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〈Teacher's Corner〉

A Statistical Interpretation of "Deadly Jump"*

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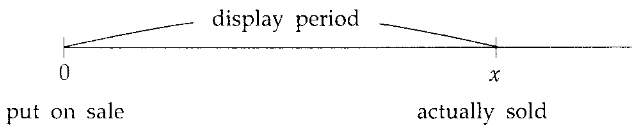
1. Suppose a commodity is put on sale at the commodity's "current" price.[1)] The commodity is supposed to be sold immediately, but may take time to be actually sold. How long? Let X be defined as the waiting time until it is sold as is indicated in the figure below.



X is probably a memoryless random variable, because it can be reasonably assumed that the length of display period does not affect the prospect of the commodity's actual sale. "Deadly Jump" is equivalent to the statement: "The length of display period is very large on average and *accordingly* very volatile."[2)]

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* K. Marx allegedly claims that the transformation of a commodity into money is like a deadly jump.

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1) This means that the market for the product is already established and competitive.

2) The case of a long average display period with mild volatility is hardly a "deadly jump", because uncertainty is somehow reduced; the case of a short average display period with violent volatility can very likely be a "comfortable jump", because the distribution is much skewed to the left; the case of a short

The simplest probability density function $f(x)$ which satisfies the above conditions is :

$$f(x) = \lambda e^{-\lambda x}, \quad x > 0$$
$$= 0, \ otherwise.$$ [3]

Then, X is memoryless, and $\mu_x = \frac{1}{\lambda}$, $\sigma_x^2 = \frac{1}{\lambda^2}$. "Deadly Jump" means very small λ.

2. Suppose a new product is introduced into the market.[4] It can hardly be expected that the product is immediately sold. It may remain unsold in the market for quite a long time. How long? As time goes on, information concerning the product accumulates among potential buyers, and hence the saleability of the product increases up to a certain time point. But beyond it the saleability decreases, because relevant information does not come by readily and the accumulated information begins to lose its effects. Let Y be a time point at which the product is sold, and α be the time point at which the saleability attains its largest value

$$\text{Then } f(y) = \frac{1}{\alpha!} y^{\alpha} e^{-y}, \quad y > 0$$
$$= 0, \ otherwise.$$

is possibly the simplest saleability probability density function

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average display period with mild volatility is : "Viva market economy!"

3) Incidentally, the exponential distribution can be desired from the poisson process. Let X be the first anival time. Then, $P(X > x) = P\{no\ anivals\ in\ (0, x)\} = e^{-\lambda x}$, where λ is the mean anival rate; so $F(x) = P(X < x) = 1 - P(X > x) = 1 - e^{-\lambda x}$, $x > 0$, and hence $f(x) = \lambda e^{-\lambda x}$, $x > 0$. Mood, A. M., F. A. Graybill and D. C. Boes, *Introduction to the Theory of Statistics*, 3rd edition. McGraw Hill, 1974, pp. 121～123.

4) Unlike the case of I., the market emerges with the product.

describing the above situation.[5)]

$mode = \alpha$, $\mu_y = \alpha + 1$, $\sigma_y^2 = \alpha + 1$. Large α means "deadly jump".

5) The gamma distribution is derived based on *mode* without recourse to the exponential distribution. Let $f(y)$ be a probability density function such that

$$f(y) > 0 \text{ for } y > 0$$
$$= 0, \textit{ otherwise}, \text{ and } \textit{mode } y = \alpha$$

What is the simplest form of $f(y)$? Let $h(y) = y^k g(y)$ such that $k > 0$, $g(y) > 0$ and differentiable for $y \geq 0$.

$$\text{Then } \left.\frac{dh(y)}{dy}\right|_{y=\alpha} = \left[ky^{k-1}g(y) + y^k g'(y)\right]\Big|_{y=\alpha}$$
$$= k\alpha^{k-1}g(\alpha) + \alpha^k g'(\alpha) = 0$$

Let $k = \alpha$; then, $k\alpha^{k-1}g(\alpha) + \alpha^k g'(\alpha) = 0$ reduces to a simpler form $g(\alpha) = -g'(\alpha)$. The simplest $g(y)$ satisfying $g(\alpha) = -g'(\alpha)$ is possibly $g(y) = e^{-y}$.

$$\text{Then } h(y) = y^\alpha e^{-y}, \quad y > 0$$
$$= 0, \quad \textit{otherwise},$$

$$\text{and } f(y) = \frac{1}{\alpha!} y^\alpha e^{-y}, \quad y > 0$$
$$= 0, \quad \textit{otherwise}.$$