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A Strategic Model of Product Location Choice by Incumbents to Entry with Application to the U.S. Automobile Industry

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This paper describes a game-theoretic model that suggests why the dominant U.S. firms in the U.S. automobile industry did not respond to Japanese imports by themselves adding product lines close to the product space location of the Japanese imports. Numerical method is applied to find the solution for additional location by incumbents. Result shows that incumbents do not have incentives to add one more location because of reduced profit which is caused by competition effect and expansion effect, in addition to sunk establishment cost for new location.

I. Introduction

Entry in a growing market of Japan during late 1950s and early 1960s changed the incumbents' (two largest firms−Toyota and Nissan) combined market share during this period. But since late 1960s, these two incumbents regained their combined market shares in Japan's domestic market. However, (foreign) entry into the mature, saturated market of the U.S., did not change incumbents' (two largest firms−G.M. and Ford) behaviors as much as that in the Japanese one. It was found that foreign entry (especially from Japan) in the previously unoccupied segment of product space in the U.S. automobile industry did not have a significant impact on

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incumbents' behaviors represented by the absolute growth rate difference of the two largest firms.[1)]

In the U.S., the Japanese imports occupied a segment of the U.S. automobile product space unoccupied by the U.S. firms, and because of the oil crises and high gasoline prices, there was a substantial shift of demand on this segment. In the Japanese domestic market, the middle size firms occupied a position in the Japanese product space, experiencing a relatively long-term increase of their shares than the two dominant firms.

This paper describes a strategic model that suggests why the dominant U.S. firms did not respond to the Japanese imports by themselves adding product lines close to the product space location of the Japanese imports.

Taking into account these facts, it will be shown theoretically how entry into product differentiated and mature industry will disturb pricing and quality location choice behavior of incumbent. This paper will demonstrate how true incumbents' equilibrium choices of pricing and qualities change when an entrant appears and produces a lower quality product than the incumbents in a vertically differentiated product model. More specifically, post-entry response by incumbents will be discussed: Will incumbents plan to stay at the existing location, or move to another location (relocation), or add one more location to existing location?

It seems appropriate to model automobile industry as an oligopoly with product differentiation in both the U.S. and Japan. This industry has been led by the two largest firms in each country (G.M. and Ford in the U.S. automobile industry, Toyota and Nissan in Japan) in production, sales and in Japan, exports.

The basic difference between adding a location and relocation is significant. If firm relocates its quality location, it must consider the huge relocation cost. On the other hand, when a firm adds one more location to its existing locations, it must consider two effects — one is an expansion effect and the other is a competition effect — in addition to the sunk cost for the additional location.

These two effects are carefully analyzed by Sutton [6]. According to Sutton, the market expansion effect measures the extent to which the addition of new products expands total industry sales, prices being fixed. And the competition effect measures

1) This is analyzed in Kim [9]. In the analysis of the role of market share of smaller firms in fluctuations of absolute growth rate difference of the two largest firms, market share of import was used in the U.S. regressions, and the share of the three medium-size Japanese firms in the Japanese regressions. It was not expected that the effect of these competitors to be the same.

the extent to which, given the list of available products, prices are lower when each of these products is owned by a different firm, as opposed to the situation where all products are owned by a monopolist.[2)]

This study will attempt to show how these two effects depend on locations of additional new products and parameters by two previous incumbents when entry occurs in a modified Prescott-Visscher model [3].

Prescott and Visscher deal with sequential entry under the assumption of infinite relocation cost. In their model, once a firm enters the market and locates at the certain point on the product space, they can not relocate, even though market conditions change, since the relocation cost is infinite. They analyzed sequential entry with perfect foresight in a model of single location for each firm (*i.e.*, there are no multiproduct firms). However, in reality, firms respond to changes in market condition through either relocation (even though it takes time and large cost), or by addition of one (or more) location(s) to existing locations with establishment costs that are sunk.

In the automobile industries of the U.S. and Japan, the two largest (or three in the U.S.) firms (incumbents) tried to add one more locations (small-size cars in the U.S. and bigger-size cars in Japan) to existing locations when market conditions changed because of such factors as entry and fuel price changes during the 1970s.

Prescott and Visscher mention that noncooperative equilibrium of the quality choices has an existence problem with the Hotelling solution concept in an example using three firms. That is why they consider firms locating in sequence once and for all. On the other hand, Shaked and Sutton [4] develop a model in which noncooperative equilibrium of quality choices is possible only with two firms possessing both positive market shares and profits. These two firms locate far away from each other to avoid price competition. They also show how the entry of additional firms leads to a configuration in which the top quality product is available at price zero, assuming no production costs, while all firms earn zero profits.

Shaked and Sutton's [4] result depends on the assumption of a very limited diversity of tastes or limited range of income level. They state, "This reflects the fact that competition between the surviving high quality products drives their prices

2) Shaked and Sutton [5] give a slightly different definition. They measure the expansion effect by the increase in monopoly profit. In other words, for the monopolist, the incentive to introduce a new variety is determined by the demand for the new product net of any loss of sales incurred by existing products.

down to a point at which not even the poorest consumer prefers the (excluded) low quality products even at price zero. This number of products reflects *inter alia* the utility functions of consumers and the shape of the income distribution."(Shaked and Sutton [4], p. 12).

In other words, in a particular form of utility function, Shaked and Sutton [4] assume a uniform distribution of incomes on where $[a, b]$ where $2a < b < 4a$; whence their upper bound for number of firms with positive profit was 2. However, they commented that it can be shown that this upper bound rises as the range of income increases.

In this paper, a uniform distribution of tastes is assumed. But b is not restricted to $2a < b < 4a$, as Shaked and Sutton [4]. In reality, upper bound of consumer tastes in passenger car market could be more than 4 times of the lower bound.[3]

What is learned from Prescott and Visscher, and Shaked and Sutton is that firms differentiate products as much as possible to avoid price competition in a differentiated industry. This means that if firms locate near each other, it is strongly possible for the firms to collude in prices in a differentiated industry since, otherwise, they will end up with negative profits resulting from Bertrand competition, because of the fixed costs.

This paper will show how incumbents (actually, the two largest firms) react to the entry through the addition of new locations after an entry occurs. This aspect of analysis could reflect the major events of the U.S. automobile industry during the 1970s and 1980s after two times of oil crisis. Through this model how the incumbents behave, or should behave can be explained.

To find out additional new locations by incumbents, numerical methods will be used. Given numerical values for the existing two locations of incumbents and the entrant's location (x_1, x_2 and x_e), it is possible to find out the numerical solutions of additional two locations of incumbents.[4] Once the numerical values are found for the perfect Nash equilibrium of additional new locations (x_{1e}, x_{2e}) of the incumbents, prices of five products (p_1, p_2, p_{1e}, p_{2e}, p_e), and profits (π_1, π_2, π_e), market shares (s_1, s_2, s_e),[5] and revenues (R_1, R_2, R_e) of two incumbents and one entrant can also be

3) When the variety of passenger cars offered by manufacturers is analyzed, the diverse consumer tastes and income levels can be seen.

4) To make calculation easier and to see how incumbents react after entry occurs, the three locations are given numerical values.

5) Market share of each firm is measured by the number of cars sold.

found.[6]

With these numerical values of perfect Nash equilibria of locations, prices, profits, and revenues of each firm, expansion and competition effects can be analyzed. In addition to these two effects, a "share effect" that is measured by change in each firm's market share is proposed.

These numerical values of perfect Nash equilibria depend on the predetermined values of existing locations of two incumbents and entrant location, in addition to parameter values such as maximum values of consumer's evaluation of quality V, cost parameter c, and consumer density N.

How these numerical values of perfect equilibria change as the predetermined values of locations and parameter values are changed will be explained. This theoretical reasoning will be applied to the U.S. automobile industry in an analysis of how incumbents behaved when there is an entry in the market.

Section II considers specifying assumptions of theoretical model for game, firms, consumers and demand for each firm. Section III discusses perfect Nash equilibrium of two stage game and reports the numerical result of this perfect Nash equilibria of prices and location choices of two incumbents and entrants. Section IV discusses the implications and concludes with overall evaluation of the findings.

II. Assumptions

This model assumes that before entry, there are two firms in the market and these two incumbents produce one product each. After entry, each incumbent chooses to add one more location of product. The assumptions on incumbents, entrants, product space and consumers are as follows:

1. Game

1) Two incumbents before entry
2) One entrant
3) This is a two stage game: Given location of entrant at the lower quality than the given two locations of incumbents, two incumbents choose additional lo-

6) Details of the game will be given in section II and III.

cations in the first stage. In the second stage, two incumbents and entrant compete on prices for 5 products.

2. Firms

1) Product space is one-dimensional in quality x, $x=[0, 1]$.
2) Higher quality firm charges higher price. *i.e.*, if

$$x_1 > x_2 > \cdots > x_n$$

and, then

$$p_1 > p_2 > \cdots > p_n$$

where x_i is quality of the product of firm i and p_i is price of the product i charged by firm i.
3) Firm i chooses which quality of car it will produce and the price it will charge for its product to maximize its profit given other firms' decisions of qualities and prices.
4) Entrant pays a fixed entry cost K which is sunk. K is large enough to make it unprofitable for an entrant to change its location (quality).
5) Addition of one more location by incumbent after entry occurred is possible, but with a sunk cost K.
6) Marginal cost of product i is constant and is increasing in its quality x_i: $c(x_i)$.
7) Each firm has a complete information on the quantity demanded for its product as a function of all the products' qualities and prices in the market.

3. Consumers

1) Each consumer has his/her own evaluation of quality as v. This v is uniformly distributed on the interval $[0, V]$ with density N.
2) Each consumer purchases one unit from the firm that offers the smallest subjective total price.
3) Subjective total price of product i for consumer type v is $p_i - vx_i$ where p_i is the price offered by firm i and v is consumer's subjective evaluation of product quality and x_i is product quality of firm i.
4) Consumers have a complete information on quality specifications and prices of

all the existing firms in the market.

4. Demand for Firm i

Assume

$$x_h > x_i > x_j$$

$$p_h > p_i > p_j$$

then, marginal consumers $-v_{hi}$, $v_{ij}-$will have following equalities:

$$p_h - v_{hi} x_h = p_i - v_{hi} x_i$$

$$p_i - v_{ij} x_i = p_j - v_{ij} x_j$$

So, demand for firm i with x_i and p_i is

$$q_i^d = N[v_{hj} - v_{ij}] = N\left[\frac{p_h - p_i}{x_h - x_i} - \frac{p_i - p_j}{x_i - x_j}\right]$$

It can be seen from q_i^d, that if p_i increases, v_{hi} will be decreased and v_{ij} will be increased, and q_i^d will be decreased. But if p_h and/or p_j increase, then v_{hi} will be increased and v_{ij} will be decreased, and likewise q_i^d will be increased. Hence,

$$\frac{\partial q_i^d}{\partial p_i} < 0, \quad \frac{\partial q_i^d}{\partial p_h} > 0, \quad \frac{\partial q_i^d}{\partial p_j} > 0$$

So, price competition is possible in this model.

Where there are n firms in the industry, quantity demanded for firm i, q_i^d is

$$q_i^d = N\left[\frac{p_h - p_i}{x_h - x_i} - \frac{p_i - p_j}{x_i - x_j}\right]$$

when there is no other firm offering a higher quality then,

$$q_i^d = q_1^d = N\left[V - \frac{p_i - p_j}{x_i - x_j}\right]$$

when there is no other firm offering a lower quality then,

$$q_i^d = q_n^d = N\left[\frac{p_{n-1} - p_n}{x_{n-1} - x_n} - 0\right] = N\left[\frac{p_{n-1} - p_n}{x_{n-1} - x_n}\right]$$

where 0 might be necessary since consumers might use public transportation or used car. Here it is assumed to be 0 just for computational convenience.

Ⅲ. Price and Quality Choices

1. Perfect Nash Equilibrium of Two Stage Equilibrium

In the model, there are two incumbents and one entrant. This model is a two stage noncooperative game. Before game starts, entrant is in the market at the location much lower than the two locations of incumbents. In the first stage, incumbents decide whether to add one more location or not. If the incumbents decide to add one more location, then an additional new location is chosen. In the second stage, after finding out rivals' quality locations, each firm (two incumbents and one entrant) will choose prices for each of its locations.

The solution concept is a subgame-perfect Nash equilibrium. This is a noncooperative equilibrium that maintains noncooperative equilibrium property from any decision point in the game. For example, when incumbents decide to add one more location each to their existing locations, their price strategies are assumed to be Nash equilibrium in the final stage of the price game. To solve subgame-perfect Nash equilibrium, backward induction is employed.

Hence, Nash price strategies are solved in the second stage, assuming quality choices are given. Plugging the Nash price strategies expressed as a function of quality locations of 5 products (two locations by each incumbent and more location by entrant, and parameter values are already given) into profit function, then the profit function is expressed by only quality locations and parameters. From these profit functions, it is possible to obtain the Nash strategy for additional new locations of two incumbents if an additional location is more profitable for a firm than not having it. If we have Nash values of new locations of incumbents, then Nash price, profit, revenue and share by including the new lcoations also can be found.

When incumbents face entrant's price cutting strategy, they must decide whether to stay with previous prices and qualities or reduce their prices to maximize their profits. Firms are basically concerned with profits. So, they are trying to maintain or even improve its profit through another choice variable when facing entries.

There is another choice variable when incumbents face an entrant. It is location choice-here it is called quality choice. When there is an entrant that erodes incumbents' market shares, incumbents can preserve their market share by upgrading their qualities with previous prices. However, quality change means location change. Location change accompanies relocation cost. So, incumbents could consider an additional new location instead of relocation even though new location incurs establishment cost K. In a highly product-differentiated industry, incumbents do not want to lose customers in the existing locations.

By adding one more location to existing location, incumbents can limit entrant's market segment which is assumed to be located at the much lower segment of the market than existing incumbents' locations. Also they can consider expansion effects by new locations, although there are competition effects and establishment cost K.

Since two choice variables—price and quality of product—are availabe, firm's market share and profit is not only affected by its price and the other firms' prices, but also, by its quality and the other firms' qualities. This is why we need to analyze the joint effects of prices and qualities.

In reality, an automobile manufacturer sets a suggested retail price for the dealers, although the final price charged by dealers depends on market conditions. Prices change frequently. However, quality of the car does not change very often. Once the quality of a car is determined, it is not easily changed during the model year. Usually, it undergoes slight change at least once a year.[7] Now, solutions to the question of how incumbents behave in price and location choices in response to given entry location?' become more important.

As shown before, incumbents will decide on pricing and quality locations depending on their payoff functions which in turn depend on demand function and cost function.

In the following sections, the second and the first stage game will be analyzed briefly with the firms' payoff functions to find price and quality choices of incumbents in a noncooperative equilibrium framework depending on several different cost functions.

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7) Here, style change is not assumed as quality location change.

2. Nash Equilibrium in the Second Stage Price Game

In this final stage, each firm i will choose its price p_i to maximize profit given quality location choices of second stage as if firms act only in this stage.

1) Two Products by Two Incumbents only

Before the entrant is included in the analysis, the noncooperative equilibrium of price with given locations of two incumbents (x_1, x_2) when there are only two incumbents will be found.

Then, demand functions for incumbents, firm 1 and firm 2, when $x_1 > x_2$ is assumed and so $p_1 > p_2$ are

$$q_1 = N\left[V - \frac{p_1 - p_2}{x_1 - x_2}\right]$$

$$q_2 = N\left[\frac{p_1 - p_2}{x_1 - x_2}\right]$$

where q_1 and q_2 are demand functions for firm 1 and firm 2; p_1 and p_2 are prices for products of firm 1 and firm 2; x_1 and x_2 are existing locations (qualities) of products of firm 1 and firm 2.

The profit function of each incumbent, firm 1 and firm 2, is

$$\begin{aligned}\pi_1 &= [p_1 - c(x_1)]q_1 \\ &= N[p_1 - c(x_1)]\left[V - \frac{p_1 - p_2}{x_1 - x_2}\right]\end{aligned}$$

$$\begin{aligned}\pi_2 &= [p_2 - c(x_2)]q_2 \\ &= N[p_2 - c(x_2)]\left[\frac{p_1 - p_2}{x_1 - x_2}\right]\end{aligned}$$

The first order conditions of the profit maximization problems of each firm with respect to p_1 and p_2 are

$$\frac{\partial \pi_1}{\partial p_1} = \left[V - \frac{p_1 - p_2}{x_1 - x_2}\right] - \frac{1}{x_1 - x_2}[p_1 - c(x_1)] = 0 \qquad (1)$$

$$\frac{\partial \pi_2}{\partial p_2} = \left[\frac{p_1 - p_2}{x_1 - x_2}\right] - \frac{1}{x_1 - x_2}[p_2 - c(x_2)] = 0 \qquad (2)$$

The second order conditions for the maximization problems are satisfied because the profit function of each firm i is strictly concave in its price.

From (1) and (2), the noncooperative equilibrium prices of firm 1 and firm 2 are as follows:

$$p_1 = \frac{1}{3}[(2V)(x_1 - x_2) + 2c(x_1) + c(x_2)] \tag{3}$$

$$p_2 = \frac{1}{3}[(V)(x_1 - x_2) + c(x_1) + 2c(x_2)] \tag{4}$$

It is assumed that $c(x_i)$ is increasing in x_i.

From (3) and (4), it can be found,

$$p_1 - p_2 = \frac{1}{3}[(V)(x_1 - x_2) + c(x_1) - c(x_2)] \tag{5}$$

By plugging numerical values in (3) and (4), perfect Nash equilibrium prices can be found for two incumbents applying Prescott-Visscher's revised Hotelling structure in a vertically differentiated model. Looking at the U.S. automobile industry, two largest firms (G.M. and Ford) produce almost the same products and compete. So, in the sample problem, it is assumed that $x_1 = .6$, $x_2 = .5$ in a product space where $0 < x < 1$.

Before foreign entry, most American automobile firms produced big cars, and almost no small cars. But after foreign entry (since the introduction of the Beetle by Volkswagen in the late 1950s), American automobile firms started paying (a little) attention to the smaller-size cars. This is why $x_1 = .6$, $x_2 = .5$, $x_e = .2$ are assumed in the sample problem where x_e denotes entrant's location.

The cost function we used is of the form

$$C(x_i) = cx_i$$

where c is constant.[8]

The equilibrium prices, profits, quanties, revenues, and shares of two incumbents found in the sample problem are unique. These numerical values of equilibria are in

〈Table 1〉 When Incumbents only are in the Market

Firm	Location	Price	Quantity	Revenue	Profit	Share
Firm 1	$x_1 = .6$	$p_1 = .9$	$q_1 = 3$	$R_1 = 2.7$	$\pi_1 = .9$	$s_1 = .6$
Firm 2	$x_2 = .5$	$p_2 = .7$	$q_2 = 2$	$R_2 = 1.4$	$\pi_2 = .4$	$s_2 = .4$

Note: Firm 1 and Firm 2 represent incumbents.

8) In the sample problem, $V = 5$, $N = 1$, $c = 1$, $x_1 = .6$, $x_2 = .5$ and $0 < x < 1$.

the Table 1.

Several different sample problems were tried with different parameters. If V increased, market shares of two firms were unchanged. However, prices, profits and revenues increased.

An intuitive explanation is that since there were only two firms, even though V increased, its share, which is a relative concept between two incumbents, would not be changed, but prices, profits, and revenues would be increased because consumer's evaluation of quality was increased.

When c increased, market share of firm 2 (lower quality firm) increased (of course, firm 1's share decreased). This is because price difference (p_1-p_2) gets larger as (cx_1-cx_2) gets larger. Hence, lower relative price of firm 2 adds its market share than before. When c increased, profits and revenues of incumbents were reduced because of higher cost than before. But, prices increased.

2) Three Products by Two Incumbents and One Entrant

Noncooperative equilibrium prices in the second stage of this game will be determined with given locations of incumbents and entrants (x_1, x_2, x_e), when there is entry.

Assume

$$x_1 > x_2 > x_e$$

$$p_1 > p_2 > p_e$$

Then, demand functions for incumbents firm 1 and firm 2, and entrant e are,

$$q_1 = N\left[V - \frac{p_1-p_2}{x_1-x_2}\right]$$

$$q_2 = N\left[\frac{p_1-p_2}{x_1-x_2} - \frac{p_2-p_e}{x_2-x_e}\right]$$

$$q_e = N\left[\frac{p_2-p_e}{x_2-x_e}\right]$$

Since there are two incumbents and one entrant, there are a total of three firms. Then, the maximization problem for incumbent is

$$\max\ \pi_i = [p_i - c(x_i)]q_i \qquad \text{for } i = 1, 2 \text{ incumbents}$$

Maximization problem for entrant is

$$\max \pi_e = [p_e - c(x_e)]q_e - K \qquad \text{for } e = \text{entrant}$$

where K is fixed cost neeed to establish location at x_e.[9]

Then, noncooperative Nash equilibrium prices—p_1, p_2, p_e—will be the ones that jointly solve these three maximization problems for $i = 1$, 2 and e. (In this vertical differentiation model, price undercutting is possible. And so, market share of individual firm could be volatile.)

Considering these facts, profit functions of incumbents 1 and 2, and entrant e are as follows:

$$\begin{aligned}\pi_1 &= [p_1 - c(x_1)]q_1 \\ &= N[p_1 - c(x_1)]\left[V - \frac{p_1 - p_2}{x_1 - x_2}\right]\end{aligned}$$

$$\begin{aligned}\pi_2 &= [p_2 - c(x_2)]q_2 \\ &= N[p_2 - c(x_2)]\left[\frac{p_1 - p_2}{x_1 - x_2} - \frac{p_2 - p_e}{x_2 - x_e}\right]\end{aligned}$$

$$\begin{aligned}\pi_e &= [p_e - c(x_e)]q_e - K \\ &= N[p_e - c(x_e)]\left[\frac{p_2 - p_e}{x_2 - x_e}\right] - K\end{aligned}$$

The first order conditions of the profit maximization problems of each firm with respect to p_1, p_2 and p_e are

$$\frac{\partial \pi_1}{\partial p_1} = \left[V - \frac{p_1 - p_2}{x_1 - x_2}\right] - \frac{1}{x_1 - x_2}[p_1 - c(x_1)] = 0 \qquad (5)$$

$$\frac{\partial \pi_2}{\partial p_2} = \left[\frac{p_1 - p_2}{x_1 - x_2} - \frac{p_2 - p_e}{x_2 - x_e}\right] - \left[\frac{x_1 - x_e}{(x_1 - x_2)(x_2 - x_e)}\right][p_2 - cx_2] \qquad (6)$$

$$\frac{\partial \pi_e}{\partial p_e} = \left[\frac{p_2 - p_e}{x_2 - x_e}\right] - \left[\frac{1}{x_2 - x_e}\right][p_e - c(x_e)] = 0 \qquad (7)$$

The second conditions for the maximization problems are satisfied. This is derived form (5), (6) and (7).

From (5), (6) and (7), the noncooperative equilibrium prices of firm 1, firm 2 and entrant e are as follows:

9) Incumbents do not need this fixed cost because they already committed this cost. So, in their cost functions, K is not included.

$$p_1 = \frac{1}{2}[V(x_1-x_2)+c(x_1)] + \frac{1}{6(x_1-x_e)}[V(x_1-x_2)(x_2-x_e)] + \frac{1}{6(x_1-x_e)}[c(x_1)(x_2-x_e)+2c(x_2)(x_1-x_e)] + \frac{1}{6(x_1-x_e)}[c(x_e)(x_1-x_2)] \quad (8)$$

$$p_2 = \frac{1}{3(x_1-x_e)}[V(x_1-x_2)(x_2-x_e)] + \frac{1}{3(x_1-x_e)}[c(x_1)(x_2-x_e)+2c(x_2)(x_1-x_e)] + \frac{1}{3(x_1-x_e)}[c(x_e)(x_1-x_2)] \quad (9)$$

$$p_e = \frac{1}{6(x_1-x_e)}[V(x_1-x_2)(x_2-x_e)] + \frac{1}{3(x_1-x_e)}[c(x_1)(x_2-x_e)+2c(x_2)(x_1-x_e)] + \frac{1}{3(x_1-x_e)}[c(x_e)(7x_1-x_2-6x_2)] \quad (10)$$

Here,

$$p_1-p_2 = \frac{1}{2}[V(x_1-x_2)+c(x_1)] - \frac{1}{6(x_1-x_e)}[V(x_1-x_2)(x_2-x_e)] - \frac{1}{6(x_1-x_e)}[c(x_1)(x_2-x_e)+2c(x_2)(x_1-x_e)] - \frac{1}{6(x_1-x_e)}[c(x_e)(x_1-x_2)]$$

As in the two incumbents case, plugging parameter values into equations (8), (9) and (10), the perfect Nash equilibrium can be obtained for prices, profits, revenues, and shares of two incumbents and entrant in the sample problem.[10] And these equilibria are unique, too. These numerical values of equilibria are shown in the Table 2.

Comparing result of Table 2 with that of Table 1, what is found is that both incumbents, firm 1 and firm 2 suffer from an entrant, since their prices, revenues, and profits are decreased.

10) In the sample problem, $V=5$, $N=1$, $c=1$, $x_1=.6$, $x_2=.5$, $x_e=.2$ and $0<x<1$. Sunk cost K is not added to the profit function of entrant.

〈Table 2〉 **When Entrant Joins the Market with Two Incumbents**

Firm	Location	Price	Quantity	Revenue	Profit	Share
Firm 1	$x_1=.6$	$p_1=.86$	$q_1=2.63$	$R_1=2.26$	$\pi_1=.69$	$s_1=.53$
Firm 2	$x_2=.5$	$p_2=.63$	$q_2=1.67$	$R_2=1.04$	$\pi_2=21$	$s_2=.33$
Firm *e*	$x_e=.2$	$p_e=.41$	$q_e=0.71$	$R_e=0.29$	$e_e=.15$	$s_e=.14$

Note: Firm 1 and Firm 2 represent incumbents.
Firm e represents entrant.

Similarly with the two incumbents only case, when there are three firms because of new entrant, as V increases, prices and profits of three firms are increased. However, unlike the two incumbents only case where the share of two firms are unchanged regardless of size of V, in this three firms case, if V increases, share of firm 1 increases at the loss of entrant, while firm 2's (another incumbent−second highest quality firm) share remains the same as only two incumbents case. However, in terms of revenue, all firms benefit from increased V because of higher price and higher quantity sold. So, expansion of V creates the positive expansion and competition effects to firm 1 and 2.

3) When Incumbents Add One More Location Each after Entry

Numerical values of perfect Nash equilibrium of three firms before incumbents take an action responding to entry will be changed if incumbents add one more location to existing locations.

In this section, additional new locations (x_{1e}, x_{2e}) will be found with predetermined values of x_1, x_2 and x_e. By finding these two additional locations, it can be seen how these two incumbents react strategically to entry once it occurred.

Up to now, most of entry analysis has been focused on how incumbent(s) can strategically react to potential entrant before it occurs. However, in the analysis of the U.S. automobile industry, it is more interesting to find out how incumbents react (or should have reacted) to an entrant (here, foreign entry) that moves into lower quality market segment.

This is why it is important to find out additional new locations of two incumbents in the sample problem with given vlaues of three locations −x_1, x_2 and x_e. With these two new loation values, prices and quantities of five products (locations), and profits, revenues and shares of three firms can be derived. Then, the competition and expansion effects campared with case in which each incumbent did not add one more location after it faces entrant can be analyzed.

When new locations are (x_{1e}, x_{2e}) and corresponding prices are (p_{1e}, p_{2e}), profit functions of 3 firms are as follows:

$$\pi_1 = [p_1 - c(x_1)]q_1 + [p_{1e} - c(x_{1e})]q_{1e} - K$$
$$= N[p_1 - c(x_1)]\left[V - \frac{p_1 - p_2}{x_1 - x_2}\right]$$
$$+ N[p_{1e} - c(x_{1e})]\left[\frac{p_2 - p_{1e}}{x_2 - x_{1e}} - \frac{p_{1e} - p_{2e}}{x_{1e} - x_{2e}}\right] - K$$

$$\pi_2 = [p_2 - c(x_2)]q_2 + [p_{2e} - c(x_{2e})]q_{2e} - K$$
$$= N[p_2 - c(x_2)]\left[\frac{p_1 - p_2}{x_1 - x_2} - \frac{p_2 - p_{1e}}{x_2 - x_{1e}}\right]$$
$$+ N[p_{2e} - c(x_{2e})]\left[\frac{p_{1e} - p_{2e}}{x_{1e} - x_{2e}} - \frac{p_{2e} - p_e}{x_{2e} - x_e}\right] - K$$

$$\pi_e = [p_e - c(x_e)]q_e - K$$
$$= N[p_e - c(x_e)]\left[\frac{p_{2e} - p_e}{x_{2e} - x_e}\right]$$

where K in π_1 and π_2 represents establishmen cost for new location x_{1e} and x_{2e}, and so it is considered to be sunk.

These are the profit functions when new products are located as follows:

$$x_1 > x_2 > x_{1e} > x_{2e} > x_e$$

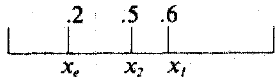
$$p_1 > p_2 > p_{1e} > p_{2e} > p_e$$

This indicates that higher quality product charges higher price.

In this case, perfect Nash equilibrium prices for existing locations (p_1, p_2) of this five products case are different from that of previous three products case (p_1, p_2), even though these price pairs correspond to the same locations.

Location ordering of $(x_1, x_2, x_{1e}, x_{2e}, x_e)$ is just one of twenty possible location cases of (x_{1e}, x_{2e}). These possible locations of (x_{1e}, x_{2e}) are shown in Figure 1 in the next page. Depending on each case, the demand function for each product will be different and so is the profit function for each firm.

⟨Figure 1⟩ Possible Location Orderings of (x_{1e}, x_{2e})



(1) $x_e < x_{2e} < x_{1e} < x_2 < x_1$

(2) $x_e < x_{1e} < x_{2e} < x_2 < x_1$

(3) $x_{2e} < x_e < x_{1e} < x_2 < x_1$

(4) $x_{1e} < x_e < x_{2e} < x_2 < x_1$

(5) $x_{2e} < x_{1e} < x_e < x_2 < x_1$

(6) $x_{1e} < x_{2e} < x_e < x_2 < x_1$

(7) $x_e < x_{2e} < x_2 < x_1 < x_{1e}$

(8) $x_e < x_{1e} < x_2 < x_1 < x_{1e}$

(9) $x_{2e} < x_e < x_2 < x_1 < x_{1e}$

(10) $x_{1e} < x_e < x_2 < x_1 < x_{1e}$

(11) $x_e < x_2 < x_1 < x_{2e} < x_{1e}$

(12) $x_e < x_2 < x_1 < x_{1e} < x_{2e}$

(13) $x_e < x_{2e} < x_2 < x_{1e} < x_1$

(14) $x_e < x_{1e} < x_2 < x_{2e} < x_1$

(15) $x_{2e} < x_e < x_2 < x_{1e} < x_1$

(16) $x_{1e} < x_e < x_2 < x_{2e} < x_1$

(17) $x_e < x_2 < x_{2e} < x_1 < x_{1e}$

(18) $x_e < x_2 < x_{1e} < x_1 < x_{2e}$

(19) $x_e < x_2 < x_{2e} < x_{1e} < x_1$

(20) $x_e < x_2 < x_{1e} < x_{2e} < x_1$

In this final case of two incumbents and one entrant, where the entrant already chose its location at x_e, the game is reduced to a two-stage game: at the first stage, incumbents choose new locations (x_{1e}, x_{2e}) given existing locations of incumbents and entrant (x_1, x_2, x_e). Then at the second stage, firms choose prices of these all five products (p_1, p_2, p_{1e}, p_{2e}, p_e). Hence, it is expected that there will be perfect Nash equilibrium of locations (x_{1e}, x_{2e}) and prices (p_1, p_2, p_{1e}, p_{2e}, p_e). With these equilibrium values, profits, revenues and shraes of three firms can be determined.

Numerical solutions of these perfect Nash equilibrium with given parameters and predetermined location values for x_1, x_2 and x_e were attempted through Mathematica [8] program package.

When predetermined values of ($V=5$, $c=1$, $N=1$, $x_1=.6$, $x_2=.5$ and $x_e=.2$) were given to each profit function, solutions can be found to cases 1 through 6 only, but

⟨Table 3⟩ When Incumbents add One more Location Each after Entry

Firm	Location	Price	Quantity	Revenue	Profit	Share
Firm 1	$x_1=.6$	$p_1=.85$	$q_1=2.46$	$R_1=2.18$	$\pi_1=.61$	$s_1=.58$
	$x_{1e}=.2$	$p_{1e}=.20$	$q_{1e}=0.43$			
Firm 2	$x_2=.5$	$p_2=.50$	$q_2=1.24$	$R_2=0.82$	$\pi_2=.12$	$s_2=.33$
	$x_{2e}=.2$	$p_{2e}=.20$	$q_{2e}=0.43$			
Firm *e*	$x_e=.2$	$p_e=.20$	$q_e=0.43$	$R_e=0.09$	$\pi_e=.00$	$s_e=.09$

Note: Firm 1 and Firm 2 represent incumbents.
Firm *e* represents entrant.

not to cases 7 through 20. Perfect Nash equilibrium new locations are $x_{1e}=x_{2e}=.2$ to cases 1 through 6. This is the same location as entrant's location. Corresponding to these location values, perfect Nash equilibrium prices and quantities of five products, and profits, revenues and shares of three firms which are shown in Table 3 can be found.[11]

Compared with three products (x_1, x_2, x_e) case (before addition of new locations by incumbents), profits of two incumbents are drastically reduced. Profit of firm 2 is reduced more than firm 1. Entrant's profit is reduced to zero.

However, firm 1 (incumbent with higher quality) increased its market share while firm *e* (entrant) lost its market share. So, through the competition effect, due to establishing one more location, profits for both incumbents were reduced; however through the expansion effect, only firm 1 increased its market share. Here, profit of each firm is the value before establishment cost (sunk cost) *K* is added to each profit function. If *K* is added to each profit function of three firms, entrant's profit will be definitely negative.

IV. Conclusion

This study set about to show how entry in a product differentiated and mature industry would disturb pricing and quality location choice behaviors in an industry where a segment of product space was not occupied by incumbents before entry in a saturated market such as the U.S. passenger car market.

Questions were answered by using numerical methods, and by changing the predetermined values of locations and parameter values.

In summary of numerical values of perfect Nash equilibria, the results are as follows:

i) The addition of one more location by each incumbent gives firm 1 and firm 2 negative expansion and competition effects.

ii) However, the addition of one more location gives a positive share effect to firm 1; on the other hand, firm 2's market share is not changed.

iii) Sunk cost is not specified to entrant and incumbent for the new location yet. If sunk cost is added to entrant, after incumbents add one more location each, the

11) This is the value before sunk cost *K* is added.

entrant faces negative profit due to positive sunk cost. If sunk cost is added to the incumbents' profit functions, profit will be much more reduced, too. Depending on the size of sunk cost, it could be negative.

From this sample problem, it is understood that if incumbents add one more location each to existing locations, they can have entrant face negative profit when sunk cost is added to the profit function. However, because of competition and expansion effects, their profits would be reduced compared with the case in which incumbents do not add one more location each after entry. These profits will be much more reduced if sunk cost K is added to the profit function. This fact could explain why incumbents do not have incentives to add one more location to existing locations.

Consequently, this numerical example shows why (foreign) entry did not affect the behaviors of the two largest firms of the U.S. automobile industry: According to the numerical example, incumbents do not have incentives to add one more location to existing locations, even though an entrant gains market share thus reducing the incumbents' profits. Thus, even with an entrant in the market, incumbents remain at their initial locations. Sunk cost which will be accompanied when incumbents add new locations to the existing locations becomes a important factor in restricting incumbent's behavior, too. Further research on sunk cost is required in a study on post-entry response by incumbents.

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