v(0) - v(0,0) < p(1,0) \tag{5}$$
$$p(1,1) - p(0,1) < v(1,0) - v(0,0). \tag{6}$$

To start, note that

$$\begin{aligned} \pi_2^F - \pi^N &= p(1,1)(v(1,1) - v(1)) - p(0,1)(v(0) - v(0,0)) \\ &= (p(1,1) - p(0,1))(v(1,1) - v(1)) > 0, \end{aligned}$$

5) Cambell and Levin (2000) have established the same result relying on a numerical method.

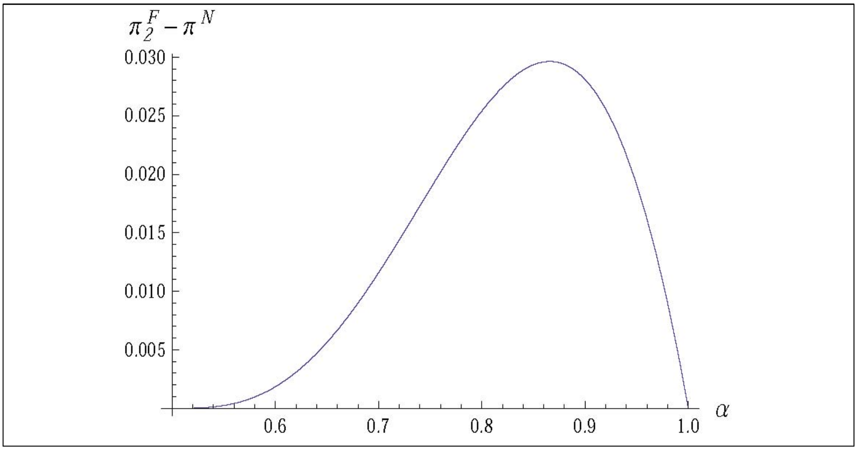
where the second equality follows from (5) and the inequality from $p(1,1) > p(0,1)$. So bidder 2's payoff is higher in I^F. Next,

$$\begin{aligned} 2\pi^N - \pi_2^F &= p(1,0)(v(1,0) - v(0,0)) + p(0,1)(v(0) \\ &\quad - v(0,0)) - p(1,1)(v(1,1) - v(1)) \\ &= p(1,0)(v(1,0) - v(0,0)) - (p(1,1) - p(0,1))(v(0) \\ &\quad - v(0,0)) > 0, \end{aligned}$$

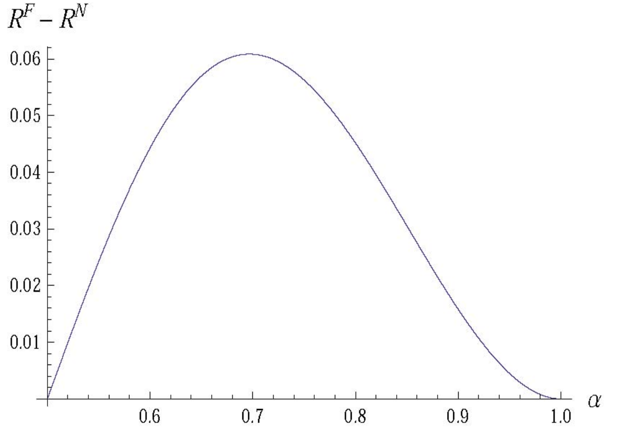
where the second equality follows from (5) and the inequality from the inequalities in (5) and (6). So the total payoff of bidders is lower in I^F than in I^N, which implies that the seller's revenue is higher in I^F since $R^F - R^N = (0.5 - \pi_2^F) - (0.5 - 2\pi^N) = 2\pi^N - \pi_2^F > 0$. ■

So both bidder 2 and seller benefit from the former's enhanced information. In contrast, bidder 1, whose information gets revealed to the rival bidder, earns zero payoff in I^F. The following two graphs give a more quantitative picture of how bidder 2's payoff and seller's revenue are compared across two information structures.

【Figure 1】 Payoff Difference between I^N and I^F



【Figure 2】 Revenue Difference between I^N and I^F



One can see that the maximum increase in revenue is about 0.06 around $\alpha = 0.7$, which is 12 % percent of total surplus generated (i.e. 0.5). On the other hand, the maximum increase in bidder 2's payoff is about 0.03 around $\alpha = 0.88$, which is 6 % percent of total surplus generated.

In the next section, we will see that both seller's revenue and bidder 2's payoff can fall as a result of bidder 2 getting better informed.

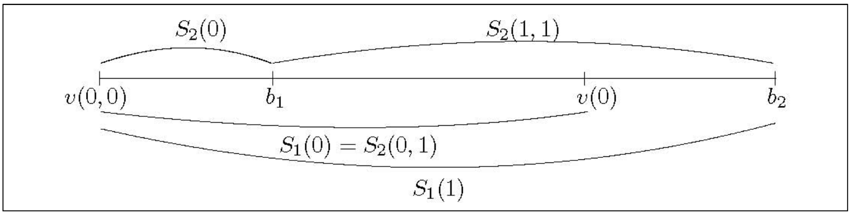
2. Information Structure I^P

Let us now turn to the information structure I^P in which bidder 1's information remains the same as in I^N and I^F while bidder 2 observes s_1 as well as s_2 if $s_2 = 1$ but only s_2 if $s_2 = 0$.

There is a (unique) mixed strategy equilibrium with the supports described in the Figure 3, where $S_i(x)$ denotes the support for the strategy of bidder i with $s_i = x$, and $S_2(x,y)$ the support for the strategy of bidder 2 with $(s_1, s_2) = (x, y)$.[6)]

6) A heuristic argument for the equilibrium uniqueness is provided in the Appendix.

【Figure 3】 Supports of equilibrium bidding distributions in I^P



Here, b_1 and b_2 satisfy $v(0,0) < b_1 < v(0) < b_2 < v(1)$ and are determined to satisfy the following set of equations: Letting $H_1^P(\bullet \mid 0)$ and $H_1^P(\bullet \mid 1)$ denote the bidding distribution of bidder 1 with $s_1 = 0$ and 1, respectively,

$$p(0|0)(v(0,0)-b_1)H_1^P(b_1|0)+p(1|0)(v(1,0)-b_1)H_1^P(b_1|1)=0 \tag{7}$$

$$p(0|1)(v(1,0)-b_1)=v(1)-b_2 \tag{8}$$

$$(v(0,1)-b_1)H_1^P(b_1|0)=v(0,1)-v(0) \tag{9}$$

$$(v(1,1)-b_1)H_1^P(b_1|1)=v(1,1)-b_2. \tag{10}$$

The equations (7) to (10) correspond to the following equilibrium conditions, respectively: bidder 2 with $s_2 = 0$ earns the equilibrium payoff equal to zero by bidding b_1; bidder 1 with $s_1 = 1$ is indifferent between b_1 and b_2; bidder 2 with $(s_1, s_2) = (0,1)$ is indifferent between b_1 and $v(0)$; bidder 2 with $(s_1, s_2) = (1,1)$ is indifferent between b_1 and b_2. The following lemma guarantees the existence of such b_1 and b_2, whose proof is in the Appendix.

Lemma 1 *There exist* $b_1 \in (v(0,0), v(0))$ *and* $b_2 \in (v(0), v(1))$ *that solve (7) to (10) along with the corresponding values of* $H_1^P(b_1|1)$ *and* $H_1^P(b_1|0)$.

Given b_1 and b_2 that solve (7) to (10), we can construct the equilibrium bidding distributions, H_1^P's and H_2^P's, for each type of bidders such that its rival bidder is indifferent over the support of equilibrium bidding distribution, which is straightforward but tedious, and thus omitted.

The resulting equilibrium payoffs of bidder 1 and 2 are

$$\pi_1^P := p(1,0)(v(1,0) - b_1) \tag{11}$$

$$\pi_2^P := p(0,1)(v(0,1) - v(0)) + p(1,1)(v(1,1) - b_2), \tag{12}$$

respectively. Then, the seller's revenue is given by $R^P := 0.5 - \pi_1^P - \pi_2^P$.

Let us first compare this equilibrium to that under I^F. To do so, note that when the realized signal profile is $(s_1, s_2) = (1,0)$, bidder 2 bids less aggressively in I^P since he bids $v(1,0)$ for certain in I^F while bidding at most $b_1 < v(0) < v(1,0)$ in I^P. It is because in I^P, bidder 2 with $s_2 = 0$, who is not informed of bidder 1's type, bids more cautiously or less aggressively than in I^F. This induces bidder 1 with $s_1 = 1$ to be less aggressive also, which in turn causes bidder 2 with $(s_1, s_2) = (1,1)$ to bid less aggressively. As a result, both bidders enjoy a higher expected payoff in I^P than in I^F, which implies that the seller's revenue is lower in I^P. To summarize,

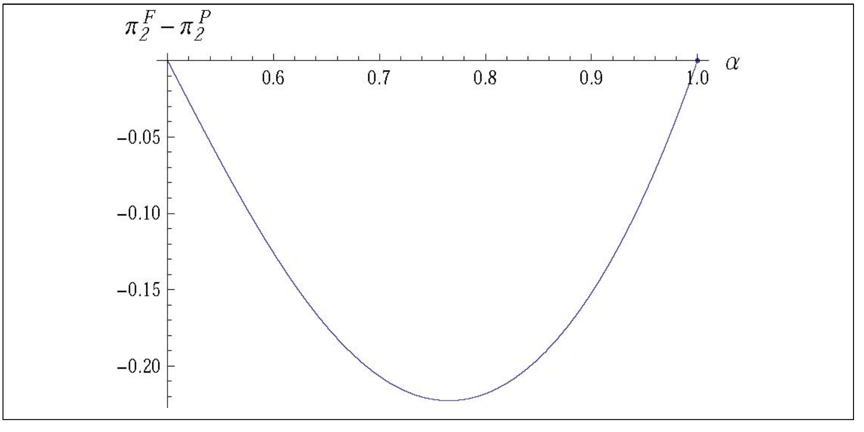
Proposition 2 *Both bidder 1 and 2's payoffs are lower in I^F than in I^P (i.e. $0 = \pi_1^F < \pi_1^P$ and $\pi_2^F < \pi_2^P$). So the seller's revenue is higher in I^F than in I^P (i.e. $R^F > R^P$).*

Proof of Propostion 2 The only nontrivial part is to compare π_2^P

and π_2^F: Using (4) and (12), we have $\pi_2^F - \pi_2^P = p(1,1)(b_2 - v(1,1)) < 0$ since $b_2 < v(1) < v(1,1)$. ■

Thus, bidder 2 suffers from the enhanced information in I^F. Figure 4 shows that bidder 2's payoff loss in I^F, compared to I^P, can be substantial and more than 40 % of total surplus around $\alpha = 0.8$.

【Figure 4】 Payoff Difference between I^P and I^F



Next, we turn to the comparison between I^N and I^P. First, the bidder 2 getting informed benefits himself but hurts his rival:

Proposition 3 *Bidder 1's payoff is lower in I^P than in I^N (i.e. $\pi_1^P < \pi^N$) while the opposite is true for bidder 2 (i.e. $\pi_2^P > \pi^N$).*

Proof of Propostion 3 Bidder 1's payoff is lower in I^P since

$$\Delta_1 := \pi_1^P - \pi^N = p(1,0)(v(0,0) - b_1) < 0, \tag{13}$$

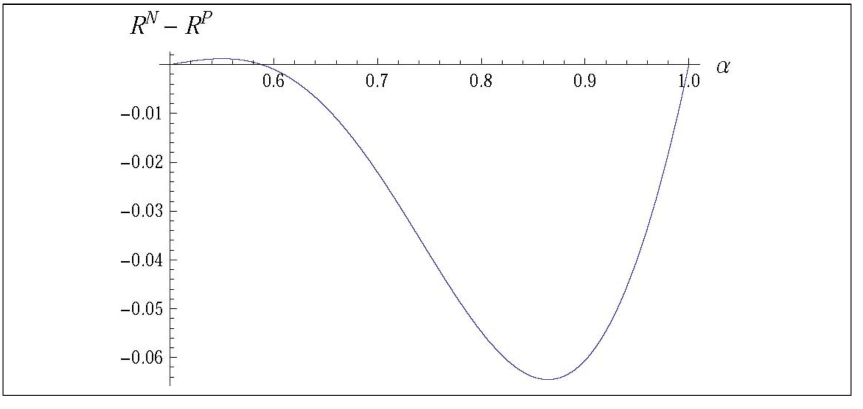
since $b_1 > v(0,0)$. Bidder 2's payoff is lower in I^N since

$$\begin{aligned}\Delta_2 := \pi_2^P - \pi^N &= p(1,1)(v(1,1) - b_2) - p(0,1)(v(0) - v(0,0)) \\ &> p(1,1)(v(1,1) - v(1)) - p(0,1)(v(0) - v(0,0)) \\ &= (p(1,1) - p(0,1))(v(1,1) - v(1)) > 0, \qquad (14)\end{aligned}$$

where the inequality follows from $b_2 < v(1)$ and the last equality from $v(1,1) - v(1) = v(0) - v(0,0)$. ■

The comparison of seller's revenues turns out to be ambiguous. As Figure 5 shows, the revenue is higher in I^N only if α is close to 0.5 while the opposite holds otherwise. Also, the revenue fall from I^N to I^P is the greatest at relatively high α.

【Figure 5】 Revenue Difference between I^N and I^P



This pattern can be explained by taking a closer look at how two bidders' payoffs compare in I^N and I^P. To do so, note that the seller's revenue is lower in I^P when the increase in bidder 2's payoff, Δ_2 in (14), dominates the decrease in bidder 1's payoff, Δ_1 in (13), and vice versa. We also note that the expression before the last inequality in equation (14) sets a lower bound for Δ_2 and approximates Δ_2 if $\alpha \approx 0.5$. Let us first assume $\alpha \approx 0.5$ or the signals are quite uninformative about the underlying value of the item, in which case $b_1 \approx b_2 \approx v(s_1, s_2) \approx 0.5$ for all s_1, s_2 and $p(1,1) \approx p(1,0) \approx 1/4$. Thus, the negative value Δ_1 is close to zero in the first order while the positive value Δ_2 is close to zero in the second order, which explains the slightly higher revenue in I^P around $\alpha = 0.5$. The opposite holds for very high values of α since $p(1,1) \gg p(1,0) \approx 0$ and $0 \approx v(1) - v(1,1) = v(0,0) - v(0) < v(0,0) - b_1 < 0$, so Δ_2 is close to zero in the first order while Δ_1 is close to zero in the second order, which makes the seller's revenue fall in I^P. It is at the intermediate values of α that the revenue fall is the greatest. To see why, note that the decrease in the seller's revenue is mostly due to the increase in the payoff of bidder 2 observing $(s_1, s_2) = (1,1)$, which amounts to $p(1,1)(v(1,1) - b_2)$.[7] This is maximized at relatively high values of α with which $p(1,1)$ is high and, at the same time, $v(1,1) - b_2$ is not too small, which explains the hump just below $\alpha = 0.9$ in Figure 5.

To summarize, the seller's revenue is likely to fall as bidder 2 gets partially informed ($R^P < R^N$). Combining this observation with Proposition 1, we find that it is quite likely to have $R^P < R^N < R^F$. In other words, the seller's revenue changes nonmonotonically with bidder 2's information, attaining the lowest when bidder 2 is partially informed.

Ⅳ. Conclusion

In this paper, we have shown that both seller and bidder can

7) Note that in I^P, bidder 2 with $(s_1, s_2) = (1,1)$ earns $(v(1,1) - b_1) H_1^P(b_1|1) = v(1,1) - b_2$ by bidding the lowest equilibrium bid b_1. By contrast, in I^N, bidder 2 with $s_1 = 1$ earns zero payoff when bidding the lowest equilibrium bid $v(0,0)$ against bidder 1 with $s_1 = 1$.

get hurt as the latter becomes better informed of his rival's signal. Also, the seller's revenue is likely to change nonmonotonically with the bidder information, achieving the lowest level in the intermediate case where the bidder is partially informed. As mentioned earlier, the case of partially informed bidder is likely when the bidder makes an endogenous decision to acquire his rival's information after observing his own signal, which is not modeled in the current paper, though. In this regard, we put forward important research agenda for future study in the following questions: what are the incentives for bidders to acquire information in auctions; what information structures endogenously arise from such information acquisition; what are the revenue and payoff consequences?

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Appendix: Equilibrium Analysis for I^P

Proof of Lemma 1: Using (8) to (10), one can express b_2, $H_1^P(b_1|0)$, and $H_1^P(b_1|1)$ in terms of b_1 and substitute them into (7) to obtain

$$\begin{aligned} & p(0|0)(v(0,1)-v(0))(v(0,0)-b_1)(v(1,1)-b_1) \\ & \quad + p(1|0)(v(1,0)-b_1)^2[v(1,1) \\ & \quad - v(1) + p(0|1)(v(1,0)-b_1)] = 0, \end{aligned} \tag{15}$$

which gives us a cubic equation to solve for b_1. We show that the LHS of (15), denoted as a function $g(b_1)$, is positive if $b_1 = v(0,0)$ and negative if $b_1 = v(0)$, which implies by the intermediate value theorem that there is $b_1 \in (v(0,0), v(0))$ we look for. First, it is straightforward to see that $g(v(0,0)) > 0$. Next,

$$\begin{aligned} g(v(0)) &= p(0|0)(v(0,1)-v(0))(v(0,0)-v(0))(v(1,1)-v(0)) \\ & \quad + p(1|0)(v(1,0)-v(0))^2[v(1,1)-v(1)+p(0|1)(v(1,0)-v(0))] \\ & < p(0|0)(v(0,1)-v(0))(v(0,0)-v(0))(v(1,1)-v(0)) \\ & \quad + p(1|0)(v(1,0)-v(0))^2[v(1,1)-v(0)] \\ & = (v(0,1)-v(0))(v(1,1)-v(0))[p(0|0)(v(0,0) \\ & \qquad - v(0)) + p(1|0)(v(1,0)-v(0))] \\ & = 0, \end{aligned}$$

where the last equality holds since

$$p(0|0)+p(1|0)=1 \ \text{ and } \ p(0|0)v(0,0)+p(1|0)v(1,0)=v(0).$$

Given that $b_1 \in (v(0,0), v(0))$, we have from (8)

$$\begin{aligned} b_2 - v(0) &= v(1) - v(0) - p(0|1)(v(1,0) - b_1) \\ &> v(1) - v(0) - p(0|1)(v(1,0) - v(0)) \\ &> (1 - p(0|1))(v(1) - v(0)) > 0, \end{aligned}$$

and also

$$b_2 - v(1) = - p(0|1)(v(1,0) - b_1) < 0,$$

as desired. ■

We now provide a heuristic argument to characterize the equilibrium in Section 3.2. Let us begin with a few observations (without proof). First, each bidder i of type $s_i = 0$ earns zero payoff while other types earn positive payoffs. Also, the support for the equilibrium bidding strategy of each type of bidders must be a connected interval. Further, bidder 1 has to randomize his bid since his signal is known to bidder 2 with $(1,1)$. These observations imply that (i) $S_1(0) = S_2(0,1) = [v(0,0), v(0)]$, (ii) bidder 1 with $s_1 = 0$ does put a mass on $v(0,0)$ while bidder 2 of type $(s_1, s_2) = (0,1)$ does not, and (iii) the equilibrium payoff of bidder 2 with $(s_1, s_2) = (0,1)$ is $v(0,1) - v(0)$. Also, since bidder 1 with $s_1 = 1$ can face bidder 2 with either $s_2 = 0$ or $(1,1)$, we must have $S_1(1) = S_2(0) \cup S_2(1,1)$ with $S_2(0)$ and $S_2(1,1)$ being disjoint.

Using theses observations, we locate other supports for the mixed strategies in equilibrium. First, it must be that $S_2(0) = [v(0,0), b_1]$ for some $b_1 > v(0,0)$. To see this, note that the lower end of $S_2(0)$, denoted $\overline{b}$, must be equal to $v(0,0)$. Clearly, $\overline{b} \geq v(0,0)$. If $\overline{b} > v(0,0)$, then bidder 2 with $s_2 = 0$ would earn a negative payoff by bidding slightly above $\overline{b}$ since such bid has a nontrivial winning probability only against bidder 1 with $s_1 = 0$ (due to (ii) above). Also, it must be that $b_1 > v(0,0)$, which is because $b_1 = v(0,0)$ means that $S_2(0) = \{v(0,0)\}$ so $v(0,0)$ is the lower end of $S_2(1,1)$. Since bidder 1 with $s_1 = 1$ must earn a positive payoff, he never puts a mass at $v(0,0)$, implying that by bidding $v(0,0)$, bidder 2 with $(s_1, s_2) = (1,1)$ must earn zero equilibrium payoff, a contradiction. Furthermore, letting b_2 denote the upper end of $S_2(1,1)$ (namely $S_2(1,1) = [b_1, b_2]$), it must be that $b_1 < v(0) \leq b_2 < v(1)$. To see this, if $b_1 \geq v(0)$, then bidder 2 with $s_2 = 0$ would earn a negative payoff by bidding b_1 unless $b_2 = b_1 = v(0)$, which is not possible. Also, if $b_2 < v(0)$, then bidder 2 with $s_2 = 0$ can earn a positive payoff by bidding some $b \in (b_2, v(0))$, a contradiction. Lastly, if $b_2 \geq v(1)$, then bidder 1 with $s_1 = 1$ would earn a non-positive payoff by bidding b_2, a contradiction.

We now show that no type of any bidder has an incentive to submit a bid outside the support. First of all, it is clear that any type of bidder 1 and bidder 2 with $(s_1, s_2) = (0,1)$ do not have an incentive to bid outside the support. It remains to prove that (a) bidder 2 with $(s_1, s_2) = (1,1)$ has no profitable downward deviation to some $b \in [v(0,0), b_1)$ and (b) bidder 2 with $s_2 = 0$ has no profitable upward deviation to some $b \in (b_1, v(0)]$. We only prove (a) since proving (b) is similar. To begin, using (7) and (9), we can obtain for $b \in [v(0,0), v(0)]$,

$$H_1^P(b|1) = \frac{p(0|0)(b - v(0,0))(v(0,1) - v(0))}{p(1|0)(v(1,0) - b)^2}.$$

Given the equilibrium payoff in (10), a deviation by bidder 2 to some $b \in [v(0,0), b_1)$ would be unprofitable if

$$(v(1,1)-b)H_1^P(b|1) \le v(1,1)-b_2$$

or

$$f(b) := \frac{(b-v(0,0))(v(1,1)-b)}{(v(1,0)-b)^2} \le \frac{p(1|0)(v(1,1)-b_2)}{p(0|0)(v(0,1)-v(0))} =: c, \quad (16)$$

where $c>0$ is a constant. We know that

$$f(v(0,0)) = 0 < f(b_1) = c < f(v(0)).^{8)} \quad (17)$$

Since $f(b)=c$ corresponds to a quadratic equation regarding b and thus can have at most two solutions, (17) implies that $f(b)=c$ at $b=b_1$ and possibly some $b>v(0)$ while $f(b)<c$ for $b<b_1$, as required by (16).

8) The inequality $f(v(0))>c$ corresponds to the fact that bidder 1 with $s_1=0$ gets a negative payoff by bidding $v(0)$.

공통가치 일차가격경매에서 입찰자가 가진 정보가 미치는 영향에 관하여

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논문초록

본고는 공통가치를 가진 재화를 판매하는 경매에서 입찰자들이 가진 정보가 점차적으로 증가할 때 입찰자들의 보수와 판매자의 수입이 어떤 영향을 받는지를 연구한다. 이를 위해 두 입찰자들로 구성된 일차가격상황에서 세가지 정보체계를 상정하되, 세 정보체계에 걸쳐 어느 한 입찰자의 정보는 단계적으로 증가하는 반면에 다른 입찰자의 정보는 변함이 없다고 가정한다. 주요결과로, 입찰자가 가진 정보가 증가함에 따라 해당 입찰자의 보수와 판매자의 수입이 모두 감소할 수 있다는 사실을 보인다. 아울러, 판매자의 보수는 입찰자의 정보가 중간수준에 있을 때 가장 낮아질 수 있음을 보인다. 이는 판매자의 보수가 입찰자들이 가진 정보가 증가함에 따라 단조적으로 증가하지 않을 수 있음을 의미한다.

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