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Dynamic Stability in a Nonlinear Model of Hyperinflation

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Abstracts

Cagan's studies on the monetary dynamics of hyperinflations were marred by his selective truncation of the data series. Towards the end of the explosive hyperinflations real balances rose rapidly. The linear specification of the demand for money function is justified as a first-order Taylor series approximation to the true, nonlinear function. This may be appropriate for small variations in the independent variables but it is likely to be inappropriate in hyperinflations. Therefore, we estimate a second-order approximation and find that the squared expected inflation term is highly significant.

We then address the issue of stability of the inflation process and also find stability in all of the cases addressed by Cagan. The monetary dynamic analysis of the model provides insights into various qualitative results which are different to Cagan's study, though some are in accord with studies of Goldman and Frenkel.

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I. Introduction

It was the study of the German hyperinflation that led Cagan to construct the prototype of the money demand function in which the only argument was the expected rate of inflation. His foundation has long played an important role in the dynamic analysis of money and inflation. Cagan dealt with questions concerning the role of money in generating inflation. He has shown that a self-perpetuating inflation was irrelevant to real world situations such as well-known episodes of hyperinflation. As is well-known, the dynamic stability condition depends in an essential manner on both the expected inflation elasticity of the demand for money and the coefficient of expectations adjustment. Thus, econometric estimates of these two parameters provide vital evidence on the stability of the inflation process.

Cagan's semi-log, linear model of money and inflation, however, does not fit the last part of the German hyperinflation.[1] During this period, inflation and money growth both reached their highest rates yet the stock of real money balances increased. It was this observation that led Flood and Garber (1980) to introduce the probability of monetary reform to explain the rationale for the last part of the German hyperinflation.

Instead, we suggest that the linear model used by Cagan is probably only a linear approximation of the true nonlinear model of money demand behavior. The main purpose of this paper is to propose an alternative model of hyperinflation which can account for the rising demand for real money balances in the face of an accelerating hyperinflation. In view of this, we shall try to fit the

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1) A large number of subsequent studies, thus, have truncated the turbulent part at the end of the German hyperinflation [see Barro (1970), Sargent and Wallace (1975), Frenkel (1977), and Sargent (1977)].

entire sample period using a nonlinear, semi-log specification. Given that this specification is compatible with the data, it raises the issue of the stability of the inflationary process given money growth. Traditionally, it has been observed that the estimates obtained by Cagan indicate a stable money demand function in the sense that it does not result in a self- perpetuating inflation. This accepted fact needs to be reviewed if the true money demand function is nonlinear. Thus, we shall discuss the dynamic stability of the proposed portfolio schedule.

This model is shown to be consistent with the data for the entire sample period. The resultant estimates indicate that there is little evidence of a self-generating inflation even if the turbulent part of the hyperinflation is included. The dynamic analysis presented here provides some insights into various qualitative results which are different to Cagan's study, though some are in accord with studies of Goldman (1972) and Frenkel (1973).

The rest of this paper is organized as follows. Section two reviews the literature on Cagan's classic study and section three develops the basic model. Section four and five analyze the dynamic interactions between money growth, inflation and stability in the short-run and in the long-run. In section six the empirical results of the estimation are presented. The final section offers some implications of the results and concluding remarks.

II. Review of Previous Studies

Inflation, even if fully anticipated, reduces money's usefulness as a store of value. Expectations of inflation will, therefore, induce individuals to economize their holdings of real cash balances. In this context, Cagan (1956) constructed a prototype of the money demand function in which the demand for real cash balances is specified as a semi-log linear function of the expected rate of change of the price level alone.[2] This is justified by the fact that, during a period of hyperinflation, fluctuations in the price level and the rate of inflation are some astronomical that their effect completely dominates the effects of the changes in the other variables which are important determinants of the transactions-cost models of money (e.g., the Baumol-Tobin model)

In the money demand functions where expectations of future inflation play a central role, the fundamental difficulty is how to infer expectations which cannot be directly observed. Thus, extra modeling or theorizing is needed at this stage. Introducing an adaptive expectations hypothesis, Cagan developed a well-known model of money and hyperinflation. In recent years, it has been prominent in the literature on rational expectations models in macroeconomics. This semi-log, linear model was empirically tested and confirmed by the data drawn from the six European countries after World War II which experienced an episode of hyperinflation. In particular, Cagan found the following major results:[3]

(i) A predominant role of the expected rate of future inflation,

(ii) A stable money demand function in both senses: a functional stability emphasized by monetarists and a dynamic stability in the senses that it does not result in a self-sustaining inflation, and

(iii) A strong relationship between money and prices.

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2) The reason for using the semi-log form is that during some months early in the sample period, inflation was negative and the logarithm of a negative figure is not defined. After estimating the double-log form using the Box-Cox transformation, however, Frenkel (1977) finds that the semi-logarithmic form demand function for money is superior.

3) These results provide empirical support for the modern Quantity Theorist's view.

Cagan's seminal study has raised many questions as to the proper econometric specification and application of rational expectations to this problem and has led to a large body of literature. Cagan's empirical methods were criticized by Jacobs (1975), who questioned Cagan's orthogonality assumption. He suggests that the regression equation should be modified by solving real money balances as a function of the rate of growth of the money supply, instead of using past rates of inflation. Jacobs obtained unstable money demand functions for some countries, i.e., Germany, Hungary, and Greece. However, his estimate of expectation adjustment coefficient turned out to be negative for the German, Greek and Hungarian data. This result makes no economic sense in the hyperinflations and may reflect the inadequacy of the model. In addition, a failure of the orthogonality assumption does not appear to be significant because there is substantial evidence that inflation caused money creation in the Granger sense during that period.[4)]

The mechanism of adaptive expectations introduced by Cagan has been widely used in the literature with respect to the behavior of the demand for money under inflationary conditions. Some empirical results seem to contradict the explicit assumption that the adjustment coefficient remains constant over time. Thus, instead of a fixed adjustment, Khan (1977) allowed the coefficient of expectation adjustment to vary with the level and variability of the rate of inflation. Using this, he estimated the Cagan model. The results obtained indicate that the impact effect of expected rate of inflation on real money balances is, in general, lower than Cagan's estimate. He also finds that the speed of adjustment is influenced by the extent to which inflation is changing.

4) For a more detailed discussion of causality tests, see Sargent and Wallace (1973a), Frenkel (1977) and Sargent (1977).

Since the simple adaptive expectations scheme is a backward-looking solution, future events do not affect the adaptive process for expectations. Thus, it seems a restrictive form and is sometimes referred to as an ad hoc scheme. Recently, it has been argued that there are serious weaknesses inherent in the adaptive expectations hypothesis.[5] Instead, it may be preferable to adopt Muth's (1961) concept of rational expectations. In the context of rational expectations, Sargent and Wallace (1973a) dealt with the money and hyperinflation question when expectations are rational and the money supply is endogenous. Their empirical tests reported substantial evidence on the existence of feedback from price inflation to monetary expansion with no reverse feedback from money creation to inflation. This evidence is necessary to show that Cagan's adaptive expectations are, in fact, formed rationally.[6]

They point out that, under such a framework, Cagan's estimate for the slope parameter is not statistically consistent because the single equation procedure will generally suffer from a simultaneous equations bias problem. As a result, Sargent (1977) constructed a money supply equation which depends only on past rates of inflation. He obtained maximum likelihood estimates of the semi-elasticity of the demand for real balances with respect to the expected rate of inflation for the seven European hyperinflations. These estimates are seemingly consistent with a relatively large standard error.[7]

5) The adaptive expectations model is, in general, too mechanistic and also makes systematic errors. In addition, this scheme cannot deal with jumps in a variable perceived by economic agents.

6) They did not conduct any direct empirical test of their model but, instead, reported results of indirect tests of feedback from current inflation to future rates of money creation. For this reason, Noh (1988) dealt with some questions of whether their model is consistent with the data.

7) The estimation is carried out under the assumption that innovation in the money demand equation is uncorrelated with innovation in the money

Frenkel (1977) has pointed out the problem of empirically measuring unobservable expectations of inflation. He proposed a direct measure of expectations based on the time series data from the foreign exchange market. This measure does not depend on any arbitrary expectations model but rather uses observable market prices to infer expectations. He used the forward premium on foreign exchange as a direct measure of the expected cost of holding money. This may be justified by the fact that during the hyperinflation foreign exchange market functions efficiently. Thus, expectations reflected in market behavior will be optimal forecasts using all available information. Frenkel's estimates of the slope parameter for the seven hyperinflations are shown to be somewhat lower than Cagan's and Khan's estimates.

However, Abel, et al. (1979) points out that even if the forward premium is treated as a proxy for the expected rate of inflations, Frenkel's formulation raises some questions as to what the precise role of the forward premium in the money demand functions is. If domestic money were a bad substitute for goods and a close substitute for foreign assets, the expected rate of currency depreciation could be the only true explanatory variable. However, if both goods and foreign assets were good substitutes for domestic money, the expected rate of inflation and the anticipated rate of depreciation of foreign exchange should both be argument s.[8] In addition, Salemi (1980) has argued that the forward exchange premium does not contain all the relevant information available to predict inflation and thus leads to underestimating the extent of changes of the price level that actually take place.

Recent studies of Cagan's model have, for the most part,

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supply equation. Without such an arbitrary restriction it is not possible to obtain a consistent estimator for α(Sargent, 1977, p.73).

8) However, these two variables were shown to be closely related. For more details, see Abel, et. al. (1979).

truncated the data for the explosive part of the German hyperinflation. However, Flood and Garber (1980) extended the estimation to the whole sample period in order to draw something qualitatively different from any previously studied money demand behavior. They hypothesized that during the hyperinflation rumors of monetary reform may encourage the belief that prices will not continue to rise rapidly after some months and this widespread belief leads individuals to hold higher amounts of real cash balances in order to avoid certain costs involved in holding lower levels of money balances.

Using a computed probability of reform, they transformed data for the whole sample period and reestimated the Cagan model. Their work showed that "a rising expected rate of inflation conditional on no monetary reform could be consistent with a rising demand for real balances, if the subjective probability of monetary reform is rising and the expected rate of inflation conditional on monetary reform is rising and the expected rate of inflation conditional on monetary reform is sufficiently less than the expected rate of inflation conditional on no reform."[9] They also find that instability of the money demand function at the end of the hyperinflation was reduced somewhat by accounting for the probability of currency reform. However, the weakness of this approach may be that it requires accurate measurement of the unobserved subjective probability of monetary reform. This probability depends partly on Cagan's structural parameter.

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9) A monetary reform is defined to be a change in the class of stochastic process governing the supply of money such as linear in levels, linear in logarithms and linear in the second logarithms of money growth. The question is how agents could form expectations of the timing of monetary reform. For more details, see Flood and Garber (1980).

Ⅲ. Model Specification

As mentioned earlier, Cagan's linear model of money and inflation does not explain the explosive part of the German, Greek, and second Hungarian hyperinflations. These observations were the highest rates of both inflation and money growth and yet showed the rising real money demand. Thus, Cagan's linear regression model tends to over-predict the stock of real balances in the beginning months, but to under-predict in the last turbulent part of hyperinflation. This implies that there is a powerful curvature effect of expected inflation on real cash balances demanded and thus the linear model used by Cagan is probably only a linear approximation of the true nonlinear model of money demand behavior. For this reason, we suggest the hypothesis that, for small variations in variables of interest, this linear approximation may be quite accurate, but for large changes it is likely to be seriously misspecified.

The preceding discussion is of some relevance for the regression model in which the explanatory variable enters in the form of a power function. The reason can be also given by the Taylor's expansion series of $f(\pi)$ which can be written as

$$\frac{M^D}{P} = f(\pi) = f(a) + f'(a)(\pi - a) + \frac{f''(a)}{2!}(\pi - a)^2 + \cdots + \frac{f^q(a)}{q!}(\pi - a)^q + R_{q+1} \qquad (1)$$

where a denotes any fixed number in the domain of π, and R_{q+1} represents the remainder. If q is sufficiently large, the remainder R_{q+1} will be negligible. Therefore, we obtain an approximation of $f(\pi)$ by a suitable choice of q. In view of this, the demand for real cash balances in period t is now assumed to be specified as a semi-log, second-order polynomial function of the expected rate of inflation in the same period.10)

One important type of curvilinear response model is given by

$$\frac{M_t^D}{P_t} = \exp(-\alpha_0 - \alpha_1\pi_t + \alpha_2\pi_t^2) \tag{2}$$

where M_t^D= the quantity of nominal money balances demanded in period t, P_t = the general price level in period t, $\alpha_i (i = 0, 1, 2)$ are exogenously given positive constants.

Equation (2) implies an intrinsically linear money demand function which is nonlinear with respect to expected inflation but linear with respect to the parameters to be estimated. The basic common characteristic of this model is that it can be converted into standard linear model by a suitable transformation of the variables. This specification may call first for a decrease and then an increase in the demand for real balances in respect to increases in the expected rate of inflation.

It will be convenient to transform equation (2) by taking natural logarithms. Adding a stochastic term u_t, we can rewrite equation (2) as

$$m_t^D - p_t = -\alpha_0 - \alpha_1\pi_t + \alpha_2\pi_t^2 + u_t \tag{3}$$

where m_t^D = the natural logarithm of the nominal money balances

10) Let $f(\pi) = \alpha_0 + \alpha_1\pi + \alpha_2\pi^2$ and let $q=2$. Suppose we expand $f(\pi)$ around $\pi = 0$. Then we have $f(0) = \alpha_0$, $f'(0) = (\alpha_1 + 2\alpha_2\pi)_{\pi=0} = \alpha_1$, $f''(0) = 2\alpha_2$ so that $f''(\pi) = \alpha_0 + (\pi - 0)\alpha_1 + \frac{(\pi-0)^2}{2}2\alpha_2 = \alpha_0 + \alpha_1\pi + \alpha_2\pi^2$ which is exactly correct.

demanded, p_t = the natural logarithm of the general price level, π_t = the expected rate of inflation, u_t = a random error term with white noise properties.

If we let $\pi_{1t} = \pi_t$ and $\pi_{2t} = \pi_t^2$ then the semi-log, second-order polynomial monetary equation (3) can be transformed into a linear form as follows,

$$m_t^D - p_t = -\alpha_0 - \alpha_1\pi_{1t} + \alpha_2\pi_{2t} + u_t \qquad (4)$$

Equation (3) illustrates a curvilinear money demand equation in which the response function is quadratic,11) whereas equation (4) looks like a particular case the standard linear model. The regression coefficients in the polynomial regression model are frequently written in a slightly different manner to reflect the pattern of the exponents.

$$m_t^D - p_t = -\alpha_0 - \alpha_1\pi_{1t} + \alpha_{11}\pi_{2t} + u_t \qquad (5)$$

where the regression coefficient α_1 is often called the linear effect coefficient, while α_{11} is called the curvature effect coefficient.12)

We now assume that the money market clears at each moment in time.

$$m_t - p_t = m_t^D - p_t \qquad (6)$$

where $m_t^S = m_t$, the natural logarithm of the nominal money

11) A quadratic form specification is widely used in economic analysis. For example, a quadratic loss function or a quadratic cost function etc..

12) The elasticity of demand for money with respect to expected inflation is $-(\alpha_1 - 2\alpha_{11}\pi_t)\pi_t$.

supply in period t which is determined directly by government monetary policy.

Substituting equation (6) into equation (5), we have a simple stochastic, quadratic money demand function that incorporates a curvature effect term that could have been overlooked from Cagan's linear money demand model.

$$m_t - p_t = -\alpha_0 - \alpha_1 \pi_{1t} + \alpha_{11} \pi_{2t} + u_t \qquad (7)$$
$$= -\delta - \alpha \pi_t + \gamma \pi_t^2 + u_t$$

were $\delta = \alpha_0$, $\alpha = \alpha_1$, $\gamma = \alpha_{11}$.

In order to complete the model, the expected rate of inflation π_t is assumed to be generated by an adaptive expectations mechanism introduced by Cagan. It can be specified in a continuous-time case as follows.

$$\dot{\pi}(t) = \beta [\chi(t) - \pi(t)], \quad \beta > 0 \qquad (8)$$

or, alternatively, in a discrete-time case,

$$\pi_{t+1} = \beta \chi_t + (1-\beta)\pi_t = \sum_{i=0}^{\infty} \beta (1-\beta)^i \chi_{t-i}, \; 0 \leq \beta \leq 1 \qquad (9)$$

where $\chi(t)$ denotes the current actual rate of inflation, β represents the constant coefficient of expectation, $d\pi/dt = \dot{\pi}$.

Equation (8) states that if actual inflation is revealed to exceed what was anticipated, expectations about the future rate of inflation are adjusted upwards; conversely, if the actual rate of inflation falls short of what was expected, the expected rate of inflation is adjusted downwards. The coefficient of expectations adjustment β implies how fast expectations of future inflation adjust to new information.[13]

By substituting equation (9) into equation (7), we complete the semi-log, nonlinear monetary model of hyperinflation. This quadratic form of money demand function will be empirically estimated in the section 5. As this is usually estimated by time series methods, the coefficients of α_1 and α_{11} are, in general, not identified econometrically. Often, the ad hoc identifying restriction, $\sum_{i=0}^{\infty}\beta(1-\beta)^i = 1$ has been assumed. Since the optimal predictor for many economic variables does not follow that restriction, the smoothness impositions may be employed instead.[14] This procedure is, however, far from mechanical. In working with this problem, we employ a two-step procedure. In the first step, we generate proxies for the π_t series corresponding to any given values of β. Using these proxies in the second step, we pick the estimates of the parameters that obtain the highest value of the coefficient of determination (R^2). Since β is unknown, this search procedure would yield OLS estimates for the parameters that have asymptotically maximum likelihood properties.

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13) In the discrete-time case, the coefficient will, in general, fall short of unity. If this is case, current information will be partially discounted in adjusting expectations. In other words, since β is a fraction, agents will always be lagging, i.e., if we assume that inflation will be continually increasing, then systematic errors exist. As β approaches 0, information about recent inflation is increasingly ignored in revising expectations. Then expectations formulation is static expectations ($\pi_{t+1}=\pi_t$). While if β=1, it becomes regressive expectations ($\pi_{t+1}=\chi_t$).

14) Each π_t can be computed by the following form of formula, $\pi_t = \sum_{i=0}^{\infty}\beta(1-\beta)^i x_{t-1}$. In this procedure, an appropriate lag i is set such that $\beta(1-\beta)^i < 0.0005$. This is slightly different from Cagan's procedure (see Cagan, 1956, p.39), but does not change any of the qualitative results.

Ⅳ. The Short-run Equilibrium Analysis

All markets are assumed to be clear at each moment in time so that price changes are determined exclusively by the money market equilibrium condition. The assumption of the rapid adjustment of prices to disequilibrium is reasonable for a period of hyperinflation. It is well recognized that hyperinflation is fundamentally a monetary phenomenon in the sense that hyperinflations cannot be sustained without continuously accelerating money growth.[15)]

The main model developed in the previous section will be rewritten in continuous-time as follows.

$$m^D(t) - p(t) = -\delta - \alpha\pi(t) + \gamma\pi^2(t) \qquad (5')$$

$$m(t) - p(t) = m^D(t) - p(t) \qquad (6')$$

The short-run equilibrium is defined as equilibrium in the money market with given expectations about future rates of inflation. Substituting equation (6′) into equation (5′) and differentiating with respect to time yields,

$$\mu(t) - \chi(t) = -\alpha\dot{\pi}(t) + 2\gamma\pi(t)\dot{\pi}(t) \qquad (10)$$

where $\mu(t) = dm/dt$, $\chi(t) = dp/dt$, and $\dot{\pi}(t) = d\pi/dt$.
Solving for $\chi(t)$, one obtains

$$\chi(t) = \mu(t) + [\alpha - 2\gamma\pi(t)]\dot{\pi}(t) \qquad (11)$$

In order to eliminate $\dot{\pi}(t)$ from equation (11), we substitute

15) with real output given at full employment.

equation (8) into equation (11). Then we have the short-run equilibrium rate of inflation.

$$\chi(t) = \mu(t) + [\alpha\beta - 2\beta\gamma\pi(t)][\chi(t) - \pi(t)] \tag{12}$$

or, equivalently,

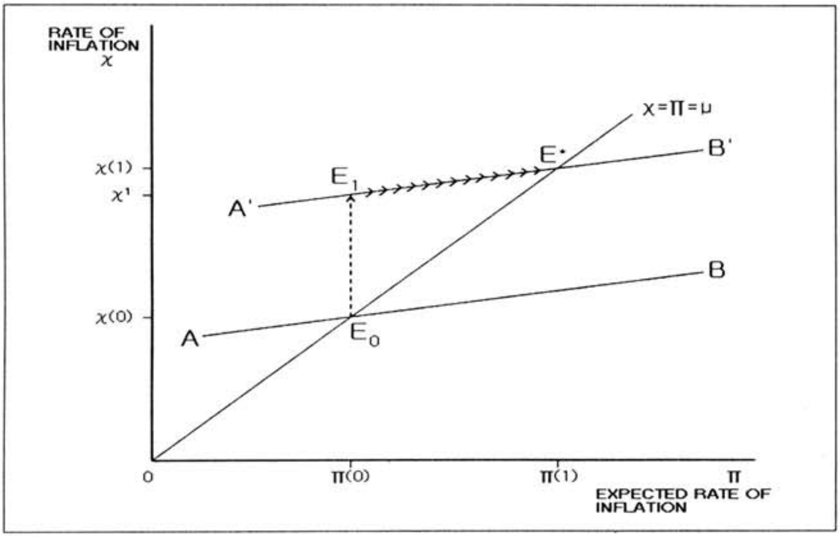
$$\chi(t) = \frac{\mu(t) - [\alpha\beta - 2\beta\gamma\pi(t)]\pi(t)}{1 - \alpha\beta + 2\beta\gamma\pi(t)} \tag{12'}$$

Equation (12′) states that the rate of inflation is determined by the exogenous money growth rate and by expectations of the future inflation which are predetermined at each moment.

Suppose that the rate of growth of the money supply is doubled at $t = t_0$. What would happen to the short-run rate of inflation? This is illustrated in Figure 1 as an upward shift of the AB schedule. To see this, we assume that the economy is initially in equilibrium characterized by the initial condition, $\mu(0) = \chi(0) = \pi(0)$, at a point E_0, E^* is also assumed to be a long-run equilibrium point or a steady state solution. Since the adaptive expectations hypothesis implies that $\pi(t)$ is pinned down already by the history of inflation, there will not be any movement in expectations of the future inflation at the moment, $t = t_0$ at which the rate of money growth is actually doubled.

Thus, with expectations given, actual inflation must jump instantaneously by $2\mu(0)/[1 - \alpha\beta + 2\beta\gamma\pi(0)]$, i.e., to a point E_1 so as to keep the money market in equilibrium at the moment. The degree of an instantaneous jump now depends both upon the parameters, α, γ, β, and the initial condition of the expectations $\pi(0)$.

【Figure 1】 Dynamics of adjustment to an increase in the rate of money growth (when $2\gamma\pi > \alpha$)



In particular, the initial expectations $\pi(0)$ play a critical role in dampening the inflation rate overshooting after the rate of money growth is altered. In contrast to the Cagan model, a one-shot increase in the rate of money growth has a different influence on the short-run rate of inflation, depending upon the extent to what expectations of inflation are predetermined. In Cagan's model, inflation rate overshooting depends only on the magnitude of $1/(1-\alpha\beta)$ which is greater than unity.

This can be easily established by differentiating equation (12) with respect to time,

$$\frac{d\chi}{dt} = \frac{\dfrac{d\mu}{dt} - [\alpha\beta - 2\beta\gamma\pi + 2\beta\gamma(\chi - \pi)]\dfrac{d\pi}{dt}}{1 - \alpha\beta + 2\beta\gamma\pi} \tag{13}$$

Dividing both sides of equation (13) by $d\mu/dt$, yields

$$\frac{d\chi}{d\mu} = \frac{1 - [\alpha\beta - 2\beta\gamma\pi + 2\beta\gamma(\chi - \pi)]\frac{d\pi}{d\mu}}{1 - \alpha\beta + 2\beta\gamma\pi} \tag{14}$$

with $d\pi/d\mu = 0$ by construction, equation (14) can be rewritten as

$$\frac{d\chi}{d\mu} = \frac{1}{1 - \alpha\beta + 2\beta\gamma\pi} < 1, \quad \text{if } \pi > \frac{\alpha}{2\gamma} \tag{15}$$

In general, the inflation rate overshooting appears again in a formulation involving the quadratic form demand function for money. For π extremely high, however, it does not occur. Thus, equation (15) implies that the impact effect of a rise in the rate of monetary expansion by $d\mu$ raises the rate of inflation by less than $d\mu$ in the short run.[16)]

Dividing equation (13) by $d\pi/dt$ yields

$$\frac{d\chi}{d\pi} = \frac{\frac{d\mu}{d\pi} - [\alpha\beta - 2\beta\gamma\pi + 2\beta\gamma(\chi - \pi)]}{1 - \alpha\beta + 2\beta\gamma\pi} \tag{16}$$

Equation (16) implies that when $t = t_0 + \Delta$,

$$\frac{d\chi}{d\pi} = 1 - \frac{1 + 2\beta\gamma(\chi - \pi)}{1 - \alpha\beta + 2\beta\gamma\pi} \tag{17}$$

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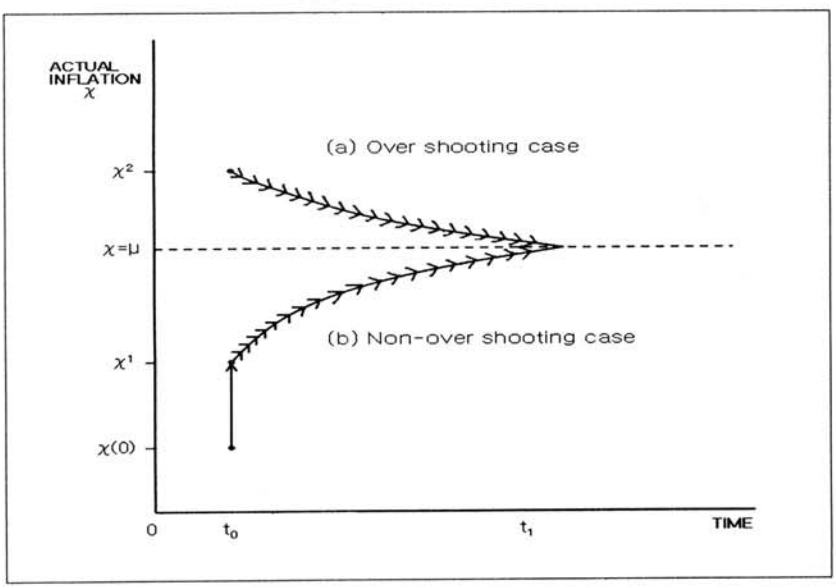
16) This implication is also established by Frenkel (1975) in which the demand for money depends upon the short-run expected rate of inflation which is, in turn, influenced by two elements given by $\dot{\pi}_n = \gamma(\chi - \pi_n)$, $\gamma > 0$, $\dot{\pi} = \delta(\pi_n - \chi) + \beta(\chi - \pi)$, $\delta > \beta$,
where π_n = expectations on the long-run average rate of inflation, π = the expected short-run rate of inflation.
Solving for χ_t yields $\chi_t = 1/a[\mu(1) + \alpha\delta\pi_n - \alpha\beta\pi]$ where $a = 1 + \alpha(\delta - \beta) > 1$. Thus, an arbitrary restriction of $\delta > \beta$ is necessary to ensure that $d\chi/d\mu = 1/[1 + \alpha(\delta - \beta)] < 1$.

Note that the short-run schedule has a positive slope which is smaller than unity unless the second term on the right-hand side of equation (16) is greater than unity. Since $\mu(1)=\chi(1)=\pi(1)$ is a new steady state solution, it is straightforward to show from equation (16) that the long-run slope is unity. From a point E_1 expectations of inflation are revised upwards over time until the economy arrives at the new steady state represented by the $\mu(1)=\chi(1)=\pi(1)$. Following the initial jump, actual inflation will continue to accelerate as expectations of inflation change. In such a situation, real values of the money stock will rise initially and then decline monotonically. It is a special case that the behavior of real balances is consistent with empirical evidence of the short-run effects of an acceleration of the monetary growth rate.17)

【Figure 2】 Time path of inflation after a once-and-for-all increase in the rate of money growth



17) In the Cagan model, after inflation rate overshooting, actual inflation will continue to decelerate toward a new steady state. Therefore, real values of the money balances will call first for a decrease and then an increase. This is a strange implication.

Cagan's empirical work revealed that $\alpha\beta < 1$.[18] This implies that the rate of inflation will eventually equal the rate of increase of the money supply and that the possibility of a self-perpetuating inflation does not occur in real world. Nonetheless, Cagan's theoretical model does not exclude the possibility that $\alpha\beta > 1$. In this respect, Goldman (1972) points out a paradoxical implication of Cagan's model in the sense that an increase in the rate of growth of the money supply calls for a decrease rather than an increase in the rate of inflation. This implication can be easily illustrated from equation (15) with $d\pi/d\mu = 0$ by construction in the short run and $\gamma = 0$.

$$\frac{d\chi}{d\mu} = \frac{1}{1-\alpha\beta} \tag{18}$$

which is always negative if $\alpha\beta > 1$. Only in the short-run analysis does it say that the actual rate of inflation is inversely related to the rate of growth of monetary expansion. In contrast, it is clear that, in equation (17), $d\chi/d\mu$ is always positive in a hyperinflation even if $1-\alpha\beta < 0$. Thus, the presence of nonlinearity in the portfolio balance schedule can overcome the paradoxical short-run implication of Cagan's model.[19]

V. The Long-run Equilibrium and Dynamic Stability

As individual agents start to discount their new information about the time path of inflation, the long-run equilibrium rate of

18) This result is in accord with studies of Sargent and Wallace (1973a), Sargent (1977), Khan (1975), Frenkel (1977), and Flood and Garber (1980).

19) Dropping the assumption that the money market is always in equilibrium, such a paradoxical implication of Cagan's model does not occur. For more details, see Goldman (1972).

inflation will be doubled at a point E^* if and only if the dynamic stability condition to be discussed below is (locally) satisfied. The time path toward the long run equilibrium is characterized by the arrow along the line $E_1 - E^*$. This is usually justified on the grounds that it yields a unique solution path for the present model along which the expected rate of inflation depends exclusively on past values of inflation.

The differential equation for $\pi(t)$ can be obtained by substituting equation (11) into equation (8) to get

$$\dot{\pi}(t) = \beta\{\mu + [\alpha - 2\gamma\pi(t)]\,\dot{\pi}(t) - \pi(t)\} \tag{19}$$

Rearranging the terms, we get

$$\dot{\pi}(t) = \frac{\beta(\mu - \pi)}{1 - \alpha\beta + 2\beta\gamma\pi} \tag{20}$$

From equations (8) and (20), a steady state for expectations occurs when $\dot{\pi}(t) = 0$. Given the rate of growth of the money supply μ it is easy to establish that the only steady state occurs when $\mu = \chi = \pi$. Thus, the long-run equilibrium will be reached at a point E^* represented by $\mu(1) = \chi(1) = \pi(1)$. Equation (20) can be slightly modified in such a way to get,

$$\frac{d\pi}{dt}(a + b\pi) = \beta(\mu - \pi) \tag{21}$$

where $a = 1 - \alpha\beta$, and $b = 2\beta\gamma$.

Dividing and multiplying both sides of equation (21) by $\beta(\mu - \pi)$ and dt, respectively, one obtains

$$d\pi\left\{\frac{a+b\pi}{\beta(\mu-\pi)}\right\}=dt \tag{22}$$

Taking an integral of both sides of equation (22), we arrive at

$$\frac{1}{\beta}\int\frac{a+b\pi}{\mu-\pi}\,d\pi=\int dt \tag{23}$$

Using the fact that $\int dt = t +$ constant(=0) and that $(a+b\pi)/(\mu-\pi)=-b+(a+b\mu)/(\mu-\pi)$, equation (23) can be written as

$$-\frac{1}{\beta}\int b\,d\pi+\frac{1}{\beta}\int\frac{a+b\mu}{\mu-\pi}\,d\pi=\int dt \tag{24}$$

Solving equation (24) reveals,

$$-\frac{b}{\beta}\pi(t)-\frac{a+b\mu}{\beta}\log[\pi(t)-\mu]=t \tag{25}$$

Solving for $\pi(t)$, we obtain

$$\pi(t)=\mu+\exp\left[-\frac{2\beta\gamma\pi(t)+\beta t}{1-\alpha\beta+2\beta\gamma\mu}\right] \tag{26}$$

Since $\pi(t)$ appears on both the left-hand and right-hand sides of equation (26), we cannot solve for $\pi(t)$ explicitly. However, the dynamics of the model can be studied with the aid of the phase diagram depicted in Figure 3. It is quite straightforward to draw the AB schedule whose slope is negative in the following way.

Differentiation of equation (20) with respect to time reveals

$$\frac{d\dot{\pi}}{dt} = \frac{\beta\, d\mu/dt}{1-\alpha\beta+2\beta\gamma\pi} - \frac{\beta(1-\alpha\beta+2\beta\gamma\mu)\, d\pi/dt}{(1-\alpha\beta+2\beta\gamma\pi)^2} \tag{27}$$

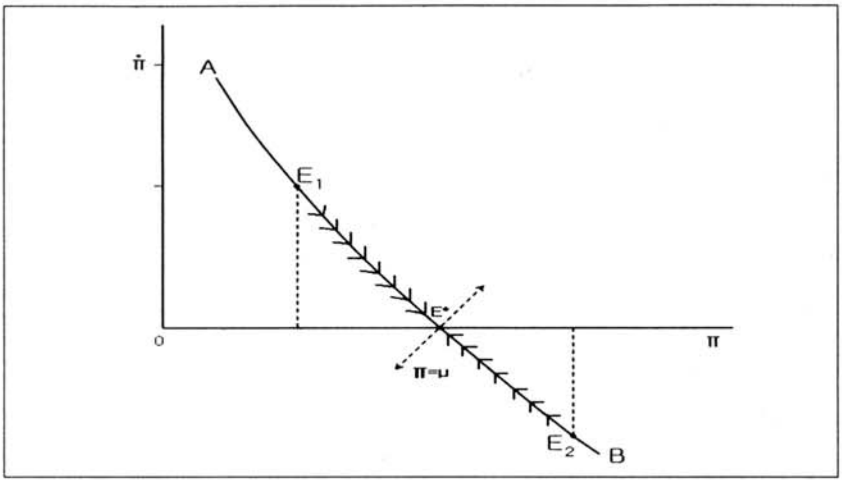
Dividing both sides of equation (27) by $d\pi/dt$ yields

$$\frac{d\dot{\pi}}{d\pi} = -\frac{\beta(1-\alpha\beta+2\beta\gamma\mu)}{(1-\alpha\beta+2\beta\gamma\pi)^2} < 0 \tag{28}$$

since $d\mu/d\pi = 0$ by the assumption that there is only a once-and-for-all increase in the rate of money growth.

This means that the sign of equation (28) depends in an essential manner upon the sign of $1-\alpha\beta+2\beta\gamma\mu$. Whether or not $1-\alpha\beta+2\beta\gamma\mu$ is positive plays a uniquely important role in studying the stability of equilibrium points and also the local behavior of the solution. Thus, the equilibrium is stable for $1-\alpha\beta+2\beta\gamma\mu > 0$ and unstable for $1-\alpha\beta+2\beta\gamma\mu < 0$. Notice that the sign of $1-\alpha\beta+2\beta\gamma\mu$ in equation (28) coincides with that of $1-\alpha\beta+2\beta\gamma\pi$ in equation (20) because μ and π move in the same direction. As can be seen, the derived stability condition is less stringent than that of Cagan's model. It now becomes dependent upon the rate of growth of the money supply. Using the phase diagram, if the economy starts with $\mu > \pi$, then expectations of inflation continue to accelerate along the line $E_1 - E^*$ until $\mu = \pi$. Conversely, if initially $\mu < \pi$, then the expected rate of inflation continues to decelerate along the arrow line $E_2 - E^*$. However, if $1-\alpha\beta+2\beta\gamma\mu < 0$ then $d\dot{\pi}/d\pi > 0$. In this case the equilibrium is unstable. Expectations of inflation will continue to accelerate and thus lead to a reduction in the demand for money. These amounts of money go into the creation of demand for goods which, in turn, reinforce a further inflation. The process continues explosively.

【Figure 3】 Stability of the Long-run Equilibrium



Given the condition $1-\alpha\beta+2\beta\gamma\mu$, there is no question about the stability of the monetary equilibrium in the case of either $1-\alpha\beta>0$ or $1-\alpha\beta<0$ with small values. But for γ or μ sufficiently small, $1-\alpha\beta+2\beta\gamma\mu$ has the sign of $1-\alpha\beta$, the same as that in Cagan's linear model; otherwise, its sign is clearly positive. In the special case of $\alpha\beta>1$ with large values, increasing the rate of monetary expansion may rule out a self-perpetuating inflation.[20] Therefore, when a nonlinearity factor is present into the portfolio balance schedule, the system has been shown to remain more likely stable. However, it is important to note that this dynamic analysis must be partial because it did not deal with some important issues about multiple equilibria inherent in models with money.

20) Using an alternative form of price adjustment, Goldman (1972) also shows the same result when money illusion is present into his model.

Ⅵ. Empirical Results

We estimated the parameters of the curvilinear money demand function in equation (7) using the maximum likelihood method on Cagan's data. The *OLS* estimates of the parameters, except for Greece, are characterized by a serious problem of autocorrelation. Thus, we need to eliminate serial correlation from the residuals since the standard test statistics assume serially uncorrelated error terms. To correct for this problem, we employed the maximum likelihood method with adjustment for autocorrelation. Table 1 gives the results of maximum likelihood estimation method used here for hypothesis testing.

【Table 1】 Polynomial Money Demand Equation

Country	δ	α	γ	γ_1	β	ρ	R^2	DW
Germany[a] (DF=35)	-1.247 (4.88)	-2.732 (24.15)	1.463 (28.33)	-	0.35	-0.986 (31.31)	0.959	1.788
Greece[b] (DF=20)	-1.257 (65.29)	-3.786 (28.03)	1.686 (14.57)	-	0.15	-	0.926	1.583
Hungary[c] (DF=8)	-2.091 (44.59)	-3.303 (13.86)	2.697 (11.14)	-0.600 (11.11)	0.15	0.943 (11.00)	0.994	2.095

Note: The numbers in parenthesis below the coefficients are the absolute values of t-statistic.

a) Maximum likelihood estimation with adjustment for a first-order autocorrelation over the monthly period September 1921~November 1923.

b) OLS estimation over the monthly period January 1921~November 1922.

c) Maximum likelihood estimation with adjustment for a fifth-order autocorrelation over the monthly period July 1945~July 1946.

In each country, the coefficients of π and π^2 have the hypothesized sign. The t-statistics for all coefficients are all significant beyond the 99.9% confidence level. As shown in Tables 3 and 4, the residuals obtained from the estimated equation do not show any significant detectable pattern of serial correlation or heteroscedasticity. The results obtained provide empirical support for the curvilinear money demand behavior which was developed in Section 2. Table 2 shows that the estimated regression coefficients generally satisfy the dynamic stability condition derived in the previous section. Thus, there remains little evidence of self-perpetuating inflation even if the turbulent part of the hyperinflation is included.

【Table 2】 Estimates of the Coefficients and the 95% Confidence Intervals of Stability Condition

Coefficients	Germany			Greece			Hungary2		
	Est.	Confi.	Int.	Est.	Confi.	Int.	Est.	Confi.	Int.
$\hat{\alpha}$	2.73	2.51	-2.95	3.79	2.79	-4.79	3.30	2.83	-3.77
$\hat{\gamma}$	1.46	1.36	-1.56	1.69	0.53	-2.85	2.70	2.23	-3.17
$\hat{\gamma_1}$							0.60	0.49	-0.71
$\hat{\beta}$	0.35	0.30	-0.40	0.15	0.10	-0.30	0.15	0.10	-0.30
$\hat{\alpha}\hat{\beta}$	0.96	0.89	-1.00	0.57	0.48	-0.84	0.50	0.38	-0.85
$2\hat{\gamma}\hat{\beta}$	1.02	0.94	-1.09	0.51	0.32	-0.57	0.40	0.63	-1.34

First, for all hyperinflations the curvature effect coefficient γ is, as expected, positive and appears to be sizable and significant at the 99.9% confidence level. In the case of the second Hungarian hyperinflation, the cubic term coefficient also appears significantly. Secondly, the model explains about 96% for Germany, around 93% for Greece and about 99% for Hungary2, of the total variation in the real balances. Thirdly, introducing a curvature effect into Cagan's linear model, the Durbin-Watson statistic is also significantly improved after adjusting for the first-order autocorrelation. It does not indicate the presence of any higher-order serial correlation.

In addition, the Box and Pierce Q-statistic for the first 12 autocorrelations, which is approximately chi-square distributed, has a value of 5.76, 8.46 and 7.63 for Germany, Greece and Hungary2, respectively. The reported Q statistics are consistent with the null hypothesis that the residuals are not serially correlated. The probability of the null hypothesis is 0.928, 0.747 and 0.813, respectively. This means that there is a 93% chance for Germany, a 75% chance for Greece, a 81% chance for Hungary2 that the residuals come from a white noise process. As shown in Table 4, the Breush-Pagan test for heteroscedasticity suggests that there is little indication of non-homogeneous error variance in the model. However, Greece may not be free of a heteroscedasticity problem.

In light of these results we conclude that a linear approximation will be seriously misspecified for the entire sample period. Here our estimate of the linear effect coefficient α is greater than that of the linear regression model for all cases. Therefore, it is quite clear that the linear regression estimator of the important structural parameter α is not only biased toward zero but also inconsistent.

【Table 3】 Autocorrelation Check for White Noise

Country	To Lag	Chi-Square	DF	Prob.	Autocorrelations					
Germany	6	2.99	6	0.818	-.045	0.223	0.045	0.066	0.097	0.007
	12	5.76	12	0.928	0.152	-.037	0.128	-.031	0.062	-.082
	18	8.72	18	0.966	-.011	-.061	-.117	-.082	-.130	-.037
	24	13.76	24	0.952	-.136	-.062	0.007	-.029	-.060	-.161
Greece	6	2.28	6	0.892	-.145	0.117	-.009	0.063	-.143	-.131
	12	8.46	12	0.747	-.306	-.205	-.146	-.003	-.076	0.033
	18	8.98	18	0.960	0.024	0.045	0.019	0.040	0.027	0.032
Hungary2	6	5.04	6	0.539	0.418	0.013	-.133	-.185	-.190	-.090
	12	7.63	12	0.813	-.024	0.015	-.057	-.159	-.070	-.038

[Table 4] Test for Heteroscedasticity

Germany	Greece	Hungary2
$x^2(2)^a = 5.90^*$	$x^2(2)^a = 24.61$	$x^2(3)^{aa} = 2.97^{***}$
$x^2(1)^b = 4.51^*$	$x^2(1)^b = 24.48$	$x^2(2)^a = 1.11^{***}$
$x^2(1)^c = 2.76^{**}$	$x^2(1)^c = 21.16$	$x^2(1)^b = 0.28^{***}$

* significant at the 1 percent level.
** significant at the 5 percent level.
*** significant at the 25 percent level.
$a_z = (1\ \pi\ \pi^2)$, $aa_z = (1\ \pi\ \pi^2\ \pi^3)$, $b_z = (1\ \pi)$, and $c_z = (1\ \pi^2)$.

Note: The Breush-Pagan test for heteroscedasticity is as follows:

$$\hat{u}_t^2/\bar{\sigma}^2 = z_t'\alpha + v_t,$$

where $\hat{u}_t^2$ = the tth OLS residual from equation (7), $\bar{\sigma}^2 \int_{t=1}^{T} \hat{u}_t^2/T$, z_t = a vector of the independent variables in equation (7). Since the first element of z is unity, the null hypothesis of homoscedasticity $\alpha^* = (\alpha_2, \alpha_3 \cdots \alpha_p) = 0$ can be tested, if u_t is normally distributed.

Let $y_t = \hat{u}_t^2/\bar{\sigma}^2$, $\bar{y} = \Sigma y_t / T$, and $\hat{y}_t = z_t'\hat{\alpha}$. Then RSS $= \Sigma_{t=1}^{T}(\hat{y}_t - \bar{y})^2$.

Breush and Pagan test shows, if $\alpha^* = 0$, then (1/2) RSS is asymptotically chi-square distributed with the degrees of freedom (p-1).

[Table 5] Comparisons of Money Demand Regression Models in Germany

Model	$\hat{\delta}$	$\hat{\alpha}$	$\hat{\gamma}$	R^2	DW	SSR
Nonlinear Model (Noh)	-1.2475 (-15.58)	-2.7321 (-21.93)	1.4627 (23.49)	0.959	1.788	0.0401
Cagan	-1.3421 (-20.50)	-0.9851 (-5.871)	-	0.482	0.342	4.7349
Flood & Garber	-2.6555 (-22.56)	-4.016 (-11.07)	-	0.768	1.855	0.5509

Note: t-values are in the parentheses below the coefficients.

Table 5 compares Flood and Garber's result with ours. Our model explained about 96% of the total variation in the real money balances, while around 77% is explained by Flood and Garber's specification. In other words, the sum of squared residuals obtained from the Flood and Garber regression equation seems to be 14 times that of our regression model. As to the DW statistic, they are similar. Our estimate of α turns out to be lower than their estimate. These results imply that a quadratic form specification could be justified as an approximation to the true nonlinear money demand function for the entire sample period. Thus, the powerful curvature effect plays an important role in explaining higher real money balances demanded in the face of the explosive inflation that occurred at the end of hyperinflation.

Ⅵ. Concluding Remarks

All previous studies, with the exception of Flood and Garber (1980), have been forced to truncate from the data the turbulent observations at the last part of the German hyperinflation. This paper has attempted to extend the estimation to fit the entire sample period using a quadratic form specification. Thus, our focus was to investigate the two effects of expected inflation on demand for real balances over the entire sample period: both linear and curvature effects. We note that the curvilinear effect might be associated with Keynes' precautionary motive under uncertainty of future monetary policy. Since this idea is not derived explicitly from a choice theoretic model, its justification is empirical.

The proposed model was shown to be consistent with the data covering the entire sample period. As inflation evolves rapidly, the linear effect reduces individuals' holdings of real balances, meanwhile the curvature effect increases their holdings. This effect tends partly to offset the primary negative effect of expected inflation on the demand for money. It is found that the overall effects provide alternative evidence to account for the money demand events during the last explosive part of the hyperinflation. It is thus concluded that, for large variation in the independent variables, agents' demand function for money is not linear but quadratic. The coefficient of expectations β is estimated as a value of 0.35 in the German case. Although this value is slightly higher than Cagan's estimate, an interesting result is that inflationary expectations were shown to be sluggish in that period.

The stability condition derived from the proposed model would seem to be less stringent than Cagan's condition. It may become dependent upon the monetary growth rate. For example, when the rate of money growth is sufficiently low, the condition will become close to Cagan's. In the special case that $\alpha\beta > 1$ with large values, increasing the rate of monetary expansion would rule out the possibility that inflation becomes an ever-increasing process. This may be a strange implication of our analysis. It is observed that the estimated regression coefficients indicate a stable demand function for money in the sense that it does not result in self-perpetuating inflations.

In the short-run analysis, an important implication is that the degree of the discrete jump of inflation depends in an essential manner on the initial condition of expectations. Thus, a one-shot increase in the rate of money growth has a different influence on the short-run rate of inflation. Whereas in Cagan's model, the degree of jumps of inflation rests only upon the hypothetical values of α and β. With $\alpha\beta < 1$, short-run inflation overshoots in respect to an increase in the rate of monetary expansion. However, our model shows that both are plausible. For example, inflation does not overshoot in the case of sufficiently high money growth rate, but inflation, in general, overshoots as in the Cagan's model. For a non-overshooting case, inflation is linked positively to inflationary expectations. Thus, real values of the money balances will first increase and then decline monotonically after an acceleration of the monetary growth rate. The model shows that the behavior of real balances can be consistent with empirical evidence of the short-run effects of an increase in the rate of money growth. The model also overcomes the paradoxical short-run implication of Cagan's model in which money growth and inflation can be negatively related as long as the stability condition is not met.

Finally, there remains the possibility that multiple equilibria of inflation or the price level and real balances exist owing to the quadratic form of money demand function. This issue, which is important in models with money, will have to be investigated in future work.

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비선형 화폐수요함수의 동태적 안정성

노 인 철*

논문초록

Cagan (1956)의 초인플레이션에 관한 연구는 유럽의 초인플레이션 기간 중 마지막 기간을 제외하여 화폐수요함수를 추정했으며, 마지막 기간 무렵에 화폐의 실질 잔고가 급격히 증가된 현상을 설명하지 못한다는 한계점을 드러냈다.

Cagan의 화폐에 대한 선형 수요함수는 실제에 대한 Taylor 1차 전개 (First order Taylor series approximation)에 의해 정당화될 수 있다. 이 때 독립변수(기대 인플레이션)의 변동이 크지 않는 경우에는 적절하지만 초인플레이션 하에서는 적합하지 않는다. 따라서 기대 인플레이션의 Taylor 2차 전개(기대 인플레이션의 자승)를 계산하여 Cagan의 화폐수요함수에 추가 설명변수로 포함해서 추정을 시도하였다. 추정한 결과는 비선형 변수가 통계적으로 유의한 것으로 확인되었고, 또한 수정된 화폐수요모형의 설명력이 훨씬 더 좋은 것으로 나타 났다.

다음은 비선형 화폐수요함수를 토대로 초인플레이션 상승의 동태적 안정성에 관한 이슈를 다루었다. 수정된 화폐수요모형으로 단기와 장기에서 동태적 안정화 조건이 Cagan의 조건과 어떻게 다른 가를 분석하는 것이다. 분석결과로 Cagan과는 다른 여러 의미 있는 내용(insights)을 제시하였다. 인플레이션의 상승폭이 Cagan의 모형에서는 α, β 값에 의해서 결정되나 수정된 화폐수요모형에서는 α, γ, β, μ 값에 의해 결정된다.

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