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⟨Teacher's Corner⟩

Quizzes and Communications

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Question 1. "No two tigers can coexist forever, A duopoly will eventually collapse to a monopoly."

Formulate the above claim into a stochastic model.

Answer,

Suppose: duopolists A and B produce perfectly homogeneous products; there are N customers; at each and every purchase, each customer chooses A or B with the equal probability of $\frac{1}{2}$, and during each and every purchasing round each and every customer makes a purchase; as soon as any of the duopolists loses all his customers, the duopoly collapses to a monopoly, and hence each of the duopolists must retain at least one customer in order to preserve a foothold in the market.

Then, at the end of the first round, the probability that both of the duopolists retain at least one customer, that is, the probability that the duopoly survives, is $1-2\times\frac{1}{2^N}$, at the end of the second round the probability of the duopoly's survival is $\left(1-2\times\frac{1}{2^N}\right)^2$, ⋯, and at the

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end of the t^{th} round the probability is $\left(1-2\times\frac{1}{2^N}\right)^t$, $\lim_{t\to\infty}\left(1-2\times\frac{1}{2^N}\right)^t = 0$.

Question 2. Professors A and B are on such bad terms that each of them is provoked when he sees the other, and laments his bad luck. Oddly they are equally particular about food, and they patronize the same restaurants in the vicinity. They are six: American, Chinese, French, Italian, Japanese, and Korean. The two gentlemen come to school six days a week, and visit the restaurants for lunch, of course, but never together. Strangely the two gentlemen's patterns of patronage are exactly the same: they never visit the same restaurant consecutively; they visit one of the remaining restaurants totally at random.

What is the probability that the unpleasant encounter occurs during a lunch time?

Answer,

A's patronage is formulated into a Markov chain of which the transition probabilities can be described as follows:

	1	2	3	4	5	6
American 1	0	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
Chinese 2	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
French 3	$\frac{1}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
Italian 4	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{1}{5}$
Japanese 5	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	0	$\frac{1}{5}$
Korean 6	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	0

So is B's

As A patronizes the six restaurants always equally, the probability that on the previous day he fails in state i, $i=1,2,3,4,5,6$, is $\frac{1}{6}$; so is the probability for B.[1] Hence, the probability that on the previous day both A and B fall in state i is $\left(\frac{1}{6}\right)^2$, and the probability that on the subsequent day the unpleasant encounter occurs is $\left(\frac{1}{6}\right)^2 \times 5$. The probability that on the previous day A falls in state i and B in state j, $i \neq j$, is $\left(\frac{1}{6}\right)^2 \times 5$, and the probability that on the subsequent day the unpleasant encounter occurs is $\left(\frac{1}{6}\right)^2 \times 4$. The required probability is therefore

$$\left(\frac{1}{6}\right)^2 \times \left(\frac{1}{6}\right)^2 \times 5 + \left(\frac{1}{6}\right)^2 \times 5 \times \left(\frac{1}{6}\right)^2 \times 4 \times 6 = \frac{25}{216}.$$

1) Let $X(t)$ be the event that A fails into state X during period t. Then,

$$\begin{aligned} P(X(t)=i) &= P(X(t)=i,\ X(t-1)=j\neq i) + P(X(t)=i,\ X(t-1)=i) \\ &= P(X(t)=i,\ X(t-1)=j\neq i) \\ &= P(X(t)=i \mid X(t-1)=j\neq i)P(X(t-1)=j\neq i) \\ &= \frac{1}{5}\times\frac{5}{6} = \frac{1}{6} \end{aligned}$$