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[Question 2003 X. 1]의 답

The Proposer's Own Answer

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For A: Let U be the market share for Duopolist Ⅰ;

$$f(u) = 1, \qquad 0 < u < 1$$
$$\quad = 0, \qquad otherwise$$

then $f(u) = 1$, $0<u<1$; $= 0$, *otherwise*

For B : Let V be the market share for Duopolist Ⅱ;
then the simplest $f(v)$ which satisfies B's claim is

$$f(v) = 6v(1-v), \qquad 0 < v < 1$$
$$\quad = 0, \qquad otherwise$$

$f(v)$ is symmetrical around $v = \frac{1}{2}$, and has the single mode at $v = \frac{1}{2}$.

Let X be the purchase of a customer, a random phenomenon such that if he buys from Duopolist Ⅰ, $x=1$; if from Duopolist Ⅱ, $x=0$.

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Then, for A: $f(x|u) = u^x(1-u)^{1-x}, \quad x=1,0$

$$= 0, \quad \text{otherwise}$$

$$f(x) = \int_0^1 f(x|u)f(u)du = \int_0^1 u^x(1-u)^{1-x}\cdot 1\cdot du$$

$$= \frac{1}{2}, \quad x=1,0$$

$$= 0, \quad \text{otherwise}$$

for B: $f(x|v) = v^x(1-v)^{1-x}, \quad x=1,0$

$$= 0, \quad \text{otherwise}$$

$$f(x) = \int_0^1 f(x|v)f(v)dv = \int_0^1 v^x(1-v)^{1-x}\,6v(1-v)dv$$

$$= \frac{1}{2}, \quad x=1,0$$

$$= 0, \quad \text{otherwise}$$

Much fuss about nothing!

Further Remarks.

A bit more sophisticated model is presented. Suppose: there are innumerable potential buyers, but only N buyers actually make purchase during each purchasing period; at the end of each period, the sales records of the duopolists are made public. It is not unreasonable to assume that the potential buyer's preference for Duopolist A or Duopolist B depends on what other people think about A or B ; for instance, if A is very much preferred during the previous period, then he will be very much favored also during the current period.[1] $\frac{X(t-1)}{N}$, where $X(t-1)$ denotes the number of purchases from A during period $t-1$, is presented as a simple quantitative indicator of A's

1) "The bandwagon effect" works, Harvey Leibenstein, *Beyond Economic Man.*, Harvard University Press, 1976, pp. 48~67.

market share for period t; then, B's market share for period t, will be $1 - \frac{X(t-1)}{N}$.

Then, $\{X(t)\}$ becomes a Markov chain. The transition probability $\theta_{ij} = \Pr(X(t) = j \mid X(t-1) = i)$ can be approximated by $\hat{\theta}_{ij}$ as follows:

$$\hat{\theta}_{ij} = \binom{N}{j}\left(\frac{i}{N}\right)^j\left(\frac{N-i}{N}\right)^{N-j}, \quad i = 0, 1, 2, \cdots, N; \; j = 0, 1, 2, 3, \cdots, N$$

The chain contains two absorbing states, state 0 and state N, and $0 < \hat{\theta}_{ij} < 1$ for all $i = 1, 2, \cdots, N-1$; $i = 1, 2, \cdots, N-1$. So, it is possible to go to state 0 or state N from any state directly or indirectly. Hence, every state is eventually absorbed into state 0 or state N,[2)] and the duopoly collapses to a monopoly.

Of course, a duopoly can survive indefinitely. Suppose: there are two neighboring restaurants, A and B; innumerable patrons visit A or B; patron i visits them as follows:

	A	B
A	θ_i	$1 - \theta_i$
B	$1 - \pi_i$	π_i

$0 < \theta_i, \ \pi_i < 1, \ i = 1, 2, 3, \cdots$

As $\lim_{n\to\infty} \prod_{i=1}^{n} \theta_i = 0$, $\lim_{n\to\infty} \prod_{i=1}^{n} \pi_i = 0$, both A and B can retain some patrons. Hence, the duopoly can go on indefinitely.[3)]

2) Kemeny, J. G., Snell, J. H. and G. L. Thompson, *Introduction to Finite Mathematics*, Prentice-Hall, 1957, pp. 326~327.

3) Needless to say: if n is finite, $\prod_{i=1}^{n} \theta_i > 0$, $\prod_{i=1}^{n} \pi_i > 0$, and the duopoly may collapse to a monopoly as is shown in the first model above.