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<강의자료 Lecture Note>

Applications of the Pareto Distribution III

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January 1, 2003 a rabbi solemnly delares a statistician : "Messiah may come at any moment between January 1, 2103 and December 31, 2203, and will found a kingdom. The blissful kingdom will last forever." The statistician sarcastically replies : "Forever? Nothing lasts forever. Any form necessarily breaks down, and any life eventually dies out. Everything lasts only on a probability basis." The rabbi sadly rejoins : "You are an incorrigible pedant. Is statistics a science of blasphemy?" The statistician timidly counters : "No, on the contrary the Bible causes the birth of statistics. Shall I quote Ecclesiastes 9:11? 'I returned, and saw under the sun, that the race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to the men of understanding, nor favour to men of skill; but time and chance happeneth to them all.' Then, one who understands stochastic processes may rule the world."

They part with no malice, and the statistician engages in statistical interpretation of Messiah and his kingdom. Messiah may come at any moment between January 1, 2103 and December 31, 2203. Let Y be Messiah's arrival time.

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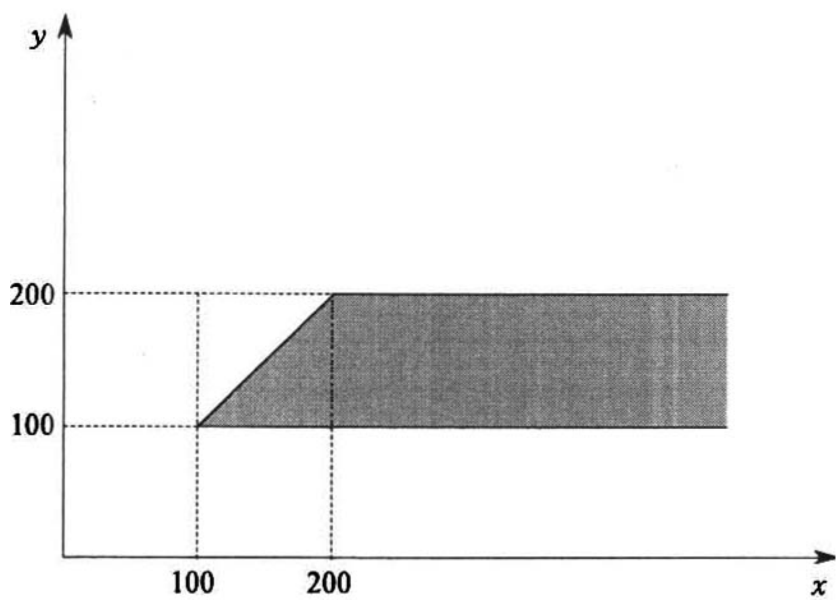
Then

$$f(y) = \frac{1}{100}, \qquad 100 < y < 200$$
$$= 0, \qquad otherwise$$

Let X be the kingdom's death time. Then, on account of the rabbi's claim, $y < x < \infty$, and X is unbounded above, and perhaps so is $E(X)$. Let $f(x \mid y)$ be conditional survival probability density function of the kingdom after Messia arrives. As aging means approaching to death, $f(x \mid y)$ is monotonically decreasing with respect to x. Hence, the simplest form of $f(x \mid y)$ is :

$$f(x \mid y) = \frac{y}{x^2}, \qquad y < x < \infty$$
$$= 0, \qquad otherwise$$

(X, Y) will cover the shaded area in the following figure.



Now we may obtain marginal $f(x)$ of X as follows:

For, $100 < y < x < 200$, $f(x) = \int_{100}^{x} f(x|y)f(y)dy$

$$= \int_{100}^{x} \frac{y}{x^2} \cdot \frac{1}{100} dy = \frac{1}{200} - \frac{50}{x^2};$$

for, $200 < x < \infty$, $f(x) = \int_{100}^{200} f(x|y)f(y)dy$

$$= \int_{100}^{200} \frac{y}{x^2} \cdot \frac{1}{100} dy = \frac{150}{x^2}$$

$$\int_{100}^{200} f(x)dx + \int_{200}^{\infty} f(x)dx = \int_{100}^{200} \left(\frac{1}{200} - \frac{50}{x^2}\right)dx + \int_{200}^{\infty} \frac{150}{x^2} dx$$
$$= \frac{1}{4} + \frac{3}{4} = 1.$$