연세경제연구 제 I 권
Yonsei Economic Studies, Vol. I (September 1994)

The Primacy and Welfare Costs of the Sub-Optimal Size Distribution of Cities under External Diseconomies

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Properties of the optimal size distribution of cities under external diseconomies are derived by using the hierarchy model, in which the agglomeration economy and the external diseconomy are explicitly considered. As a benchmark case, welfare costs of the sub-optimal size distribution of cities, WC, and the optimal size of the prime city, OSP, without external diseconomies are calculated. Since there are no data for externality costs of cities, WC and OSP under various degrees of external diseconomies are obtained by the simulation. General formulae relating benchmark WC and OSP with those under conjectured degree of external diseconomies are constructed.

I. Introduction

The strong regularities in the distribution of city size are widely observed. The equilibrium size distribution of cities are well explained by both the hierarchy model [1-5, 16, 30, 31] and the stochastic model [24, 25, 33]. But, there are only a few studies dealing with the optimal size distribution of cities [26, 27].

Analyses of optimal size distribution of cities is important in understanding the importance of national spatial policies. If the size distribution of cities is not at optimum, there will be the welfare costs due to this sub-optimality. If this welfare costs are calculated and is found to be very large, say 5% of annual GNP, it will en-

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force us to pay more attention to spatial policies of a nation. National spatial policies are not less important than fiscal or monetary policies.

For both the hierarchical and stochastic models with a Pareto distribution, the productivity of cities can be shown to be a factor explaining the Pareto coefficient [28]. This implies that the productivity of cities plays key roles in analyzing the optimal size distribution of cities.

When the agglomeration economy is considered, the size distribution of cities may be defined at optimum when the output of a nation is maximized. For this case, analyses of properties of the optimal size distribution of cities and estimations of the optimality index indicating the degree of departures from the optimum are proceeded in [27]. Welfare costs of the sub-optimal size distribution of cities(WC) and the upper bound of the optimal size of the prime city(OSP) are calculated in [26].

But, two major factors determining the optimal size distribution of cities are the agglomeration economy and the external diseconomy. In order to fully understand the optimal size distribution of cities, external diseconomies as well as agglomeration economies must be considered. In this case, the size distribution of cities may be defined at optimum when the welfare, GNP net of the externality cost, is maximized. Theoretically, there are no problems in including the externality cost in the model. But, empirically, there are some problems.

Gross Regional Product(GRP) of cities are necessary in order to consider the agglomeration economy. GRP data may be available for most nations. But, data for all cities' externality costs including all kinds of external diseconomies such as the pollution, congestion, noise, ··· etc. are not available.

Simulations will be proceeded in order to obtain WC and OSP under various degrees of external diseconomies. Degrees of external diseconomies are expressed by the ratio of the per capita externality cost of the smallest city to the per capita income(PECS) and the ratio of the per capita externality cost of the prime city to that of the smallest city(REXT).

WC and OSP without external diseconomies, benchmark WC and OSP, can be always calculated since the data are available. Therefore, it will be useful if there are some formulae relating benchmark WC and OSP with those under various degrees of external diseconomies. Based upon simulation results, general formulae will be made by using PECS and REXT. By using these formulae, it will be possible to calculate WC and OSP under conjectured degree of external diseconomies.

II. The Model

The hierarchy model of size distribution of cities will be analyzed with the explicit consideration of the productivity and external diseconomy of cities.

Let k be the hierarchy of a city and x_k be the kth commodity produced by cities: $k=1, 2, \cdots, K$. Areas with hierarchy 0 are rural areas. A city with hierarchy k produces total of k commodities, $x_1, \cdots, x_k$. $x_1, \cdots, x_{k-1}$ are produced for the local demand and x_k is produced for both the local demand and the demand of cities with hierarchies lower than k.

y_k is the GRP of a city with hierarchy k and n_k is the number of cities with hierarchy k. The GRP of cities with hierarchy k, Y_k, is $Y_k = n_k y_k$. The national demand for x_k, measured in terms of money, is assumed to be proportional to GNP, Y [31]. a_k is the exogenously given proportion of demand for x_k such that $\sum_{k=0}^{K} a_k = 1$. a_0 is the proportion of demand for rural output.

By definition of a_0, $Y_0 = a_0 Y$. Since $x_1, \cdots, x_{k-1}$ are produced for the local demand of cities with hierarchy k, the demand for $x_1, \cdots, x_{k-1}$ is $Y_k = \sum_{i=1}^{k-1} a_i$. Also, since x_k is produced for the local demand and the demand of cities with hierarchies lower than k, the volume of demand for x_k is $a_k \sum_{h=0}^{k} Y_h$. Total expenditures for commodities produced by cities with hierarchy k, the sum of volumes of demand, is the same as the income of those cities.

$$Y_k = a_k \sum_{h=0}^{k} Y_h + Y_k \sum_{i=1}^{k-1} a_i \qquad (1)$$

Even though the intercity transportation cost is not explicitly included in the model, it is assumed that the intercity transportation cost is large enough. Under this assumption, eq. (1) implies that markets are cleared for all commodities [26].

Define b_k *as* $b_k = 1 - \sum_{h=1}^{k} a_h$ and $b_0 = 1$. Since $\sum Y_h = (a_0/b_k)Y$, Y_k can be derived as follows [27].

$$Y_h = \left(\frac{a_0\, a_k}{b_k\, b_{k-1}}\right) Y$$

$$\equiv d_k Y$$

Notice that $\sum_{k=1}^{K} d_k = 1 - a_0$. Since eqs. (1) and (2) are equivalent, the market clearing condition can be written as $d_k = Y_k/Y (k = 1, \cdots, K)$. s_k is the size of a city with hierarchy k and S_k is the size of cities with hierarchy k, i.e., $S_k = n_k s_k$. Then, the exogenously given total urban population, S, is $S = \sum_k n_k s_k = \sum_k S_k p_{ik}$ is the number of workers employed at the ith industry located in a city with hierarchy k: $1 \leq i \leq k$, $1 \leq k \leq K$.

Labour force participation rate, ρ, is defined as the ratio of workers with respect to the size of a city. For simplicity, it is assumed that ρ is the same for all cities. By definition of ρ, it must be satisfied that $\sum_i p_{ik} = \rho s_k$.

In the production of commodities, the agglomeration economy is assumed to exist. But, no scale economy is assumed for each industry. Intercity transportation cost is assumed to be large enough so that the degree of agglomeration economy does not surpass the transportation cost. Thus, there does not exist the possibility of the largest city's producing all commodities.

$g_i(s_k, p_{ik})$ is the average productivity of the ith industry located in a city with hierarchy k, which is measured in terms of money. The existence of the agglomeration economy and the non-existence of the scale economy imply that $\partial g_i(s_k, p_{ik})/\partial s_k \equiv g_{is} > 0$ and $\partial g_i(s_k, p_{ik})/\partial p_{ik} \equiv g_{ip} < 0$.

By definition of $g_i(s_k, p_{ik})$, $y_k = \sum_i g_i(s_k, p_{ik}) p_{ik}$. Even if there does not exist the scale economy, it may be natural that the output of the ith industry, $g_i(s_k, p_{ik}) p_{ik}$, increases as p_{ik} increases. In order to guarantee this, it is assumed that the worker's elasticity of productivity is inelastic, i.e., $|(\partial g_i/\partial p_{ik})(p_{ik}/g_i)| < 1$. Under this assumption, $\partial[g_i(s_k, p_{ik})p_{ik}]/\partial p_{ik} = g_i[1 + g_{ip}(p_{ik}/g_i)] + (1/\rho) g_{is} p_{ik} > 0$. The effect of change in p_{ik} upon y_k can be derived as follows.

$$\partial y_k/\partial p_{ik} = (g_i + g_{ip} p_{ik}) + (1/\rho) \sum_i g_{is}\, p_{ik} \tag{3}$$

Here, $g_i + g_{ip} p_{ik}$ is the direct effect upon the ith industry and $(1/\rho) \sum_i g_{is}\, p_{ik}(1/\rho)$ is the indirect effect through the agglomeration economy.

Costs of all external diseconomies such as pollutions, congestions, $\cdots$, etc. are aggregated. $f(s_k)$ is the per capita cost of external diseconomies of a city with hierarchy k. It is assumed that $df(s_k)/ds_k \equiv f'(s_k) > 0$.

The externality cost of a city with hierarchy k, c_k, is $c_k = s_k f(s_k)$. Since $f'(s_k) > 0$, $dc_k/ds_k > 0$. It is assumed that c_k increases in increasing rate, i.e., $d^2 c_k/ds_k^2 > 0$.[1)] The externality cost of cities with hierarchy k, C_k, and that of all cities, C, can be expressed as $C_k = n_k c_k$ and $C = \sum_k C_k$.

The welfare of a city with hierarchy k, w_k, is defined as the GRP net of the externality cost; $w_k = y_k - c_k$. The welfare of cities with hierarchy k, W_k, and that of all cities, W, can be expressed as $W_k = n_k(y_k - c_k)$ and $W = \sum n_k(y_k - c_k) = Y - C$.

III. Properties of the Optimal Size Distribution of Cities

The size distribution of cities is defined at optimum if W is maximized under conditions such that the market clearing and the full employment conditions are satisfied and the total size of urban area is given.

From eqs. (1) and (2) and the definition of $g_i(s_k, p_{ik})$, Y can be rewritten as $Y = [1/(1-a_0)] \sum_k \sum_i n_k g_i(s_k, p_{ik}) p_{ik}$. Thus, $W = Y - C = [1/(1-a_0)] \sum_k \sum_i n_k g_i(s_k, p_{ik}) p_{ik} - \sum_k n_k s_k f(s_k)$. Constraints are $\sum_i P_{ik} = \rho s_k$, $\sum_k n_k s_k = S$, and $d_k = Y_k/Y$, which can be rewritten as $d_k = [n_k \sum_i g_i(p_{ik}, s_k)\, p_{ik}]/\{[1/(1-a_0)] \sum_k \sum_i n_k g_i(s_k, p_{ik})\, p_{ik}\}$ First order conditions of this optimization problem are constraints and the followings.

$$\partial L/\partial p_{ik} = [1/(1-a_0)] A_k n_k (g_i + g_{ip}\, p_{ik}) - \lambda_k = 0 \tag{4}$$

$$\begin{aligned}\partial L/\partial s_k = {} & [1/(1-a_0)] A_k n_k (\textstyle\sum_i g_{is}\, P_{ik}) + \rho \lambda_k \\ & - \mu n_k - n_k f(s_k) - n_k s_k f'(s_k) = 0\end{aligned} \tag{5}$$

$$\partial L/\partial n_k = [1/(1-a_0)] A_k (\sum g_i\, p_{ik}) - \mu s_k - s_k f(s_k) = 0 \tag{6}$$

Here, L is the Lagrangean function and λ_k and μ are Lagrange multipliers. Also, $A_k = 1 - (1-a_0)(1/d_k)(\phi_k d_k/Y) + [\sum_{h=1}^{K} \phi_h d_h/Y]$ and ϕ_k is the Lagrange multiplier related to the constraint of $d_k = Y_k/Y$. Notice that $-(1-a_0)(1/d_k)(\phi_k d_k/Y) + [(\sum \phi_h d_h)/Y]$ is the same as $-(\partial W/\partial d_k)(\partial d_k/\partial Y_k)$. Here, the property of Lagrange multiplier, $\phi_k = \partial W/\partial d_k$, is used. When Y_k changes by ΔY_k, W will change by ΔY_k if d_k's are given. But, when Y_k changes d_k's will be changed and this will in turn cause the change in W. Thus, the net effect of the change in Y_k upon W, dW/dY_k, will be $dW/dY_k = 1 - (\partial W/\partial d_k)(\partial d_k/\partial Y_k) = A_k$. Also, notice that $\mu = dW/dS$. By using eqs. (3)–(5) and the

1) If $f'(s_k) > 0$, $\partial^2 c_k/\partial s_k^2 > 0$ is always satisfied. If $f'(s_k) < 0$, the sign of $\partial^2 c_k/\partial s_k^2$ will be ambiguous. But, there are some cases of $\partial^2 c_k/\partial s_k^2$ being positive even if $f'(s_k) < 0$. An example of this is $f(s_k) = B s_k^e$ and $0 < e < 1$.

fact that $dc_k/ds_k = f(s_k) + s_k f'(s_k)$, $\partial y_k/\partial p_{ik}$ can be derived as follows.

$$\partial y_k/\partial p_{ik} = (1-a_0)(1/A_k)(\mu/\rho)[1+(\partial c_k/s_k)] \tag{7}$$

Since, A_k, μ, ρ and $\partial c_k/\partial s_k$ are independent of i, eq. (7) implies that $\partial y_k/\partial p_{ik} = \partial y_k/\partial p_{jk}$ This implies the following property of optimal size distribution of cities.

PROPERTY 1: In allocating workers to different industries located in the same city, it is optimal so that the marginal effects upon the GRP are the same.

$\partial W/\partial p_{ik}$ can be derived as follows since the change in d_k's must be considered when Y_k changes.

$$\begin{aligned}\partial W/\partial p_{ik} &= (dW/dy_k)\ (\partial y_k/\partial p_{ik})\\ &= n_k(dW/dY_k)\ (\partial y_k/\partial p_{ik})\\ &= n_k A_k(\partial y_k/\partial p_{ik})\end{aligned} \tag{8}$$

Eqs. (7) and (8) imply that $(\partial W/\partial p_{ik})/n_k$ can be derived as follows.

$$(\partial W/\partial p_{ik})/n_k = (1-a_0)(1/\rho)[\mu + (\partial c_k/\partial s_k)] \tag{9}$$

Cities with the same hierarchy are homogeneous. Therefore, $\partial W/\partial p_{ik}$ must be interpreted as the effect of changes in p_{ik} upon W when p_{ik} changes by the same amount for all cities with hierarchy k. Then, $(\partial W/\partial p_{ik})/n_k$ is the effect of changes in p_{ik} upon W when p_{ik} of one city with hierarchy k changes.

Since $\partial^2 c_k/\partial s_k^2 > 0$, $\partial c_k/\partial s_k$ increases as k increases. Thus, eq. (9) implies the following property.

PROPERTY 2: In allocating workers to the same industry located in cities with different hierarchies, as the hierarchy increases, the marginal effect upon the welfare must increase so as to compensate the increasing externality cost.

When only S changes, d_k's must be fixed at optimum levels due to properties 1 and 2. When only S changes and d_k's are fixed at optimum, $dY_k/dS = d_k(dY/dS)$ since $Y_k = d_k Y$. Since $\mu = dW/dS$ and $A_k = dW/dY_k$, by using eq. (2), eq. (6) can be rewritten as follows.

$$n_k s_k = [\mu Y/\sigma(1-a_0)]/[\mu + f(s_k)] \tag{10}$$

Here, $\sigma = dY/dS$, which is independent of k. Since $f'(s_k) > 0$, eq. (10) implies that $n_k s_k$ decreases as k increases.

PROPERTY 3: At optimum, weights of cities with lower hierarchies must be larger than those of cities with higher hierarchies.

This property can have the following implications for per capita GRP of cities. By using $Y_k = n_k y_k$ and $Y_k = d_k Y$, eq. (10) can be rewritten as $(y_k/s_k) = [(1-a_0)/\mu]d_k[\mu + f(s_k)]$. This says that the per capita GRP of a city with hierarchy k, y_k/s_k, is affected by two factors. The one is the importance of kth commodity in the national economy, which is revealed by the magnitude of d_k. (y_k/s_k) increases as d_k increases. The other is the environmental quality, which is inversely related to $f(s_k)$. At optimum, (y_k/s_k) must increase as the quality of life worsens.

IV. Modification of the Model for the Empirical Purpose

In order to analyze the effect of external diseconomies upon WC and OSP, it is necessary to simplify the model of section Ⅱ. For this, define $g_i(s_k, p_{ik})$ as $g_i(s_k, p_{ik}) = C_i s_k^a p_{ik}^{-b}$: $a > 0$, $1 > b > 0$, C_i is some constant. This definition of $g_i(s_k, p_{ik})$ satisfies that $g_{is} > 0$, $g_{ip} < 0$ and $|(\partial g_i/\partial p_{ik})(p_{ik}/g_{ip})| < 1$.[2)]

Also, it is necessary to simplify $A_k = 1-(1-a_0)(1/d_k)(\phi_k d_k/Y) + [\sum_{h=1}^{K} \phi_h d_h/Y] = dW/dY_k$ Since A_k is too complicated and includes Lagrangean multipliers, it is not possible to consider A_k as it is in the empirical study. Since $A_k = dW/dY_k$ and C does not directly affected by the change in Y_k, A_k may be approximated as dY/dY_k. By definition of $Y_k = d_k Y$, partial derivatives $\partial Y/\partial Y_k$ is $1/d_k$. Thus, the total derivative dY/dY_k is approximated by the partial derivative $\partial Y/\partial Y_k = 1/d_k$. Under this approximation, eq. (7) can be rewritten as follows.

2) The specification of $g_i(s_k, p_{ik})$ as $C_i s_k^a p_{ik}^{-b}$ may not take into account all properties of the optimal size distribution of cities described in section III. But, since this specification includes most characters of $g_i(s_k, p_{ik})$ and makes the empirical study simpler, it is used in this paper.

$$(1/d_k)(\partial y_k/\partial p_{ik}) = (1-a_0)(1/\rho)[\mu + (\partial c_k/\partial s_k)] \tag{11}$$

If $g_i(s_k, p_{ik}) = A_i s_k^a p_{ik}^{-b}$, $\partial y_k/\partial p_{ik}$ can be calculated as $g_i(s_k, p_{ik})(1-b) + (a/\rho)(y_k/s_k)$. Thus, eq. (11) can be rewritten as follows.

$$(1/d_k)[g_i(s_k, p_{ik})(1-b) + (a/\rho)(y_k/s_k)] - (1/\rho)(\partial c_k/\partial s_k) = \mu/\rho \tag{12}$$

Since y_k, s_k, d_k and $(\partial c_k/\partial s_k)$ are independent of i, eq. (12) implies that $g_i(s_k, p_{ik}) = \zeta_k$ at optimum. If $g_i(s_k, p_{ik}) = \zeta_k$, $y_k = \sum_i g_i(s_k, p_{ik})p_{ik} = \zeta_k \rho s_k$ and $y_k/s_k \equiv i_k = \zeta_k \rho$ at optimum. By using these facts, the following relationship can be derived by using eq. (12).

$$\zeta_k = \left[\frac{d_k}{d_1}\right]\left[\frac{\mu + (\partial c_k/\partial s_k)}{\mu + (\partial c_1/\partial s_1)}\right]\zeta_1 \tag{13}$$

(d_k/d_1) can be rewritten as $(d_k/d_1) = (Y_k/Y)/(Y_1/Y) = (i_k n_k s_k)/(i_1 n_1 s_1) = (i_k/i_1)(n_k s_k/n_1 s_1)$. By using eq. (10), $(n_k s_k/n_1 s_1)$ can be rearranged as $(n_k s_k/n_1 s_1) = [\mu + f(s_1)]/[\mu + f(s_k)]$.

Define $f(s_k)$ as $f(s_k) = Bs_k^e$ and $0 < e < 1$. Under this specification of $f(s_k)$, $f'(s_k) > 0$, $\partial c_k/\partial s_k > 0$ and $\partial^2 c_k/\partial s_k^2 > 0$ are satisfied. Also, $\partial c_k/\partial s_k = (1+e)Bs_k^e = (1+e)f(s_k)$ holds. By using these facts, eq. (13) can be rewritten as follows.

$$\begin{aligned}\zeta_k &= \left[\frac{i_k}{i_1}\right]\left[\frac{\mu + f(s_1)}{\mu + f(s_k)}\right]\left[\frac{\mu + (1+e)f(s_k)}{\mu + (1+e)f(s_1)}\right]\zeta_1 \\ &= (i_k/i_1)\,\Pi_k\,\zeta_1 \end{aligned} \tag{14}$$

Notice that Π_k is a function of μ, B, e, s_k and s_1. It can be shown that $\Pi_k > 1$ and $\partial \Pi_k/\partial e > 0$ for $k = 2, 3, \cdots, K$ (see the appendix).

Under given μ, B and e, i_k and Π_k are functions of ζ_k. Thus, values of ζ_k's can be determined by eq. (14) if the value of ζ_1 is determined: $k = 2, \cdots, K$. The value of ζ_1 must be determined so as to maximize the welfare of cities with hierarchy 1, $W_1 = n_1[\zeta_1 \rho s_1 - c_1]$, subject to the size of cities with hierarchy 1, $S_1 = n_1 s_1$. Since $p_{11} = \rho s_1$, $\zeta_1 = A_1 s_1^a p_{11}^{-b} = A_1 \rho^{-b} s_1^{(a-b)}$ and $s_1[\partial \zeta_1/\partial s_1] = (a-b)\zeta_1$. Also, the Lagrangean multiplier of this optimization problem is dW_1/dS_1. By using these facts, the first order necessary condition for this optimization problem can be derived as follows.

$$[(1+a-b)\rho]\zeta_1 = (dW_1/dS_1) + (dc_1/ds_1) \tag{15}$$

At optimum, dW_k/dS_k must be the same for all k. This implies that $dW_k/dS_k = dW/dS$.[3] Thus, eq. (15) can be rewritten as follows.

$$[(1+a-b)\rho]\zeta_1=(dW/dS)+(dc_1/ds_1) \quad (16)$$

Eqs. (14) and (16) can replace eqs. (7), (9) and (10) when $g(s_k, p_{ik})=A_i s_k^a p_{ik}^{-b}$ and $f(s_k)=Bs_k^e$. Thus, the effect of external diseconomies upon WC and OSP can be analyzed by using eqs. (14) and (16). For this, further simplification will be necessary.

For estimating the value of 'b', detailed information about employments of all industries for all cities will be necessary. The value of 'a' is estimated to be very small [28]. Since workers elasticity of productivity is assumed to be inelastic, the value of 'b' can be considered to be small. Therefore, it is assumed that $a-b=0$.

Since $W=Y-C$, $(dW/dS)+(dc_1/ds_1)=(dY/dS)-[(dC/dS)-(dc_1/ds_1)]$. dY/dS is the increase in GNP when urban population increases by one. In allocating a person to a certain city, it is assumed that all cities are equally probable.

Then, dY/dS will be the weighted average of i_k's, where the weight is given by n_k: $dY/dS=\sum_k[n_k/(\sum_{h=1}^{k} n_h)]i_k=\sum_k \psi_k\, i_k \equiv \psi$.

dC/dS is the increase in the externality cost when the urban population increases by one. By the same reasoning as above, $dC/dS=\sum_k \psi_k(dc_k/ds_k)$. By using the fact that $dc_k/ds_k=B(1+e)s_k^e=(1+e)f(s_k)$, $(dc/ds)-(dc_1/ds_1)$ can be expressed as $(dC/dS)-(dc_1/ds_1)=\sum_k \psi_k(1+e)[f(s_k)-f(s_1)]$. Now, eq. (16) can be rewritten as follows.

$$\zeta_1^*\rho=\psi[1-\sum_K \psi_k(1+e)\{[f(s_k)/\psi]-[f(s_1)/\psi]\}] \quad (17)$$

Here, '*' implies the value at optimum. Notice that eq. (17) holds only at optimum. $f(s_k)/\psi$ is the ratio of per capita externality cost of a city with hierarchy k to the weighted average of per capita GRP.

If the size of a city with hierarchy k increases, i_k will be changed. But, since the value of 'a' is very small, i_k is assumed to be fixed at current level. Then, $\zeta_k^*\rho$ can be determined by eq. (14): $k=2, \cdots, K$.

$$\zeta_k^*\rho=(i_k/i_1)\,\Pi_k\zeta_1^*\rho \quad (18)$$

If the size distribution of cities is at optimum, the size of a city necessary for producing the current level of GRP, s_k^*, will be as follows: $k=1, \cdots, K$.

3) Consider the following problem. Max $W=\sum W_k(S_k)$ $s.t.$ $\sum S_k=S$. The first order necessary condition for this problem is $dW_k/dS_k=\zeta$, where ζ is the Lagrange multiplier. By the property about the Lagrange multiplier, $\zeta=dW/dS$. Thus, $dW_k/dS_k=dW/dS$.

$$s_k^* = y_k / (\zeta_k^* \rho) \tag{19}$$

If the current size distribution of cities is sub-optimal, the current number of people necessary for producing current GNP is greater than the number which is necessary at optimum, i.e., $\sum_K n_k(s_k - s_k^*) > 0$. Notice that s_k is the current size of a city with hierarchy k. If these extra people are allocated to cities, gains of GNP will be $(dY/dS)\ [\sum_{k=1}^{K} n_k(s_k - s_k^*)] = \psi[\sum_{k=1}^{K} n_k(s_k - s_k^*)]$. Welfare cost of income(WCI) is defined as the ratio of this to the current GNP, Y.

Depending upon the sample size of cities, the sum of all sample cities' current GRP is not necessarily the same as the current GNP. Therefore, in calculating WCI, gains of GNP is adjusted by the ratio of the sum of all sample cities' current GRP's to the current GNP. Define η as this ratio: $\eta = (\sum_{k=1}^{K} n_k s_k i_k)/Y$. Then, WCI can be expressed as,

$$WCI = \eta\psi[\sum_{k=1}^{K} n_k (s_k - s_k^*)]/Y \tag{20}$$

If the size distribution of cities is at optimum, the externality cost will be smaller than the current one. Therefore, the gains of the externality cost will be $\sum_K n_k\ B[s_k^{(1+e)} - s_k^{*(1+e)}]$. $\sum_{k=1}^{K} n_k B[s_k^{(1+e)} - s_k^{*(1+e)}]$. The ratio of this, adjusted by η, with respect to GNP is defined as the welfare cost of externality(WCE).

$$WCE = \eta B[\sum_{k=1}^{K} n_k(s_k^{(1+e)} - s_k^{*(1+e)})]/Y \tag{21}$$

The welfare cost of the sub-optimal size distribution of cities(WC) is $WC = WCI + WCE$.

V. The Effect of External Diseconomies

The comparative static analyses of effects of external diseconomies upon WC and OSP, s_K^*, will be proceeded. Since $f(s_k) = Bs_k^e$, this can be done through the investigation of effects of changes in B and e upon WC and s_K^*.

If e increases, $\Delta e > 0$, $\Delta\zeta_1^*\rho < 0$ and $\Delta s_1^* > 0$ by eqs. (17) and (19) and $\Delta\Pi_K > 0$ by the appendix. Thus, as is shown by eq. (18), the sign of $\Delta\zeta_K^*\rho$ is generally uncertain. If $|\Delta\zeta_1^*\rho| > \Delta\Pi_K$, $\Delta\zeta_K^*\rho < 0$ and $\Delta s_K^* > 0$ by eqs. (18) and (19). If $|\Delta\zeta_1^*\rho| < \Delta\Pi_K$, $\Delta\zeta_K^*\rho > 0$ and $\Delta s_K^* < 0$.

The case of $|\Delta\zeta_1{}^*\rho| > \Delta\Pi_K$ implies that s_K^* increases as the degree of external diseconomies increases. This case is possible for small enough e. If e is small enough, the difference in the externality cost between the prime city and cities with hierarchy 1 will be very small. If this difference is smaller than that in the agglomeration economy, as the size of the prime city increases, gains from the agglomeration economy will be greater than losses from the external diseconomy. Also, since $\partial s_1{}^*/\partial e$ is always positive, the dominance of the agglomeration economy can be sustained for small enough e. Therefore, $\partial s_K{}^*/\partial e > 0$ is possible for small enough e. As e increases, this possibility disappeares. Thus, for large e, $s_K{}^*/\partial e < 0$ will hold.

For small enough e, since $\partial s_K{}^*/\partial e > 0$ and $\partial s_k{}^*/\partial e > 0$ for large k, it is possible that $\partial WCI/\partial e < 0$ by eq. (20). The sign of $\partial WCE/\partial e$ is uncertain. But, it is natural to expect that $\partial WCE/\partial e > 0$ for reasonable parameter values. If $|\partial WCI/\partial e| > \partial WCE/\partial e$, it will be possible that $\partial WC/\partial e < 0$. But, as will be shown in the next section, the possibility of this being held in the real world is minimal. Thus, for reasonable values of e, it is expected that $\partial WCI/\partial e > 0$, $\partial WC/\partial e > 0$ and $\partial s_K{}^*/\partial e < 0$.

If B increases, $\Delta B > 0$, $\Delta\zeta_1{}^*\rho < 0$ and $\Delta s_1{}^* > 0$ by eqs. (17) and (19). $\partial\Pi_K/\partial B$ can be derived as follows.

$$\partial\Pi_K/\partial B = (\mu^3 e/\beta^2)\{1 - e[f(s_1)/\mu][f(s_k)/\mu]\}(s_k{}^e - s_1{}^e) \qquad (22)$$

Here, β is the same as is defined in the appendix. The sign of $\partial\Pi_K/\partial\beta$ depends on $f(s_1)/\mu$ and $f(s_K)/\mu$, values of which are affected by B and e. If we concentrate our interests on B, it can be concluded that $\partial\Pi_K/\partial B < 0$ is possible for large enough B. Thus, for large enough B, it is possible that $\partial s_K{}^*/\partial B > 0$ since $\partial\zeta_1{}^*\rho/\partial B < 0$ and $\partial\Pi_K/\partial B < 0$. But, for small B, $\partial s_K{}^*/\partial B < 0$.

If $\Delta B > 0$, $\Delta WCE > 0$ by eq. (21). By the similar reasonings as before, $\Delta WCI > 0$ for small B and $\Delta WCI < 0$ for large enough B. Therefore, for large enough B, it is possible that $\Delta WC < 0$. But, again, the possibility of this being held in the real world is minimal. Thus, for reasonable values of B, it is expected that $\partial WCI/\partial B > 0$, $\partial WC/\partial B > 0$ and $\partial s_K{}^*/\partial B < 0$.

Both B and e affect WC and s_K^*. B determines the absolute size of the externality cost while e determines the relationship between externality costs of cities with different hierarchies. In case of the agglomeration economy, the major factor determining the optimal size distribution of cities is not the absolute size of the agglomeration economy but the relationship between agglomeration economies of cities with different hierarchies

[27, 28]. Therefore, it can be expected that the effect of e dominates that of B.

VI. Numerical Solutions and Interpretations of the Model

Calculating exact values of WC and s_K^* is not possible since there are no data for externality costs of cities. Therefore, numerical solutions of the model in section IV are derived for U.S. and Korea.

Data for rank, size and GRP of cities can be obtained. As for U.S., rank, size and annual earnings for 366 urbanized areas are obtained from "Rank of Urbanized Areas by Population" and "Income Characteristics in 1979 for Areas and Places, 1980: Urbanized Areas" in the 1980 census of population. As for Korea, "Municipal Yearbook of Korea, 1986", "Annual Report on the Family Income and Expenditures Survey, 1986" and "1985 Housing and Population Census Report" are used for obtaining rank, size and GRP for 57 cities.

In the model of section Ⅱ, the number of cities with hierarchy k, n_k, was endogenous. If n_k is endogenized in the empirical model, the model will be too complicated to be calculated. In order to avoid the complexity of endogenizing n_k and indirectly considering this endogeneity, WC and s_K^* will be calculated for three cases of square, hexagonal and octagonal geographical distribution of cities. $n_k = t(1+t)^{K-k-1}$, where t equals to 1, 2 and 3 when the geographical distribution of cities is square, hexagon and octagon respectively [4].[4)]

Average values of sizes and GRP's are used for representing each hierarchy. For obtaining average values of sizes and GRP's of cities with hierarchy 1, the method of extrapolation is used [26, 27].

PECS is defined as the ratio of per capita externality cost of a city with hierarchy 1 to the weighted average of per capita GRP: $PECS \equiv f(s_1)/\Psi$. Per capita externality cost might be much smaller than the weighted average of per capita GRP [13, 15, 17, 20, 23]. Therefore, five values of PECS, 0.02, 0.04, 0.06, 0.08 and 0.10, might include most cases.

4) If the geographical distribution of cities is hexagon (octagon), $n_K = 2/3$ (3/4). These are considered as 1. In case of square, $n_K = 1/2$, $n_{K-1} = 1$, $n_{K-2} = 2$, $\cdots$ etc. These are reformulated as $n_K = 1$, $n_{K-1} = 2$, $n_{K-2} = 4$, $\cdots$ etc.

If the value of PECS is given, $f(s_K)/\Psi$ can be calculated as $f(s_K)/\Psi = PECS(s_K/s_1)e$. For given value of PECS, e increases from 0.01 by 0.01 until the value of $REXT = [f(s_1)/\Psi]/PECS = (s_K/s_1)^e$ reaches to 10. Notice that REXT is the ratio of the per capita externality cost of the prime city to that of a city with hierarchy 1. If $PECS = 0.1$ and $REXT = 10$, $f(s_k)/\Psi = 1.0$. This means that the prime city's per capita externality cost is the same as the weighted average of per capita GRP. Thus, these values of e might include most cases.

Given values of PECS and e, Π_k can be expressed by using $f(s_1)/\Psi$, $f(s_k)/\Psi$ and μ/Ψ.

$$\Pi_k = \left[\frac{(\mu/\Psi) + [f(s_1)/\Psi]}{(\mu/\Psi) + [f(s_k)/\Psi]}\right]\left[\frac{(\mu/\Psi) + (1+e)[f(s_k)/\Psi]}{(\mu/\Psi) + (1+e)[f(s_1)/\Psi]}\right] \qquad (23)$$

Since $\mu = dW/dS = dY/dS - dC/dS = \Psi - \sum_K \Psi_k(1+e)f(s_k)$, (μ/Ψ) can be expressed as $(\mu/\Psi) = 1 - \sum \Psi_k(1+e)[\Delta f(s_k)/\Psi]$.

The process of simulation can be summarized as follows.

1) For each geographical distribution of cities, $n_k = t(1+t)^{K-k-1}$ is calculated.
2) Given n_k, values of s_k, y_k, i_k and $\Psi \equiv \sum \Psi_k i_k$ can be calculated.
3) Given PECS and e, $B = \Psi PECS/s_1^e$, $f(s_K)/\Psi = PECS(s_K/s_1)^e$, $REXT - [f/\Psi]/PECS$ can be calculated ($PECS = 0.02$, 0.04, 0.06, 0.08, 0.10. $e = 0.01$, $\cdots$, $0.01(1+z)$, z such that $REXT = 10.0$).
4) $\zeta_1^*\rho$ and s_1^* are calculated by eqs. (17) and (19). Π_k can be calculated by eq. (23) ($k = 2, \cdots, K$).
5) $\zeta_k^*\rho$ and s_k^* are calculated by eqs. (18) and (19). Here, the optimal size of the prime city, s_K^*, is calculated.
6) η is calculated as 0.55 for Korea and is assumed to be 1 for U.S.. Then, WCI and WCE are calculated by eqs. (20) and (21). WC is obtained as $WC = WCI + WCE$.

Results of simulation are summarized in tables 1 and 2. Since values of REXT's are obtained by increasing e by 0.01, those are not necessarily integers. For example, in square case of Korea with $PECS = 0.02$ and $e = 0.34$, $REXT = 5.22$. But, this case will be said that the per capita externality cost of the prime city is about 5 times greater than that of a city with hierarchy 1 for the purpose of explanatory

〈Table 1〉 The Effect of External Diseconomies (Korea)

PECS	square(K=6)					hexagen(K=5)					octagon(K=4)				
	e	REXT	s_k^*/s_k	WC	WCI	e	REXT	s_k^*/s_k	WC	WCI	e	REXT	s_k^*/s_k	WC	WCI
	K	BM	0.8519	0.0671	0.0671	BM		0.9308	0.0313	0.0313	BM		0.9438	0.0261	0.0261
0.02	0.01	1.05	0.8520	0.0685	0.0671	0.01	1.05	0.9309	0.0319	0.0312	0.01	1.05	0.9439	0.0266	0.0261
	0.08	1.48	0.8525	0.0686	0.0668	0.07	1.42	0.9312	0.0318	0.0310	0.06	1.31	0.9441	0.0266	0.0259
	0.14	1.98	0.8521	0.0690	0.0666	0.11	1.73	0.9309	0.0319	0.0309	0.10	1.57	0.9439	0.0266	0.0259
	0.34	5.22	0.8395	0.0746	0.0685	0.33	5.15	0.9158	0.0368	0.0337	0.36	5.12	0.9263	0.0317	0.0290
	0.48	10.31	0.8102	0.0871	0.0737	0.47	10.33	0.8819	0.0485	0.0404	0.51	10.12	0.8899	0.0433	0.0360
0.04	0.01	1.05	0.8521	0.0698	0.0670	0.01	1.05	0.9310	0.0325	0.0312	0.01	1.05	0.9440	0.0271	0.0261
	0.09	1.55	0.8532	0.0701	0.0663	0.07	1.42	0.9316	0.0324	0.0307	0.06	1.31	0.9444	0.0270	0.0257
	0.15	2.07	0.8523	0.0710	0.0661	0.11	1.73	0.9312	0.0326	0.0306	0.11	1.65	0.9439	0.0272	0.0256
	0.34	5.22	0.8317	0.0814	0.0688	0.33	5.15	0.9054	0.0418	0.0351	0.36	5.12	0.9139	0.0367	0.0307
	0.48	10.31	0.7903	0.1042	0.0757	0.47	10.33	0.8562	0.0636	0.0447	0.51	10.12	0.8609	0.0583	0.0406
0.06	0.01	1.05	0.8523	0.0711	0.0670	0.01	1.05	0.9311	0.0331	0.0311	0.01	1.05	0.9441	0.0276	0.0260
	0.09	0.55	0.8529	0.0713	0.0658	0.07	1.42	0.9321	0.0329	0.0304	0.06	1.31	0.9447	0.0274	0.0255
	0.17	2.29	0.8520	0.0735	0.0654	0.12	1.82	0.9312	0.0333	0.0302	0.11	1.65	0.9441	0.0276	0.0253
	0.34	5.22	0.8275	0.0872	0.0683	0.33	5.15	0.8985	0.0461	0.0356	0.36	5.12	0.9054	0.0410	0.0315
	0.48	10.31	0.7830	0.1181	0.0746	0.47	10.33	0.8440	0.0760	0.0458	0.51	10.12	0.8463	0.0707	0.0419
0.08	0.01	1.05	0.8524	0.0725	0.0669	0.01	1.05	0.9312	0.0336	0.0311	0.01	1.05	0.9441	0.0281	0.0260
	0.10	1.63	0.8547	0.0731	0.0652	0.07	1.42	0.9326	0.0334	0.0301	0.06	1.31	0.9451	0.0279	0.0253
	0.18	2.40	0.8520	0.0758	0.0646	0.12	1.82	0.9316	0.0339	0.0298	0.12	1.72	0.9439	0.0282	0.0250
	0.34	5.22	0.8261	0.0922	0.0671	0.33	5.15	0.8946	0.0497	0.0355	0.36	5.12	0.9001	0.0447	0.0316
	0.48	10.31	0.7839	0.1286	0.0715	0.47	10.33	0.8406	0.0853	0.0447	0.51	10.12	0.8412	0.0801	0.0410
0.10	0.01	1.05	0.8525	0.0738	0.0668	0.01	1.05	0.9313	0.0342	0.0310	0.01	1.05	0.9442	0.0286	0.0259
	0.10	1.63	0.8556	0.0745	0.0647	0.08	1.49	0.9331	0.0339	0.0297	0.07	1.37	0.9455	0.0282	0.0250
	0.20	2.64	0.8513	0.0788	0.0637	0.13	1.91	0.9316	0.0346	0.0293	0.13	1.80	0.9437	0.0288	0.0246
	0.34	5.22	0.8270	0.0962	0.0652	0.33	5.15	0.8929	0.0525	0.0347	0.36	5.12	0.8974	0.0475	0.0310
	0.48	10.31	0.7906	0.1353	0.0667	0.47	10.33	0.8434	0.0913	0.0419	0.51	10.12	0.8427	0.0862	0.0383

〈Table 2〉 The Effect of External Diseconomies (U.S.)

PECS	square(K=9)					hexagen(K=7)					octagon(K=6)				
	e	REXT	s_k^*/s_k	WC	WCI	e	REXT	s_k^*/s_k	WC	WCI	e	REXT	s_k^*/s_k	WC	WCI
	BM		0.9615	0.0339	0.0339	BM		0.9684	0.0253	0.0253	BM		0.9829	0.0149	0.0149
0.02	0.01	1.06	0.9617	0.0339	0.0332	0.01	1.06	0.9685	0.0257	0.0252	0.01	1.05	0.9830	0.0151	0.0148
	0.07	1.49	0.9621	0.0335	0.0326	0.05	1.34	0.9688	0.0255	0.0249	0.07	1.38	0.9833	0.0147	0.0143
	0.14	2.22	0.9617	0.0335	0.0323	0.20	1.80	0.9684	0.0255	0.0247	0.15	2.01	0.9824	0.0145	0.0140
	0.29	5.23	0.9484	0.0371	0.0344	0.28	5.22	0.9541	0.0299	0.0277	0.35	5.07	0.9660	0.0182	0.0167
	0.41	10.38	0.9176	0.0478	0.0418	0.39	10.00	0.9252	0.0401	0.0348	0.50	10.18	0.9277	0.0295	0.0247
0.04	0.01	1.06	0.9618	0.0345	0.0331	0.01	1.06	0.9686	0.0261	0.0251	0.01	1.05	0.9831	0.0153	0.0147
	0.07	1.49	0.9627	0.0335	0.0318	0.05	1.34	0.9691	0.0257	0.0245	0.07	1.38	0.9838	0.0144	0.0137
	0.15	2.35	0.9605	0.0335	0.0311	0.21	1.91	0.9681	0.0257	0.0241	0.16	2.10	0.9816	0.0140	0.0131
	0.29	5.23	0.9345	0.0399	0.0344	0.28	5.22	0.9439	0.0339	0.0292	0.35	5.07	0.9540	0.0206	0.0173
	0.41	10.38	0.8944	0.0594	0.0442	0.39	10.00	0.9007	0.0531	0.0400	0.50	10.18	0.3981	0.0412	0.0285
0.06	0.01	1.06	0.9619	0.0350	0.0330	0.01	1.06	0.9687	0.0266	0.0250	0.01	1.05	0.9832	0.0155	0.0146
	0.07	1.49	0.9633	0.0335	0.0310	0.06	1.43	0.9695	0.0258	0.0239	0.07	1.38	0.9842	0.0171	0.0131
	0.16	2.49	0.9597	0.0334	0.0297	0.21	1.91	0.9682	0.0258	0.0235	0.17	2.20	0.9806	0.0133	0.0120
	0.29	5.23	0.9339	0.0417	0.0333	0.28	5.22	0.9369	0.0372	0.0297	0.35	5.07	0.9460	0.0220	0.0167
	0.41	10.38	0.8834	0.0676	0.0435	0.39	10.00	0.8874	0.0637	0.0420	0.50	10.18	0.8833	0.0492	0.0281
0.08	0.01	1.06	0.9621	0.0356	0.0329	0.01	1.06	0.9689	0.0270	0.0249	0.01	1.05	0.9833	0.0157	0.0145
	0.09	1.67	0.9640	0.0330	0.0295	0.06	1.43	0.9699	0.0259	0.0234	0.07	1.38	0.9847	0.0138	0.0125
	0.17	2.63	0.9307	0.0331	0.0295	0.22	2.03	0.9677	0.0259	0.0227	0.18	2.31	0.9794	0.0125	0.0108
	0.29	5.23	0.9309	0.0424	0.0313	0.28	5.22	0.9324	0.0397	0.0294	0.35	5.07	0.9411	0.0222	0.0151
	0.41	10.38	0.8805	0.0718	0.0400	0.39	10.00	0.8314	0.0716	0.0418	0.50	10.18	0.8783	0.0530	0.0246
0.10	0.01	1.06	0.9622	0.0361	0.0327	0.01	1.06	0.9690	0.0274	0.0248	0.01	1.05	0.9835	0.0159	0.0144
	0.08	1.58	0.9648	0.0330	0.0289	0.06	1.43	0.9704	0.0259	0.0229	0.08	1.45	0.9853	0.0131	0.0155
	0.19	2.96	0.9561	0.0328	0.0264	0.03	2.15	0.9670	0.0260	0.0219	0.20	2.53	0.9768	0.0116	0.0093
	0.29	5.23	0.9302	0.0418	0.0285	0.28	5.22	0.9301	0.0410	0.0284	0.35	5.07	0.9387	1.0212	0.0127
	0.41	10.38	0.8833	0.0715	0.0342	0.39	10.00	0.8806	0.0765	0.0397	0.50	10.18	0.8800	0.0518	0.0185

convenience.

BM indicates the benchmark case, where the external diseconomies are not considered. By definition of BM, $WC = WCI$ for this case.[5]

In case of U.S. (Korea), $\partial[s_K^*/s_K]/\partial e > 0$ and $\partial WC/\partial e < 0$ if e is less than about 0.07 (0.08). This value of e indicates that REXT is less than about 1.4 (1.5) for U.S. (Korea). The size of the prime city, s_K, is 15.6 (9.5) million for U.S. (Korea).

The size of a city with hierarchy 1 is obtained through the method of extrapolation. The average s_1 of square, hexagonal and octagonal cases for U.S. (Korea) is 41,226 (101,270).[6] When we consider that the externality cost includes all kinds of external diseconomies and compare sizes of s_K and s_1, it might be concluded that the probability of REXT being less than 1.4 (1.5) for U.S. (Korea) is minimal.

If values of e are larger than mentioned above, cases of $\partial WC/\partial B < 0$ are found only in the octagonal (square) case of U.S. (Korea) with PECS equals to 0.10. In the octagonal (squaare) case of U.S. (Korea), s_1 is 29,348 (71,200). The probability of this size urban area's per capita externality cost being 10% of the weighted average of per capita GRP might be minimal. Therefore, it might be generally concluded that WC increases and s_K^* decrease as the degree of external diseconomies increases.

We can exactly calculate benchmark s_K^*/s_K and WC. But, because of lacks of data it is not possible to calculate s_K^*/s_K and WC under external diseconomies. Thus, it will be helpful to compare benchmark s_K^*/s_K and WC with those under external diseconomies. Define ROSP and RWC as $ROSP = (s_K^*/s_K)/(\text{benchmark } s_K^*/s_K)$ and $RWC = WC/(\text{benchmark WC})$.

When REXT is about 5, average ROSP has been obtained as 0.9686 and 0.9687 for U.S. and Korea. If we calculate benchmark s_K^*/s_K and conjecture that REXT is about 5, we can say that, on average, s_K^* is $0.9686 \times (\text{benchmark } s_K^*/s_K) \times s_K$ for U.S.. Notice that s_K is the current size of the prime city.

When REXT is about 10, average ROSP has been obtained as 0.9203 and 0.9174 for U.S. and Korea. Regardless of values of REXT, values of average ROSP's for

5) See [26] for the formulae calculating benchmark s_K^*/s_K and WC.

6) The reason why s_1 of U.S. is less than that of Korea is because the method of extrapolation is used and the number of hierarchies, K, of U.S. is larger than that of Korea. In octagonal case, K is 6 (4) and n_1 is 768 (48) for U.S. (Korea). In U.S., there are 110 actual urbanized areas and 658 extrapolated ones, sizes of which are smaller than the smallest actual urbanized area. In Korea, there are 41 actual cities and 7 extrapolated cities.

U.S. and Korea are quite similar.

When REXT is about 5(10), average RWC's are obtained as 1.3513(2.4463) and 1.3788(2.2109) for U.S. and Korea respectively. If we calculate benchmark s_K^*/s_K and conjecture that REXT is about 5, we can say that, on average, WC is 1.3513× (benchmark WC) for U.S.. Since absolute size of WC is fairly small, values of average RWC's for U.S. and Korea might be considered to be similar.

Based upon these findings, it is possible to construct general formulae relating benchmark s_K^*/s_K and WC with those under conjectured degree of external diseconomies. We run the regression of ROSP and RWC for U.S., Korea and the aggregated data of the two. Since factors affecting s_K^*/s_K and WC under external diseconomies are PECS and REXT, these are chosen as explanatory variables.[7] Results of regressions are summarized in table 3.

In table 3, numbers in parentheses are *t*-values and *adj*-R^2 is R^2 adjusted by the degree of freedom. Also, TOTAL means the aggregated data of U.S. and Korea. In case of ROSP, all coefficients are remarkably similar for all cases of U.S., Korea and TOTAL. *F*-value necessary for the Chow test is $F(730,647) = 1.1090$, the *P*-value of which is 0.9017. Therefore, it might be concluded that data for U.S. and

〈Table 3〉 **Results of Regressions**

	RSOP			*log* RWC		
	U.S.	KOREA	TOTAL	U.S.	KOREA	TOTAL
const	1.0280 (1157)	1.0276 (1161)	1.0278 (1636)	−0.0234 (−0.59)	0.3454 (13.4)	0.1718 (7.21)
PECS	−0.1742 (−15.6)	−0.1633 (−14.7)	−0.1684 (−21.3)			
REXT	−0.0095 (−78.5)	−0.0097 (−81.0)	−0.0096 (−112.7)			
log PECS				0.0722 (5.82)	0.1515 (18.7)	0.1142 (15.2)
log REXT				0.3692 (35.1)	0.3026 (44.1)	0.3339 (52.5)
adj-R^2	0.9081	0.9028	0.9051	0.6606	0.7587	0.6839
F	$F(730,647) = 1.1090$			$F(730,647) = 0.5815$		

7) Sample size of U.S. is $41 \times 5 + 39 \times 5 + 50 \times 5 = 650$ and that of Korea is $48 \times 5 + 47 \times 5 + 51 \times 5 = 730$.

Korea come from the same population. In other words, the relationship between benchmark s_K^* and s_K^* under external diseconomies is similar for both countries.

In case of RWC, coefficients of REXT are quite similar. Even though coefficients of PECS are somewhat different, $F(730,647)=0.5815$ with P-value 0.9999 enables us to conclude that RWC data for U.S. and Korea come from the same population. Therefore, we might use the estimated regression equations for TOTAL in order to calculate ROSP and RWC under conjectured degree of external diseconomies. Then, s_K^* and WC under conjectured degree of external diseconomies can be calculated as follows.

$$s_K^* = ROSP \times (\text{benchmark } s_K^*/s_K) \times s_K \tag{24}$$

$$WC = RWC \times (\text{benchmark } WC) \tag{25}$$

PECS determines the absolute size of the externality cost while REXT specifies the relationship between externality costs of cities with different sizes. As was expected, the effect of REXT upon s_K^* and WC is greater than that of PECS.[8]

VII. Conclusions

Two main purposes of this paper were to derive properties of the optimal size distribution of cities under external diseconomies and to analyze the effect of external diseconomies upon the welfare cost of the sub-optimal size distribution of cities (WC) and the optimal size of the prime city (OSP).

From the analyses of properties of optimum, it is now clear that both the agglomeration economy and the external diseconomy are two major factors determining the size distribution of cities.

Findings from simulation results are as follows. First of all, the degree of external diseconomies is positively related to the WC and is negatively related to the OSP. Secondly, the major factor affecting WC and OSP is not the absolute size of

8) We have the following regression equation estimated by using the aggregated data.

$$\log RSOP = \underset{(-3.54)}{-0.0069} \ \underset{(-15.4)}{-0.0094} \log PECS \ \underset{(-70.0)}{-0.0362} \log REXT$$

R^2: 0.7885

The elasticity of *REXT* is much larger than that of *PECS*.

the externality cost but the relationship between externality costs of cities with different sizes. Lastly, general formulae can be constructed, which relate the benchmark WC and OSP with those under the conjectured degree of external diseconomies.

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⟨Appendix⟩

Π_k can be rewritten as follows.

$$\Pi_k = \left[\frac{(\mu + Bs_k^e)(\mu + Bs_1 e) + B^2 e s_1^e s_k^e + B\mu e s_k^e}{(\mu + Bs_k^e)(\mu + Bs_1 e) + B^2 e s_1^e s_k^e + B\mu e s_1^e}\right]$$
$$\equiv \alpha/\beta$$

Since $s_k > s_1$, $\Pi_k > 1$ for $k = 2, \cdots, K$. $\partial\Pi_k/\partial e$ can be derived as follows.

$$\partial\Pi_k/\partial e = (1/\beta^2)(CX - DY)(s_k^e - s_1^e)$$

Here, $C = B\mu(1 + \ln e)$, $D \equiv B\mu e$, $X \equiv \mu^2 + B\mu s_1^e + B\mu s_k^e + B^2(1+e)s_1^e s_k^e$ and $Y \equiv B^2 s_1^e s_k^e + B^2(1+e)s_1^e s_k^e(\ln e) + B\mu s_k^e(\ln e) + B\mu s_1^e(\ln e)$. Notice that e is not the exponent. Since the sign of $CX - DY$ is uncertain, the sign of $\partial\Pi_k/\partial e$ is ambiguous. But, if $0 < e < 1$, the sign of $CX - DY$ can be determined as follows.

If $0 < e < 1$, $\ln e < 0$ and $B^2 s_1^e s_k^e < B^2(1+e)s_1^e s_k^e$. Therefore, $X > Y$. $C - D$ can be written as $C - D = B\mu(1 + e \ln e - e)$. If $0 < e < 1$, $d(e \ln e - e)/de = \ln e < 0$.

Thus, ($e \ln e - e$) is at infimum if $e = 1$. But, if $e = 1$, $1 + e \ln e - e = 0$. Thus, $C - D > 0$ if $0 < e < 1$.

Since $X > Y$, $C > D$ if $0 < e < 1$ and $s_k > s_l$, $\partial \Pi_k / \partial e > 0$.